POAC'11
Montréal, Canada

Proceedings of the 21st International Conference on
Port and Ocean Engineering under Arctic Conditions
July 10-14, 2011
Montréal, Canada

POAC11-030

Criterion of ice induced self excited vibration

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ABSTRACT

The research of ice induced structure's self excited vibration is briefly reviewed firstly, and the important findings based on test results are presented. According to the theoretical model that ductile-brittle transition compressive failure takes place during the loading phase in one cycle of vibration, a criterion is derived to predict the conditions for triggering ice induced self excitation, in other words, the combination of structure's stiffness, damping, width and ice floe's thickness, moving speed and compressive strength which makes self excitation easily occur. Finally, the proposed criterion is validated using the measured data from full scale and model tests, and some uncertainties of the criterion and further research are discussed.

KEYWORDS

Ice induced vibration; Criterion; Self excited vibration; Ductile-brittle transition

1. INTRODUCTION

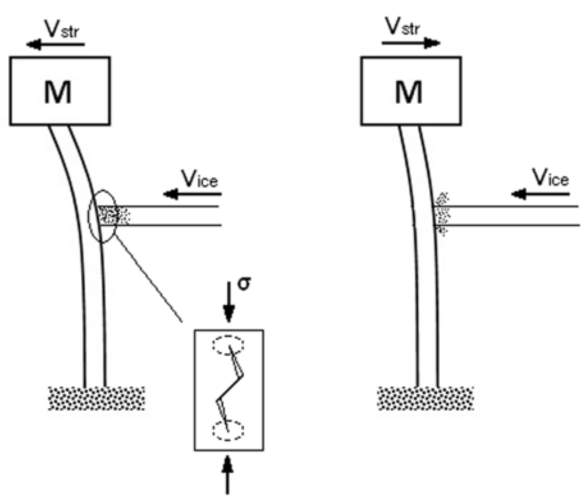
Structures with cylinder legs might vibrate significantly under the loading of a drifting level ice floe, which is termed ice induced vibration (abbreviated as IIV in this paper). Since the discovery of IIV for the first time on the drilling platforms in Cook Inlet, Alaska (Peyton, 1968), there have been lots of research on IIV using analytical methods, model scale tests or full scale measurements. Basically, there are two theoretical points of view to explain the IIV: forced vibration and self excited vibration. The theory of ice induced forced vibration suppose that ice sheet tends to fail into fragments of certain sizes, so that the "characteristic" period of dynamic ice loading is determined by ice sheet's breaking length divided by the moving speed. According to ice induced forced vibration theory, resonance arises when the characteristic period of ice loading approaches natural period of structure. The research supporting forced vibration can be found in (Matlock ,1971, Sodhi, 1988). On the contrary, some researchers believe that during IIV, fluctuation of dynamic ice loading is dominated by the structure's vibration, and the severe IIV is due to the coupling between ice loading and structure's feedback, which is a typical feature of self excited vibration. The research of ice induced self excited vibration (abbreviated as IISV in this paper) can be found in (Blenkarn, 1970,).

In the past few decades, many tests have been conducted to provide data for studying IIV. Especially, a few successful prototype test programs have made significant contribution for better understanding IIV (Määttänen, 1979, Toyama et al, 1983, Engelbrektson, 1989, Schwarz et al, 2001, Yue et al, 2001, Guo et al, 2009). The measured data indicates that there are two distinct modes of IIV: steady-state vibration and stochastic vibration, and the former do belong to self excited process in which ice force is influenced by structure's motion (Yue et al, 2001, 2009, Kärnä, 2007). Stochastic vibration caused by randomly vibrating ice force is easy to understand, and the reason is as follows: sea ice is a kind of natural material whose crystalline structure and mechanical properties are quite scattered, so that in most cases, the ice floe acting on a cylinder structure will fail in irregular brittle manner, resulting in randomly varying ice force and stochastic vibration. However under special conditions, dynamic ice force may fluctuate periodically and cause regular steady-state vibration. Normally the period of vibrating ice force is very close to the lowest natural period of structure and lead to dynamic amplification. The full scale measurement of concurrent ice force and structure's vibration on a structure in Bohai Sea indicate that long-time ice induced steady-state vibration is self excited phenomenon (Yue et al, 2001, 2009).

Unfortunately, so far our understanding of IISV is still limited: the physical origin of IISV has not been understood, as a result, no general conditions for IISV taking place has been presented, and the dynamic equation for IISV of the simplest single-degree-of-freedom (SDOF) system has not been developed.

