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ICE INDUCED VIBRATIONS OF FLEXIBLE OFFSHORE STRUCTURES: THE EFFECT OF LOAD RANDOMNESS, HIGH ICE VELOCITIES AND HIGHER STRUCTURAL MODES

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ABSTRACT

In this study a phenomenological model is proposed of Ice Induced Vibrations (IIV) and an attempt is made to answer the following questions: (i) can IIV occur at high ice sheet velocities? (ii) what are the conditions for IIV to occur at a higher natural frequency of the structure? (iii) can an initially aperiodic (you probably mean aperiodic, which is the opposite) ice loading cause IIV?

The model of crushing level ice presented in this paper may be referred to as a strip model, implying that the ice is represented by a sequence of strips parallel to the ice velocity vector. The dynamic behavior of each strip is described based on the idea by Matlock that has recently been improved by Huang and Liu. A further improvement introduced in this paper is that the load produced by each strip is randomized. The attractive elements of the strip model are that it reproduces both the descending character of the strength-indentation rate diagram and contains an inherent, though implicit, periodicity in loading on a rigid structure.

The structure that undergoes IIV is mimicked by a generalized beam with coordinate dependent parameters that allow for tuning of both the modal spectrum and modal damping.

Using the proposed model it is shown that an initially aperiodic ice loading can trigger IIV both at low and high ice velocities. IIV at higher modes can be predicted only if the modal damping of those modes is small relative to that of the lower modes.

INTRODUCTION

Occasionally vertically-sided offshore structures experience sustained vibration due to drifting ice sheets crushing against them. These vibrations, more commonly known as Ice Induced Vibrations (IIV), may lead to fatigue problems, safety issues and uncomfortable working conditions. To date only phenomenological models exist of ice failure in the course of IIV which are capable of predicting the onset and severity of these vibrations. This is a consequence of the complexity of the ice crushing process that takes place in contact with a flexible structure.

A summary of work done before the 1990's is given by T. Kärnä and R. Turunen in their paper on dynamic response of narrow structures to ice crushing (1989). Below, a short summary is proposed of the different classes of models that are used for prediction of IIV.

One class of models relies on the concepts of negative damping and self-excitation. Examples are the models by Blenkarn (1970), Määttänen (1978) and Xu and Wang (1988). Määttänen (1978) proposed that steady-state vibration of narrow vertical offshore structures is a self-excited process where the non-linear forces due to ice crushing provide an apparent negative damping effect on the structure. In his model the properties of crushing ice are related to the dynamic equations of motion of the structure through an averaged ice crushing strength curve based on quasi-static measurements.

The model by Määttänen assumes that the nominal crushing pressure shows a rapid decrease at a transitional value of the indentation rate. At low indentation rate the ice fails in a ductile manner, whereas at higher rates the ice fails in a brittle manner. It is assumed that the change in ice failure process is the cause for the drop in ice crushing pressures. However Kärnä and Jarvinen (1993) claimed that this is not supported by tests on indentation of sea ice.

A second class of models relies on the concepts of crushing length and characteristic failure frequency. At the basis of these models lies the assumption that a moving ice sheet acting on a structure tends to break into fragments of a certain size. This process determines a characteristic failure frequency assumed directly proportional to ice velocity and inversely proportional to ice thickness. In the event this characteristic frequency lies close to the natural frequency of the structure, resonant vibrations may arise. Models incorporating this approach are the models of Michel (1978) and Sodhi (1988). One problem associated with this type of models is that the values suggested for the characteristic failure frequency rely only on measurements with rigid structures where the nonlinear interaction between ice failure and structural motion is not present. (Määttänen, 1988).

A model, which contained aspects of both classes of models, was introduced in 1989 by T. Kärnä and R. Turunen. This model couples the dynamic equations for the structure and the undamaged ice field through a nonlinear element describing the crushing and clearing processes at the ice-structure interface. The ice is assumed to fail with a more or less constant nominal crushing depth. However, the general idea behind the model is that of IIV occurring because of an implicit dependence of ice strength on the indentation rate being descendent. Good correspondence with field measurements was achieved with this model for slender vertically-sided offshore structures.

