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RELIABILITY ANALYSIS OF ICE LOADS ON ARCTIC OFFSHORE STRUCTURES

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ABSTRACT

In Arctic regions, the calculation of ice loads is a key issue in the design of offshore structures. ISO 19906 provides a deterministic approach to calculate ice forces on Arctic offshore structures. The approach combines environmental forces of an ice floe with structural parameters to estimate the imposed ice load. However, owing to large uncertainties in the underlying parameters, a statistical formulation is needed. The parameters include annual number of floes, floe size, ice thickness, and ridge geometry, among others.

This paper focuses on the development of a procedure for determining design ice loads on Arctic offshore structures using a reliability approach. Level ice and ice ridges are considered as major load sources interacting with the offshore structure in the ice regime. Referring to ISO 19906 and available literatures, a deterministic methodology to calculate ice loads has been developed based on two limit mechanisms, including limit strength and limit force. In this study, ice loads on offshore structures mainly result from the first-year ridges interacting with the structure. The geometric parameters of ice floes are assumed as random variables in the probabilistic approach. A reliability-based approach using the First-Order Reliability Method (FORM) has been developed to incorporate the uncertainties in the underlying ice floe and environmental parameters. Specific site data is then used with the FORM approach to demonstrate the results which are compared to Monte Carlo simulations. The reliability-based method and the Monte Carlo simulation method are both implemented to determine design ice loads on Arctic offshore structures.

INTRODUCTION

The prediction of ice loads is a key element in the design of Arctic offshore structures. The ice load depends on many factors such as ice conditions in a particular region, structural configurations, and interaction scenarios of ice with the structure. In any geographic location, ice conditions also vary significantly in different seasons and years. Most available literature (Croasdale et al., 1994; Wright & Timco, 1994 & 2000) and ISO 19906 (ISO 19906, 2009) provide a deterministic approach to calculate ice forces on Arctic offshore structures. The formulae can be employed by combining environmental and structural parameters of an ice floe with a specified geometry. However, there are a lot of uncertainties in the ice parameters such as ice floe numbers, floe size, ice thickness, and ridge geometry, among others. A statistical description is generally required to define these parameters. Furthermore, a probabilistic approach is needed to ultimately determine the probability distribution of ice loads (Brown et al., 2001), which then can be integrated into an overall reliability assessment of offshore structures in Arctic areas.

The main focus of this study is to develop an efficient procedure for determining ice loads on Arctic offshore structures using a probabilistic approach to account for relevant uncertainties in calculating the desired design ice load (for example, the 100-year design ice load) imposed on a structure.

ICE LOAD MODELING

In ice mechanics, there are three limit mechanisms that define the net imposed ice load on a structure:

- Limit strength: An ice floe cannot sustain itself and crushes when the applied stress exceeds the material strength of ice. This strength corresponds to crushing and bending failures in the ice floe.
- Limit momentum: This is the load imposed by ice due to the floe moving with acceleration and impinging on a structure to impart its momentum as a load on the structure. However, the CSA code (CSA, 2004) indicates that the limit momentum can be neglected compared to the limit strength if the ice floe diameter is less than 5 km (which is the case of interest in this study).
- Limit force: The ice load caused by the moving ice floe, where the movement is due to wind or current force, or due to movement of surrounding ice pack.

The ice load on the structure is limited by the force necessary to fail the ice feature and by the force driving the ice feature against the structure. In the absence of sufficient environmental driving force, the ice failure force cannot be generated. Therefore, the minimum value of the environmental driving force (limit force) and the ice failure force (limit strength) is taken as the critical ice load on the structure.

Figure 1 summarizes the methodology for calculating the annual maximum ice load on an Arctic offshore structure. For an arbitrary year, the number of ice floes that would interact with the offshore structure is first calculated (specific formulas are discussed in a later section). This number depends on parameters such as ice season length, ice concentration, floe velocity, floe size, and the structure geometry.

For each floe interacting with the structure, the two force components calculated include the limit strength and the maximum ridge force across all ridges in the floe. The limit strength is calculated based on ice floe size, wind velocity, ocean current velocity, and pack ice force. The maximum ridge force is calculated by finding the maximum of each individual ridge force on the floe. Each ridge force is calculated based on ridge geometry and other ridge properties, structure geometry and the interaction scenario assumed between the ridge and the structure. As mentioned earlier, the overall ice floe load on the structure is the smaller value of the limit strength and the maximum ridge force. Such floe loads are calculated for all floes during the year and from these the annual maximum floe load is calculated.