2. PHYSICAL MECHANISM OF IISV

Yue and Guo (2009) proposed a physical mechanism to explain IISV, which can be described using the sketch of Fig.1. In the sketch, one single cycle of self excited vibration is divided into two phases: loading phase and unloading phase. In the most duration of the loading phase (as shown in Fig.1(a)) structure moves in the same direction of ice moving, and the relative displacement between ice and structure makes the ice edge compressed in ductile-brittle transition strain rates. Consequently, micro cracks continually form inside the ice edge but they do not propagate unstably due to the passivation effect at wing-crack tips (highlighted by the dashed ellipses in the ice sample), as a result, resistance of ice sheet is maintained as the number of micro cracks increase so that ice load is able to keep increasing.



(a) Loading phase (b) Unloading phase

Fig. 1 Sketch of the physical mechanism of IISV

While the density of micro cracks inside ice edge reaches a critical level, ice sheet is not able to take more loads, which trigger the coalescence of cracks and total failure of ice edge, structure moves backward and results in the unloading phase, as sketched in Fig.1 (b).

Accordingly, as described above, the key point of the physical mechanism for IISV is that during loading phase, ice edge undergoes compressive strain rate in the ductile-brittle transition range. Based on this key point, the criterion for IISV taking place can be derived, which is demonstrated in the following section.

3. DERIVATION OF THE CRITERION FOR IISV

In reality, the prototype structures are multi-degrees of freedom system, however in the case of ice sheet acting on cylinder-leg, the interaction between ice edge and structure can be focused on the loading point, i.e. the system can be simplified to a single degree of freedom (SDOF) system as sketched in Fig.2. The parameters denoted in Fig.2 are:

K	—	Structural stiffness at ice loading level
M	—	Equivalent mass of the SDOF system
C	—	Equivalent damping coefficient of the SDOF system
X	—	Reference direction of structure vibration
F_{ice}	—	Reference direction of ice loading

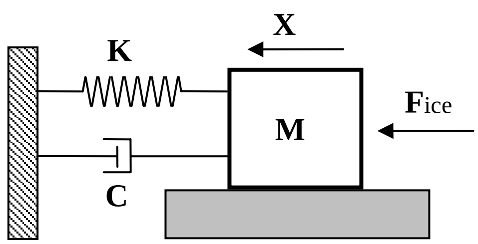


Fig.2 Single degree of freedom system representing ice-structure interaction

During the real ice-structure interaction process, the dominant period of structure vibration is normally the lowest natural period of structure, so that structure's displacement can be approximately represented by harmonic time history. On the other hand, in most cases the dynamic ice force includes an invariant static component, which means that the equilibrium position of vibration deviates from the "absolute" equilibrium position without any loading, i.e. there is a constant component involved in the structure's vibrating displacement, which can be expressed as:

$$X = A_{stat} - A_{str}\cos\omega t \tag{1}$$

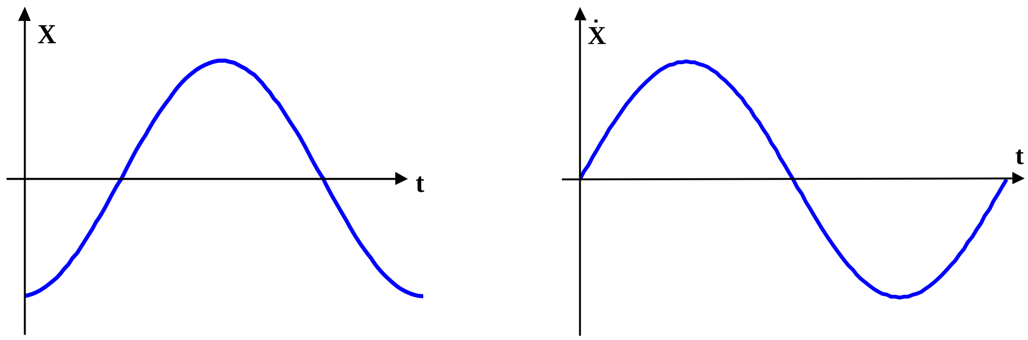
where A_{stat} is the constant component in the displacement, A_{str} is the fluctuating amplitude of vibration, and ω is the circular frequency of vibration. For simplification, the constant component is omitted in the following analysis since it has no effect on the dynamic ice-structure interaction, and the structure's vibrating displacement is:

$$X = -A_{str}\cos\omega t \tag{2}$$

which is illustrated in Fig.3(a). The differential of displacement yields structure velocity:

$$\dot{X} = A_{str}\omega \sin \omega t = V_{str} \sin \omega t \tag{3}$$

where $V_{str} = A_{str}\omega$ is the amplitude of structure's velocity. The time history of velocity in one cycle is plotted in Fig.3 (b).