Another development was proposed in 2009 by Huang and Liu also containing aspects of both classes of models. This model treats the ice failure as a series of discrete events and is applicable in the case of low ice velocities. It assumes simultaneous ice crushing over the width of the structure near the contact area. Also in this model a dependence of the ice-crushing strength on the indentation rate is assumed containing a descending branch for certain rates. The model shows to be able to capture the effect of lock-in for the ice force frequency at low ice sheet velocities.

In general, most models are part of the first class relying on the concepts of negative damping and self-excitation whereas the models of the second class are not yet found to give satisfying results when compared to field measurements.

On the way towards development of a practically applicable fracture mechanics based model for IIV, questions remain about the non-linear interaction process between drifting level ice and flexible offshore structures. The present study attempts to answer a number of these questions through modification of a Matlock-Sodhi-Huang model which can be said to be based on both schools of thought. It is investigated whether a periodical character of the ice loading is a necessity in order to obtain frequency lock-in. This is done by introducing an ice loading containing a broad spectrum of loading frequencies. Furthermore the limits of ice velocities for occurrence of synchronization in the first structural mode of vibration are briefly touched upon and the possibility of synchronization in higher structural modes is investigated.

MODEL FOR ICE STRUCTURE INTERACTION

The model presented in this paper is a modification of the Matlock-Sodhi-Huang model (2009). Figure 1 shows a top view of the model consisting of an ice sheet modelled as a set of strips and a structure mimicked by a generalized beam. The ice sheet consists of a set of strips that are not interacting with each other. Each strip is modelled as a bogie moving with a constant speed and equipped with a sequence of vertical bars as shown in Figure 2. The vertical bars are modelled as shear beams (no bending stiffness) whose strength is characterized by a strength-velocity dependence shown in Figure 3. The force exerted by the shear beams on the structure is assumed to be linear elastic with an effective contact stiffness and proportional to the relative displacement between shear beam and structure. Upon reaching the failure strength the ice force drops abruptly to a low value considered to be the residual force after crushing.

The structure that undergoes IIV is mimicked by a generalized beam with coordinate dependent parameters (mass density, cross-sectional area, moment of inertia and Young's modulus) allowing for tuning of both the modal spectrum and modal damping, see Figure 4.

Each strip in Figure 1 is modelled by a bogie, whose vertical bars are positioned quasi-periodically with a different mean-value of the period. An impression of the overall model is given by Figure 5, which indicates that each bogie has a different periodicity. The ice-structure interface is assumed to be flat so that all strips meet the structure at the same angle of incidence.

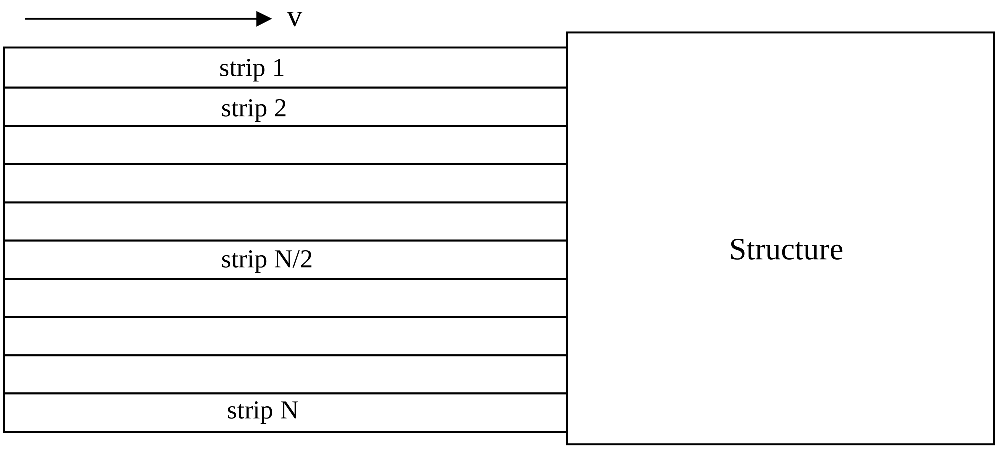

Figure 1. Top view of ice-structure system.