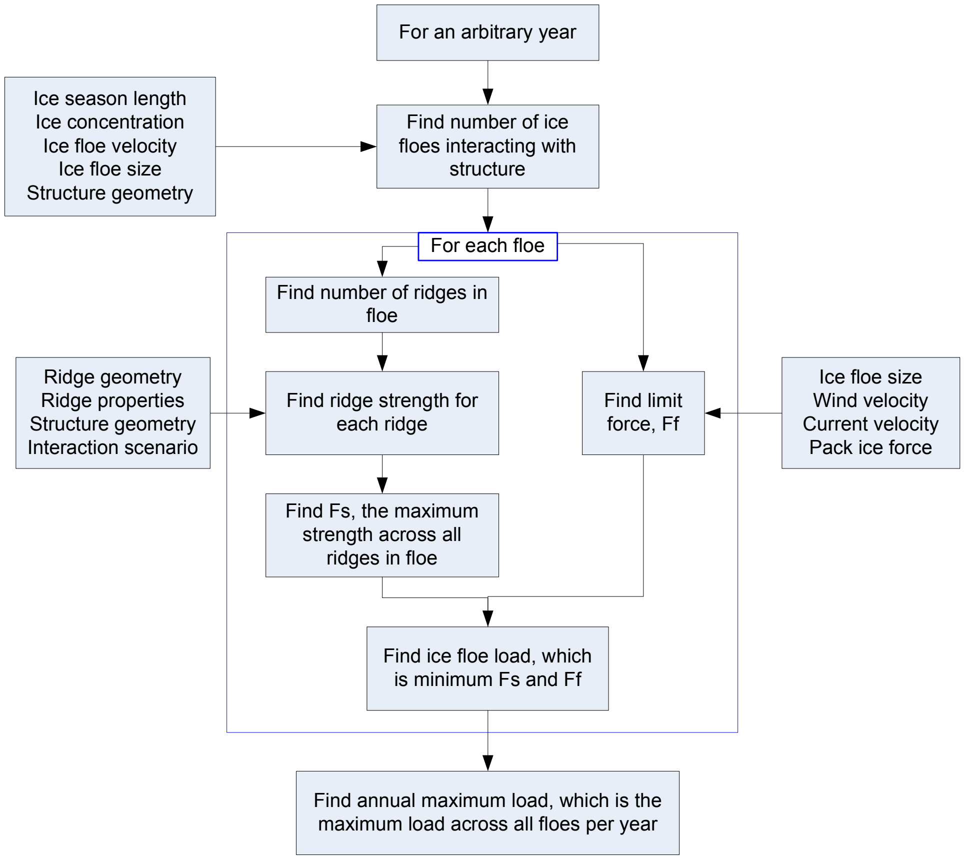

Figure 1: Flowchart of probabilistic simulation procedure

A more detailed formulation of the load components is presented next.

The ice load on a structure results from ice floes impinging on the structure. The number of ice floes per year impacting the structure is assumed here to be a variable N_F, given as

$$N_F = L_I P_I \qquad (1)$$

where

L_I is the ice season duration

$P_I = c\bar{V}(\bar{L} + d)/\bar{A}$ is the probability of impact

$\bar{A} = \frac{\pi}{4}(\bar{L}^2 + \sigma_L^2)$

c is the floe concentration

$\bar{V}$ is mean drift velocity of the floe

$\bar{L}$ is mean floe diameter

d is structure diameter or width at waterline

σ_L is standard deviation of floe diameter

The ice load per floe interacting with the structure is the lesser of two values: (1) the force due to the wave-wind environment transmitted through the floe to the structure, and (2) the limit strength at which ice crushes (and thereby cannot exert any additional force). The ice load then is defined as

$$F = \min(F_s, F_f) \quad (2)$$

where F_s is the limit strength force and F_f is the environmental driving force. Both F_s and F_f will be explained in details in the following.

The environmental driving force is given as (Loset et al. 2006)

$$F_f = F_w + F_c + F_p \quad (3)$$

where force components F_w, F_c, and F_p are wind force, current force, and pack ice force, respectively, defined as given below.