(a) (b)

Fig.3 Sketch of the time histories of structural displacement (a) and velocity (b) in one vibration cycle, which are formulated by equation (2) and (3) respectively.

Referring to the directions shown in Fig.2, the relative velocity between ice and structure is expressed as:

$$V_r = V_{ice} - \dot{X} \tag{4}$$

which can be considered as the superposition of negative structural velocity $-\dot{X}$ (shown in Fig.4 (a)) and constant ice velocity V_{ice} . Fig.4 (b), (c) and (d) illustrate the three possible conditions of ice velocity V_{ice} and structure velocity amplitude V_{str} :

1) $V_{ice} < V_{str}$ [Fig.4 (b)] A negative part of relative velocity exists in one vibration cycle, implying that structure moves faster than the intact ice sheet so that structure loses contact with ice edge. This condition does not accord with the full scale measurement, because the measured data indicate that ice force acts on structure all the time, as shown by the constant component in equation (1);

2) $V_{ice} = V_{str}$ [Fig.4 (c)] Relative velocities are always positive and the minimum value is zero, which is a possible critical condition;

3) $V_{ice} < V_{str}$ [Fig.4 (d)] Relative velocities are always positive and higher than zero, which accords with most cases of the measured ice-structure interaction.

As indicated above, the key point of the physical mechanism for IISV is that ice edge undergoes ductile-brittle transition compression during loading phase, which is the first half cycle illustrated in Fig.4 (c) or (d). It is obvious that the maximum and minimum relative velocities in the loading phase are expressed as:

$$\begin{cases} (V_r)_{\max} = V_{ice} \\ (V_r)_{\min} = V_{ice} - V_{str} \qquad (V_{ice} \geq V_{str}) \end{cases} \tag{5}$$