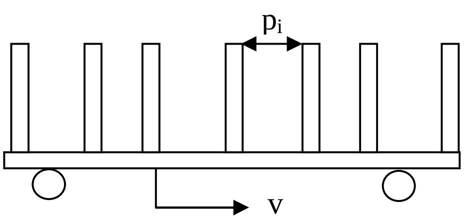

Figure 2. Side view of one strip in the ice sheet model.

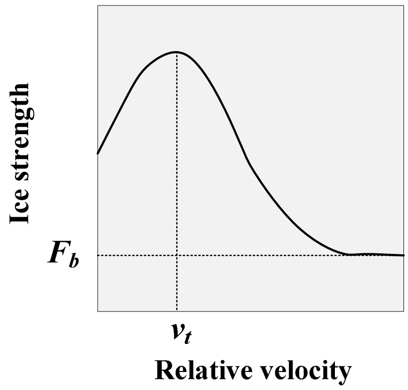

Figure 3. Dependence of failure force on relative velocity.

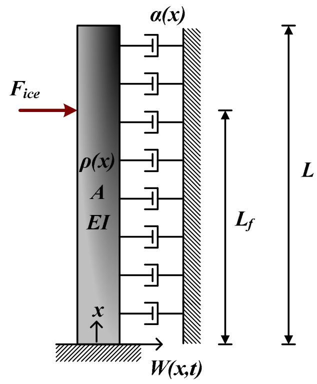

Figure 4. Structure modelled as a cantilever beam.

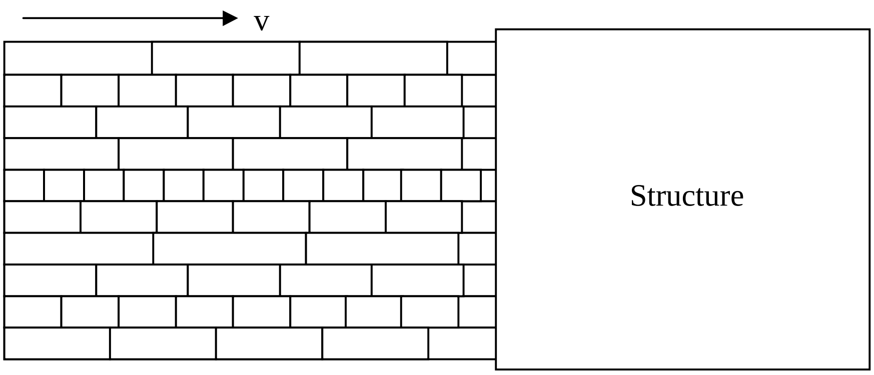

Figure 5. Top view of the model with detailed ice strips.

Dynamic equations governing the ice sheet motion

The ice sheet is modelled as a series of flexible rods on moving bogies simulating the discrete failure of ice. Figure 5 gives the top view of such model, whereas Figure 2 gives the side view of one strip. All strips move with a constant ice velocity *v*. The distance between the rods on each bogie p_i represents the length of the failure zone associated with each failure event. It is assumed to be independent of the ice sheet velocity and different for each strip. The deflection of one rod is given by:

$$\delta_i(t) = w_0 + vt - w(t) - p_i(n_i) \times (n_i - 1), \qquad (1)$$

where t is the time and n is the number of the tooth in contact with the structure on strip i. w_0 and w are the initial displacement and displacement of the structure at the ice interaction point, respectively. The force exerted by the ice on the structure until failure is assumed to be linearly elastic with an effective contact stiffness k_i . The total ice force F_{ice} acting on the structure before failure of ice in crushing is given as:

$$F_{ice}(t) = \sum_{i=1}^{N_{strips}} k_i \delta_i(t) + F_{e;i} \ , \qquad (2)$$

where N_{strips} is the number of strips taken to represent the ice sheet and $F_{e;i}$ is the residual force remaining after a crushing event has occurred. Failure of one tooth is said to occur when the failure deflection $\delta_{f,i}$ is reached:

$$\delta_{f,i} = F_f / k_i \ , \qquad (3)$$

where F_f is the failure force which is dependent on the relative velocity between the ice sheet and the structure as shown in Figure 3. This model for the ice strength takes into account the brittle, transitional and ductile crushing modes and is assumed to be given by:

$$F_f = \frac{d_i h \left(\sigma_b e^{\left(-0.9(v_r/v_t - 1)^2\right)} + \sigma_b \right)}{2} \qquad (4)$$