The wind force is given as

$$F_w = \frac{1}{2} C_w \rho_a V_w^2 A_f \quad (4)$$

where

C_w is air drag coefficient

ρ_a is air density

V_w is wind velocity

$A_f = \frac{1}{4}\pi L^2$ and L is ice floe diameter

The current force is given as

$$F_c = \frac{1}{2} C_c \rho_w V_c^2 A_f \quad (5)$$

where

C_c is current drag coefficient

ρ_w is water density

V_c is current velocity

The pack ice force is given as

$$F_p = B L h_f^{1.25} \quad (6)$$

where h_f is the consolidated layer thickness of the ice floe; and

$$B = \begin{cases} 0.6\,k & \text{for } L < 100m \\ 0.1\,k & \text{for } L > 500m \end{cases}$$ (linear interpolation for L between 100 and 500meters)

The limit strength force F_s is defined as the maximum ridge strength, where the maximum is over all the ridges in a floe. The number of ridges N_R in each floe is given as

$$N_R = L\,\rho_R \quad (7)$$

where ρ_R is the ridge density in a floe

The strength of an individual ridge in an ice floe F_R is given as (Croasdale et al. 1994 & Dolgopolov et al. 1975)

$$F_R = F_c + F_k \quad (8)$$

Where, F_c is the strength force from the consolidated layer of a ridge and F_k is the strength force from the keel portion of the ridge.

Now,

$$F_k = \mu h_k w \left(\frac{h_k \mu \gamma_e}{2} + 2c_k\right)\left(1 + \frac{h_k}{6w}\right) \quad (9)$$

where

$\mu = tan\left(45^\circ + \frac{\phi}{2}\right)$ is passive pressure coeff.

ϕ is the angle of internal friction

c_k is keel cohesion

$\gamma_s = (1 - e_k)(\rho_w - \rho_i)g$

e_k is keel porosity

The force from the consolidated part of the ridge is given as (ISO 19906, 2009)

$$F_c = F_H = \frac{H_B + H_P + H_R + H_L + H_T}{1 - \frac{H_B}{\sigma_f l_c h}} \quad (10)$$

where

$$l_c = W + \frac{\pi^2}{4} L_c$$

$$L_c = \left[\frac{Eh^3}{12\rho_w g(1 - \nu^2)}\right]^{0.25}$$

E is Young's Modulus
ν is Poisson's ratio

H_B is the breaking load; this and below load components are specified in (ISO 19906, 2009)
H_P is the load component required to push the sheet ice through the ice rubble
H_R is the load component required to push the ice blocks up the slope through the ice rubble
H_L is the load required to lift the ice rubble on top of the advancing ice sheet prior to breaking
H_T is the load required to turn the ice block at the top of the slope

To demonstrate the reliability methodology, we assume the following formulation for the variables used in the force equations above.

Deterministic variables:

Table 1: List of deterministic variables and their values selected in this study

(Timco & Frederking, 2004, Brown, Jordan, & Croasdale, 2001)

Var-iable	Description	Value	Var-iable	Description	Value
σ_f	Ice flexural Strength (Pa)	$2.4\text{x}10^5$	μ_s	Ice-structure friction coefficient	0.2
E	Young's Modulus (Pa)	$3.4\text{x}10^9$	e_r	Rubble porosity	0.3
ν	Poisson's Ratio	0.3	h_r	Rubble height (m)	5
ρ_i	Ice Density (kg/m^3)	920	θ	Rubble angle (rad.)	0.7854
W	Structure Width (m)	80	c_r	Rubble cohesion (Pa)	5000
α	Structure Slope (radian)	0.4014	ϕ_r	Angle of rubble internal friction angle (radians)	0.7854
ρ_a	Air Density (kg/m^3)	1.27	c_k	Keel cohesion (Pa)	1000
C_w	Wind Drag Coefficient	0.002	e_k	Keel porosity	0.4
ρ_w	Water Density (kg/m^3)	1020	ρ_R	Ridge density (per m)	0.01
C_c	Current drag coefficient	0.18	I_s	Ice season length (days per yr)	304
K_f	Force distribution coeff.	1.2	c	Floe concentration (unitless)	1
μ_i	Ice-ice friction coefficient	0.1	$\overline{V}$	Mean drift velocity (m/s)	0.15

Random variables: (The probability distribution type for each of the four variables below is arbitrarily assumed to be Lognormal to simply demonstrate FORM/SORM capabilities.)

Table 2: List of random variables and their specification used in this study

Variable	Probability Distribution Type	Mean μ	Standard Deviation σ	Coeff. of Variation COV= σ/μ
V_w (wind velocity)	Weibull	12 m/s	10 m/s	0.83
V_c (current velocity)	Lognormal	0.2 m/s	0.15 m/s	0.75
L (ice floe diameter)	Exponential	100m	100m	1
h_f (floe consolidated layer height)	Lognormal	0.75m	0.2m	0.27
h_c (ridge consolidated layer height)	Lognormal	1.25m	0.2m	0.16
h_k (ridge keel layer thickness)	Lognormal	7m	0.25m	0.036

The probability distribution of ice load per floe is given as

$$P_f(1) = Prob(\min\ (F_s, F_f) > x) \quad (11)$$

The average number of floes per year N_f can then be used to find the annual "failure" probability of the overall load, $\min\ (F_s, F_f)$, assuming again that loads from one floe to the next are independent of each other, and the occurrence of floes follows a Poisson model. The annual failure (exceedance) probability is given as[1]

$$P_f(N_f) = 1 - \exp[N_f \times \log(1 - P_f(1))] \quad (12)$$

Now, we discuss below how to find $P_f(1)$, using two different approaches:

- Simulation: Simple and intuitive; however, this approach is computationally intensive and may not provide all the needed insights of parameters for the design loads of interest
- Reliability methods: Namely, FORM/SORM – first- and second-order reliability methods. These methods are trickier to implement, but computationally efficient, and provide insights on importance and contributions of underlying parameters that define the design loads of interest

SIMULATION APPROACH

Figure 1 summarizes the logic for the simulation approach for finding the failure probability (exceedance probability) of a given load threshold per floe. This simulation is performed for an array of load threshold values that results in the number of failures (exceedances) for each load threshold in the array. The ratio of number of failures to the total number of simulations defines the probability of failure. From this probability distribution of overall load per floe $P_f(1)$ in Equation (11), one can then find $P_f(N_f)$ in Equation (12) for a given number of floes.

RELIABILITY-BASED APPROACH

Using the FORM approach, first we find the probability distribution of one ridge limit strength using Equation (14), which is completely defined using the random variables h_c (ridge consolidated layer thickness) and h_k (ridge keel layer thickness). Next, we find the environmental

[1] Note that for small values of $P_f(1)$, one can approximate the multi-floe probability as $P_f(N_f) = 1 - \exp[-N_f \times P_f(1)]$

force probability distribution $Prob(F_f > x)$ with the force defined in Equation (3) that is completely defined by the random variables:

- V_w (wind velocity)
- V_c (current velocity)
- L (ice floe diameter)
- h_f (floe consolidated layer thickness)

Using the probability distributions of F_s and F_f, one can then find the overall load probability distribution as for one floe as

$$\begin{aligned} P_f(1) &= Prob[\min(F_s, F_f) > x] \\ &= Prob[F_s > x \text{ and } F_f > x] \\ &= Prob[F_s > x] \times Prob[F_f > x] \end{aligned} \tag{13}$$

The last step in the above formulation assumes that F_s is independent of F_f. This assumption is not necessarily true due to the random variable L (floe diameter). The floe diameter defines the number of ridges N_R for a given floe, and in turns defines the maximum ridge force probability distribution. However, as we will later discuss, this dependence of F_s on F_f is fairly weak, and the above equation gives practically the same results as the simulation approach where even this weak dependence is not assumed.

We first find the probability distribution of one ridge force F_R as

$$P_R = Prob(F_R > x) \tag{14}$$

Assuming independent ridge forces throughout a floe, one then can find the probability distribution of the maximum ridge force F_S as

$$\begin{aligned} P_S &= Prob(F_S > x) \\ &= 1 - Prob(F_S < x) \\ &= 1 - Prob(\text{Every Ridge Force} < x) \\ &= 1 - Prob(\text{Ridge Force } 1 < x).Prob(\text{Ridge Force } 2 < x)\ldots Prob(\text{Ridge Force } N_R < x) \\ &= 1 - (1 - P_R)^{N_R} \end{aligned} \tag{15}$$

A feature of the FORM method is that it also provides for "design points" at each load threshold where the probability of failure is calculated. For F_f probability distribution then, we have the values of the more likely floe diameters that cause a given load value x_{th}. At this threshold value, FORM gives the likely floe diameter L_{th}. This floe diameter is used to calculate the number of ridges using Equation (7). Given this number of ridges, $Prob[F_s > x_{th}]$ can then be calculated from the single ridge probability distribution P_R. Using this approach, we introduce the weak dependence between F_s and F_f.