with d_i the width of strip i, h the thickness of the ice sheet, v_r the relative velocity between ice sheet and structure, v_t the transitional velocity as can be seen in Figure 3 and σ_b the approximately constant brittle failure stress at high ice sheet velocities.
Combining the above the ice force F_{ice} acting on the structure over time can be expressed as:

$$F_{ice}(t) = \begin{array}{l} \left\{ \sum_{i=1}^{N_{strips}} \left[k_i \delta_i(t) + F_{e;i} \right] \left[H(\delta_i) - H(\delta_i - \delta_{f,i}) \right] \right\} H(v - \dot{w}(t)) + \\ \quad \left\{ \sum_{i=1}^{N_{strips}} F_{e;i} \left[H(\delta_i - \delta_{f,i}) - H(\delta_i - p_i) \right] \right\} H(v - \dot{w}(t)) \end{array} \qquad (5)$$

Where H is the Heaviside step function and a dot represents the first derivative with respect to time. w is the displacement of the structure at the ice interaction point.

Equations governing small bending vibrations of the structure

The structure is modelled as a cantilever beam with coordinate-dependent mass density and damping as shown in Figure 4, which transverse motions are governed by the following equation:

$$\rho(x) A \frac{\partial^2 w}{\partial t^2} + EI \frac{\partial^4 w}{\partial x^4} + \alpha(x) \frac{\partial w}{\partial t} = F_{ice}(t, w, \dot{w}) D(x - L_f), \qquad (6)$$

where A is the cross-sectional area of the beam, $w(x,t)$ is the transverse displacement of the beam, E is Young's modulus, I is the moment of inertia, F_{ice} is the global ice action as defined by Eq. (1), D is the Dirac delta function, x is the coordinate along the beam, L_f is the point of action of the ice load, α and ρ represent the coordinate dependent damping and mass density, respectively, given by:

$$\rho(x)=\rho_0+\rho_1(x)=\rho_0\left(1+\frac{1}{\sqrt{\pi}\sigma_\rho}e^{\left(-(x-L_\rho)^2/\sigma_\rho^2\right)}\right) \tag{7}$$

$$\alpha(x)=\alpha_0+\alpha_1(x)=\alpha_0\left(1+\frac{1}{\sqrt{\pi}\sigma_\alpha}e^{\left(-(x-L_\alpha)^2/\sigma_\alpha^2\right)}\right) \tag{8}$$

The boundary conditions are given by:

$$w(0,t)=0,\ \left.\frac{\partial w(x,t)}{\partial x}\right|_{x=0}=0,\ \left.\frac{\partial^2 w(x,t)}{\partial x^2}\right|_{x=L}=0,\ \left.\frac{\partial^3 w(x,t)}{\partial x^3}\right|_{x=L}=0 \tag{9}$$

This model does not represent a real offshore structure but does give the freedom to tune the amount of modal damping and ratio of natural frequencies as preferred.

BASE CASE

A base case is defined taking only one bogie into account with a constant failure length p_1. The effect of variation of the model parameters on the obtained results will be demonstrated in the next sections by using the results of the base case as a reference. The structure and ice properties used in the computation are given in Table 1. ζ is the structural damping as a fraction of critical defined as:

$$\zeta=\frac{\alpha_0}{2\rho_0 A\sqrt{EI/\rho_0 A}} \tag{10}$$

Table 1. Properties used in computation.

w_0 [m]	p_1 [mm]	h [m]	d [m]	F_e [kN]	v_t [m/s]	δ_f [mm]	σ_b [MPa]	ρA [kg/m]	EI [MN m^2]	ζ [-]
0	2	0.4	1	40	0.1	1.8	0.5	1000	200	0.04

Figure 6 shows the results of the analysis of the base case. The dependencies are presented of the mean displacement of the structure w, the root mean square (RMS) of displacement and the frequency of vibration f_r relative to the fundamental frequency f_n of the structure on the far-field ice velocity.

Frequency lock-in is said to occur at a certain interval of velocities when two conditions are met:

1. The frequency of response equals one of the natural frequencies of the structure over the entire interval.
2. A significant amplitude increase is observed considering the motions of the structure.

Provided that the initial energy input around the natural frequency for all velocities in the interval is equal, i.e. the amplitude increase is not a result of forced response where the initial energy around the natural frequency increases with the ice sheet velocity. For the model presented here this last condition is met.

From the frequency dependence and RMS in Figure 6 it can be observed that lock-in occurs for ice sheet velocities between 0.10 m/s and 0.23 m/s approximately as in this are the frequency of response is equal to the natural frequency of the structure and the RMS, a measure for the amplitude of structural vibrations, increases significantly for increasing ice sheet velocities. The minor dip and bump in the graph around 0.15 m/s can be explained by resonance that occurs due to coincidence of the loading frequency and the natural frequency of structure, which apparently plays the governing role at this velocity.

The maximum displacement of the structure, in terms of RMS of displacement, increases until a critical value for the ice velocity (about 0.225 m/s) is reached at which the energy dissipation by structural damping exceeds the energy input into the system by the failure process.
The mean structural displacement w_{mean} shows a descending character. This is a direct consequence of the introduced velocity dependence of ice strength as given by Figure 3 where for high ice sheet velocities the strength of the ice reduces and, as a result, the mean displacement of the structure is also reduced.

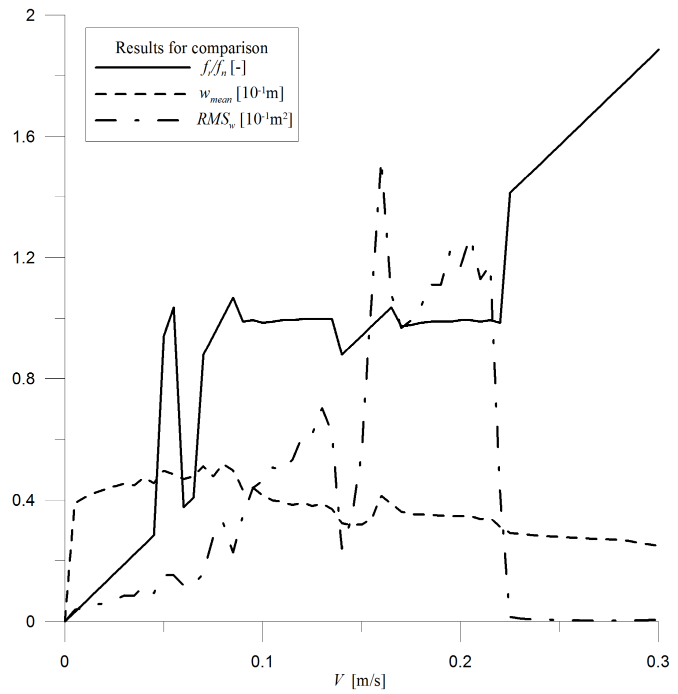

Figure 6. Results used for comparison.

APERIODIC ICE LOADING

The assumption of a constant failure length p_i in the model causes a periodic loading pattern to be observed for the case in which the structure is restricted from moving. An example of such a loading pattern is given in the left graph in Figure 7 for an ice sheet velocity of 0.12 m/s.