RESULTS AND DISCUSSION

Figure 2 shows simulation results of overall annual ice load for a range of simulation samples. Note that as the number of simulations increases from 10^3 to 10^9, the results appear to converge for large load thresholds with increasing simulations. Note that 10^9 simulations took about 3 hours of computer time on a 2.5GHz machine, with analysis time varying roughly linearly with number of simulations. The last result for the largest number of simulations is at an exceedance probability of 10^{-9} (the reciprocal of the number of simulations), so N simulations will provide one data point at 1/N exceedance probability (EP) and will not provide any results for any smaller EP values. The annual exceedance probability results were obtained by converting the per floe results to annual results using the annual number of floes per year in Equation (12). One can observe that to get converged results for 100-year load, for example, 10^6 or greater simulations are needed. For converged 1000-year results, 10^7 or greater simulations are needed.

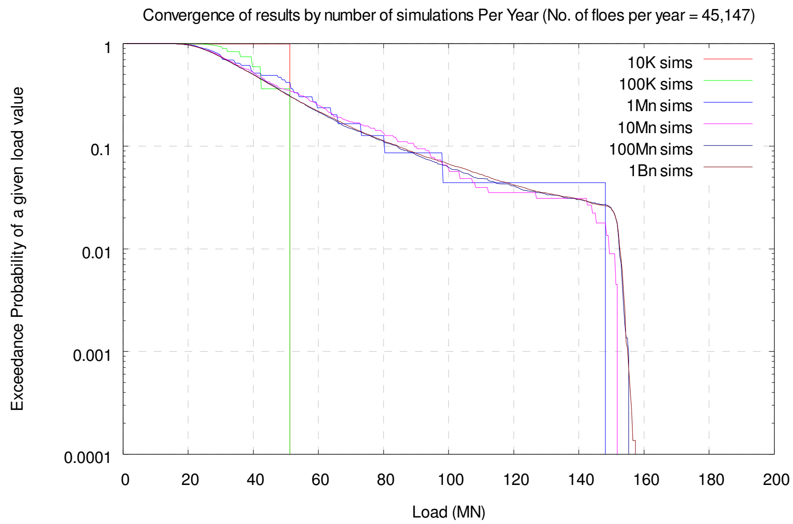

Figure 2: Convergence of simulation results per year

Figure 3 shows the exceedance probability results for environmental force (F_f, red line), limit strength (Fs, shown as F_{1R} with green line in the figure), and the combined ice load shown as min(F_s, F_f) in blue using the equations introduced earlier. The blue line shows a transition point around 150MN where the limit strength starts governing the overall load; prior to this transition point, the environmental forces essentially define the overall load.

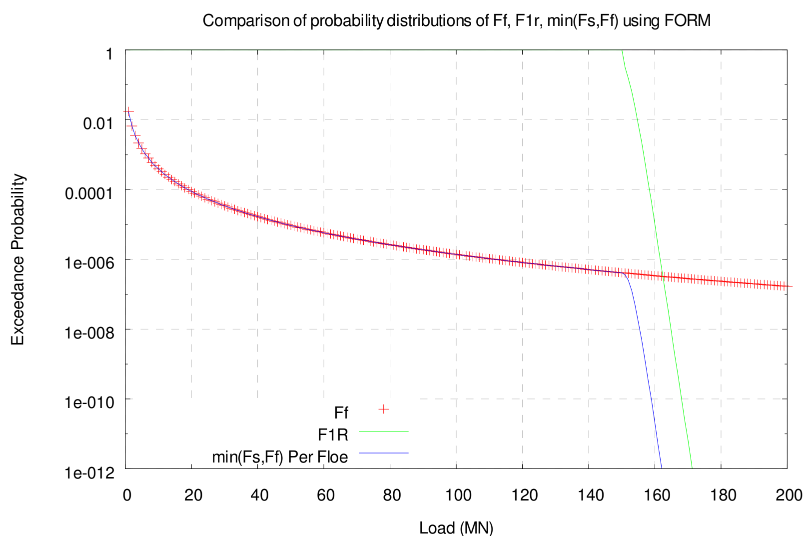

Figure 3: FORM results for single ridge force, environmental force, and overall force per floe

Figure 4 shows a very close agreement of FORM-based results with simulation for the entire load range presented. The FORM result is the annual ice load exceedance probability (EP) result, from which one can find the design load values for return periods of interest. For example, the 100-year load can be read off at about 0.01 EP value and the load is about 125MN. Similarly, the 1000-year load is about 155MN. The 100-year load is governed by environmental force and the 1000-year load is governed by the ice strength value.