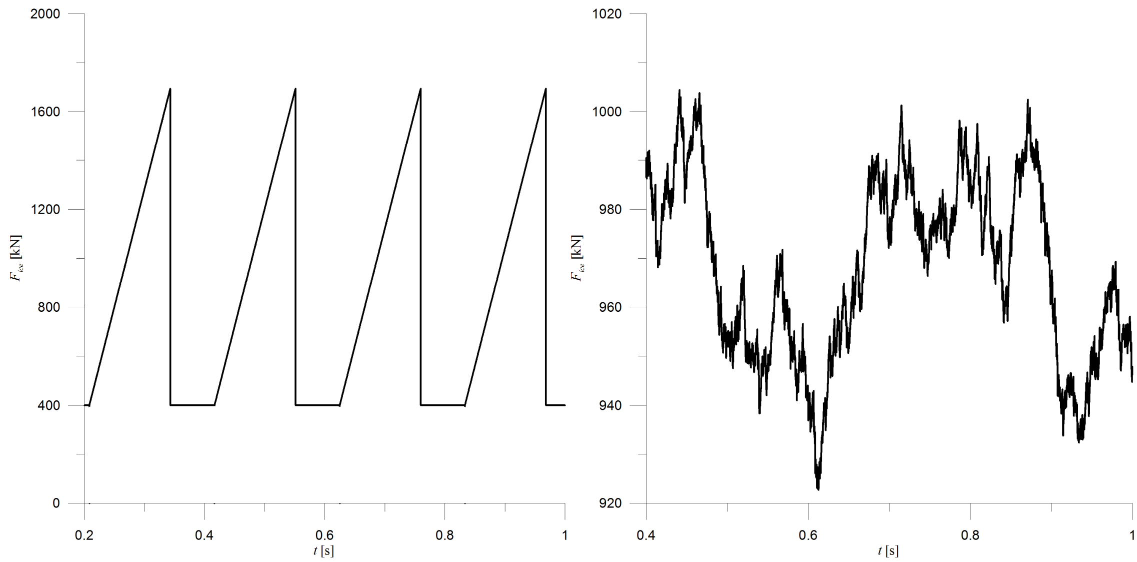

Figure 7. Left: Periodic loading pattern on non-moving structure. Right: Aperiodic loading pattern on a non-moving structure.

In order to show that an aperiodic load can also trigger IIV the amount of strips representing the ice sheet is increased. Each of the strips is taken to have a distinct failure length p_i uniformly distributed between 0.2 and 1.8 times a mean value p_{mean}. Hereby a broad spectrum of failure frequencies is introduced in the ice sheet causing an aperiodic loading pattern on the non-moving structure. The right graph of Figure 7 shows an aperiodic force pattern obtained for a velocity of 0.12 m/s with 1000 strips. When compared to the force pattern as shown in the left graph of Figure 7 it can be clearly seen that an apparently aperiodic ice load is obtained.

Introducing such an aperiodic ice force on a flexible structure and analysing the response signal it can be shown that the model still predicts synchronization to occur. The range of velocities between which the lock-in occurs is almost the same as the range predicted for the case with a periodic ice loading as can be seen in the left graph of Figure 8, where used notations are the same as in Figure 6. Differences are found in the response for higher than 0.2 m/s ice velocities where the structure is now predicted to vibrate at its natural frequency even though no increase of the response amplitude is observed. This is a typical force response of a linear damped system to a broad-band aperiodic excitation. Based on the obtained results a conclusion can be drawn that an initially aperiodic ice loading can trigger IIV.

SYNCHRONIZATION AT HIGH ICE SHEET VELOCITIES

The maximum velocity at which synchronization occurs between the ice and a certain structural mode of vibration is of major importance for the fatigue design of structures. The current model predicts that the smaller the modal damping, the larger the maximum ice sheet velocity at which lock-in occurs. However it can be shown that also the modal mass and stiffness parameters are of key importance here. It is the natural frequency of the lowest mode in combination with its modal damping that determines up to which ice sheet velocity susceptibility to IIV can be expected.

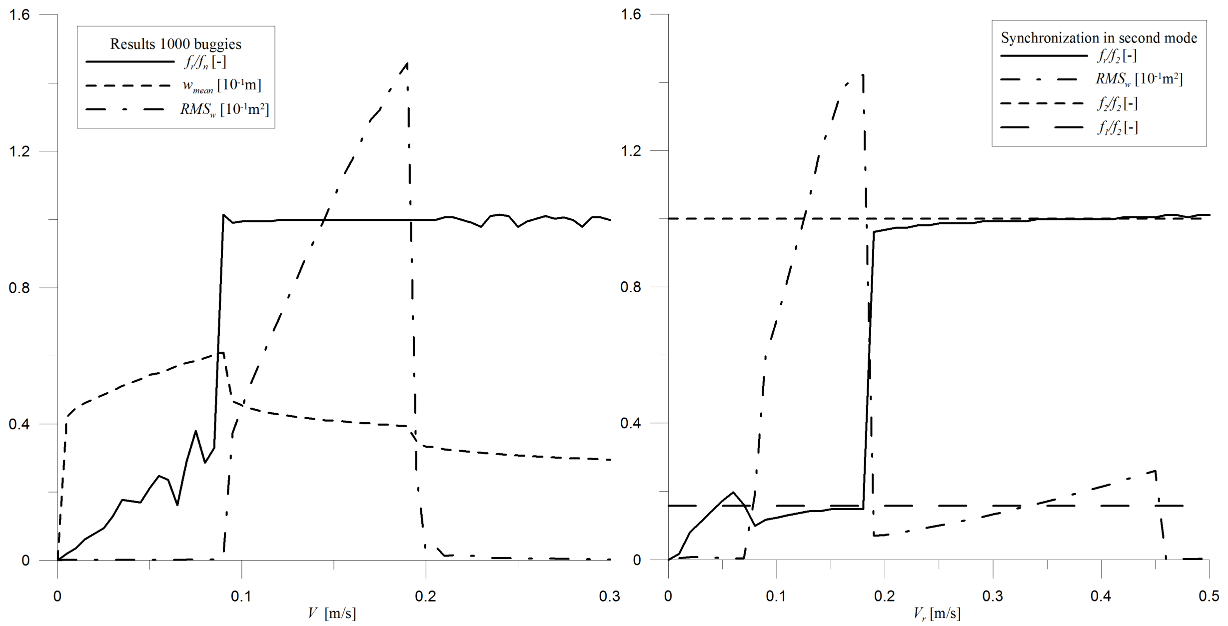

Figure 8. Left: Results for aperiodic loading. Right: Synchronization in the first and second mode of vibration.

SYNCHRONIZATION AT HIGHER STRUCTURAL MODES

Accounting for the higher structural modes the model predicts that, in general, no frequency lock-in occurs with the higher modes if the damping and mass density in equation (6) are kept constant. This corresponds to what is mainly believed to be true that the IIV in most cases seem to only occur at the lowest natural frequency of structures. In order to obtain frequency lock-in with the second mode the modal damping of this mode should be significantly reduced. Within the framework of the considered model, this can be achieved by assuming that the viscous damping in equation (6) acts mainly around the node of this mode. If this is done, the model shows that frequency lock-in occurs with this mode although for high ice velocities, between 0.20 m/s and 0.45 m/s in the right graph of Figure 8. A perceptible increase in amplitude occurs in the second mode. However the amplitude increase of the first mode, synchronization between 0.10 m/s and 0.20 m/s in the right graph of Figure 8, remains much higher.

The general conclusion is drawn that frequency lock-in is predicted to occur with higher modes only if the damping in one of those modes is small compared to the damping in the lower modes of vibration of the structure. Also it only occurs at higher ice sheet velocities which lie beyond the ice velocity interval in which frequency lock in with the first mode takes place.

CONCLUSIONS

An apparently random ice load on a non-moving structure has been reproduced by the presented model by means of increasing of the number of strips representing the drifting ice sheet. Frequency lock-in has been predicted to occur under the action of such a force. Based on that, it has been concluded that the implicit periodicity of the ice load in time is not a key factor for IIV to occur (at least in this kind of models).

Frequency lock-in with higher modes of structural vibration has been predicted at those modes whose modal damping is less than damping in the fundamental mode. The higher modes lock onto ice forcing only at relatively high velocities of the ice sheet.

The model presented in this paper has not been tuned to predict results of measurements or full-scale observations though this could have been done. We chose not do so because the model still has a number of drawbacks which should be removed before the tuning would become sensible. Yet, the authors hope that the presented results will be of interest as they show a number of qualitative results, the understanding of which is essential for the development of reliable phenomenological and fracture-mechanics-based models for IIV-prediction. The first such result is that it seems to be imperative that a successful IIV-prediction model should predict a descendent mean force on a non-moving structure as a function of ice velocity in the far field. It must be noted, however, that such dependence does not have to be introduced as a material property of ice. The second result is that the initial ice loading (the loading that takes place before the structure is set in significant motion) on the structure does not have to be periodic or even quasi-periodic to trigger frequency lock-in. It seems to be sufficient that the ice loading spectrum contained a sufficient amount of energy at the frequencies close to the natural frequencies of the structure. Finally, in order to predict frequency lock-in at higher modes of the structure, prediction model has to accurately account for all damping mechanisms. This means that the internal (material and joints-associated) damping in the structure, as well as the hydrodynamic damping and soil damping must be accounted for in the model. It may also be important to correctly account for dissipation of energy caused by ice-structure interaction associated with other than locked modes of vibrations.

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