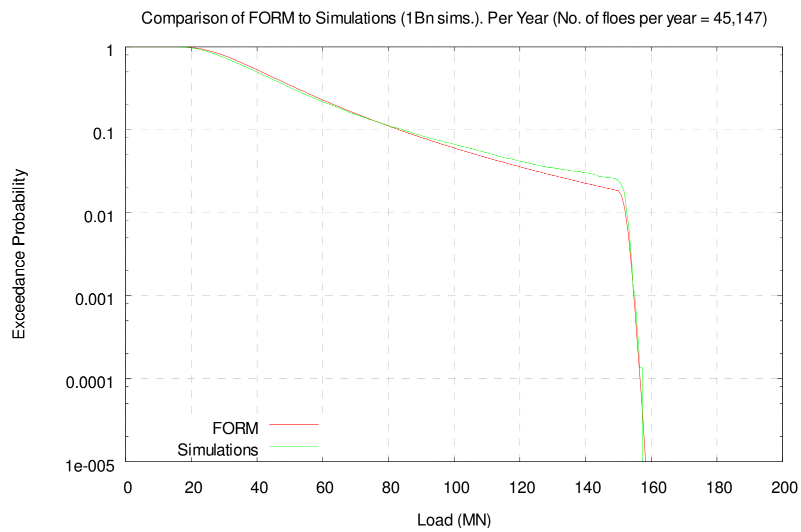

Figure 4: Comparison of FORM results to simulation (using 1Billion simulations) per year

The following tables present the design point representing the most likely "failure point" and the importance of random variables at this point for the 100- and 1000-year loads. Note that at the 100-year load, the ridge-related random variables do not govern and so are not included in the table. The importance factors provide a measure of sensitivity of the design point to the individual random variables and are the "direction cosines" of the vector for the design point. The sum of the importance factors per FORM analysis equals 1. The first four random variables represent the first FORM analysis (i.e., for environmental force) and the last two random variables represent the ice ridge strength FORM analysis. Note that for the 100-year load, the current velocity importance factor is about 64%, while the floe diameter gets 36% importance. The 1000-year load is governed by ridge properties and so notice that for the ridge FORM analysis, the ridge consolidated layer thickness is about 93% important, while the ridge keel height is 7% important.

Table 3: Design point and importance factor for 100- and 1,000-year load using FORM

	100-yr Load (125MN)		1,000-yr Load (155MN)	
Variable	**Design Point**	**Importance**	**Design Point**	**Importance**
V_w (wind velocity)	9.44	0	9.44	0
V_c (current velocity)	2.25	0.64	2.27	0.64
L (ice floe diameter)	644.	0.36	645.	0.36
h_f (floe consolidated layer thickness)	0.724	0	0.724	0
h_c (ridge consolidated layer thickness)	1.44	0.91	1.79	0.93
h_k (ridge keel layer thickness)	7.07	0.09	7.16	0.07

Molikpaq structure is taken as an example study herein. The design ice loads for 20 and 50 years return periods determined in this study are about 80 MN and 100 MN, respectively. Timco et al. (1999) measured the global load on the Molikpaq by first year ice ridge in the range of 45 – 90 MN. The comparison of these two results indicates that the methodology proposed in this study could give a reasonable prediction for the design ice load on Arctic offshore structures. The equations and the distributions of different random variables used in this analysis have got validation at a certain degree based on this case study. Further study might be necessary for more application of Arctic offshore structure examples.

SUMMARY

This paper presents a reliability-based methodology for calculation on design ice loads on Arctic offshore structures. The methodology demonstrates a reliability approach using First-Order Reliability Methods (FORM) to efficiently calculate design ice loads (extreme loads) accounting for uncertainties in several important parameters that define the ice load formulation. A computation tool that implements the FORM approach and also supports the Monte Carlo simulation approach has been developed to predict ice loads on Arctic offshore structures. However, the simulation approach is very time-consuming compared to FORM analysis. The FORM approach, on the other hand, becomes increasingly accurate for rarer events and also permits development of importance factors for the underlying parameters that define the design load. This offers insights into which parameters are most critical in the definition of ice loads.

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