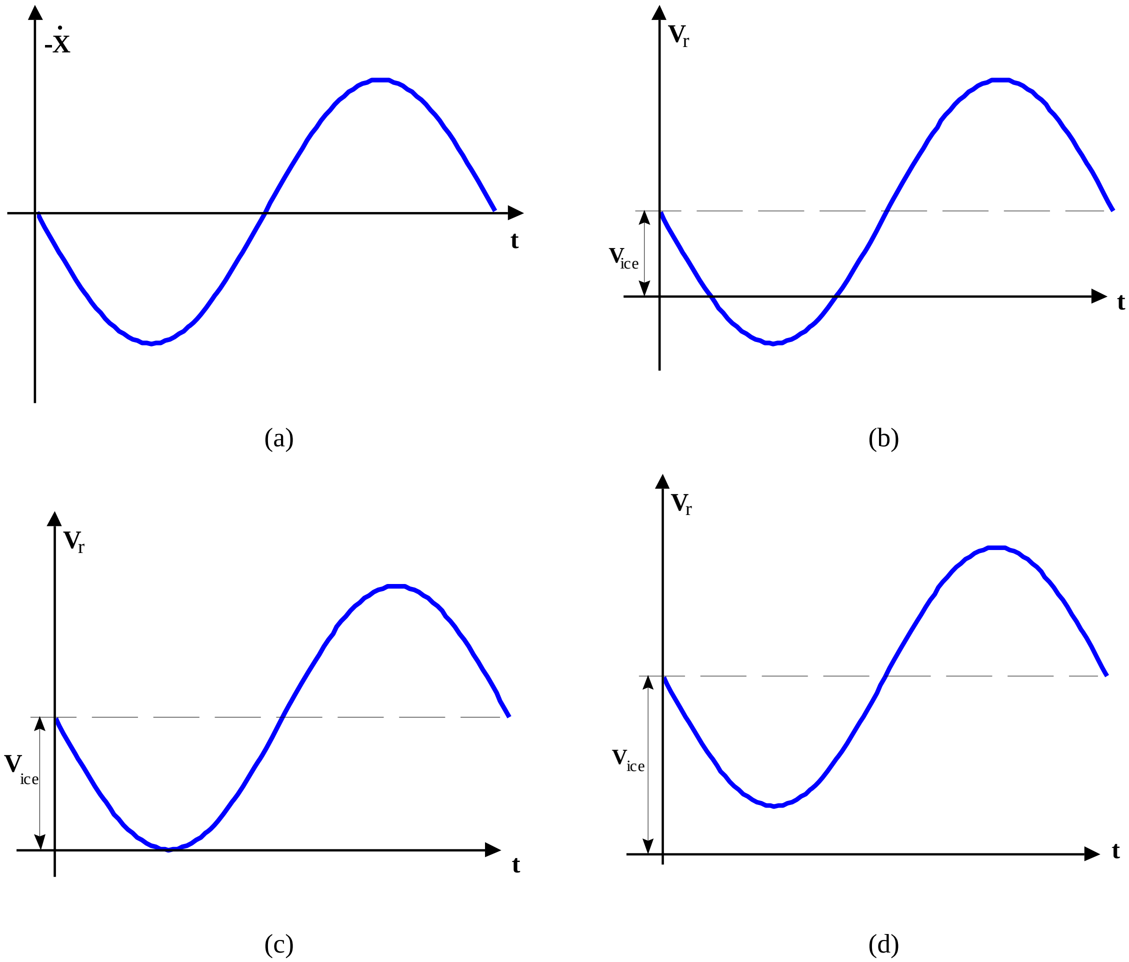


Fig.4 Sketch of negative structural velocity (a) and three possible relations between structure velocity amplitude and constant ice velocity (b), (c), (d).

It is assumed that during loading phase, there is no macro failure in ice sheet so that ice sheet close to structure can be seen as intact. In that case, the strain rate of ice sheet is a function of relative velocity V_r, structure diameter D or ice thickness h. In present analysis, structure diameter is considered much higher than ice thickness, implying that ice sheet indented by structure could be approximated as plane stress condition, as a result, length of the stress field in ice sheet is a function of structure diameter, and the "equivalent" compressive strain rate in ice sheet is formulated as:

$$\dot{\varepsilon}_e = \frac{V_r}{\alpha_L D} \tag{6}$$

where $\dot{\varepsilon}_e$ is equivalent compressive strain rate. In reality, the compressive strain rate in ice sheet is a spatial field, and the strain rate close to structure surface must be higher than that in the area far from structure. Fig.5 is a two dimensional sketch of the stress field in ice sheet, in which the strain rate at point A must be the highest and the strain rate at point B could be considered very small.

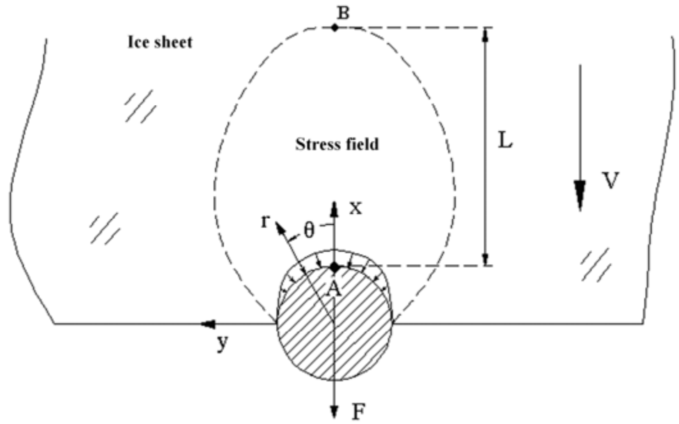


Fig.5 Two dimensional sketch of stress field in ice sheet

Equivalent strain rate $\dot{\varepsilon}_e$ in equation (6) is a value which represents a kind of "average" strain rate in the stress field in Fig.5, thus it is lower than the maximum value at point A but much higher than the infinitesimal strain rate at point B. The coefficient α_L in equation (6) represents the ratio between length of stress field L and structure diameter D. Based on the physical mechanism of IISV, the strain rate in ice sheet should fall into ductile-brittle transition range, which could be formulated by the equivalent strain rate as follows:

$$\underline{\varepsilon} \le \dot{\varepsilon}_e \le \overline{\varepsilon} \tag{7}$$

where $\underline{\varepsilon}$ and $\overline{\varepsilon}$ are the lower and upper bound of ductile-brittle transition range respectively. Substituting equation (6) into the inequation (7), the following inequation is obtained:

$$\alpha_L D \underline{\varepsilon} \le V_r \le \alpha_L D \overline{\varepsilon} \tag{8}$$

According to Schulson (2001), the following lower and upper bound of ductile-brittle transition range could be used in this analysis:

$$\underline{\varepsilon} = 10^{-4} s^{-1} \qquad \overline{\varepsilon} = 10^{-2} s^{-1} \tag{9}$$

Applying the inequation (8) and values of equation (9), the conditions in which IISV takes place could be derived. The parameters in inequation (8) are analyzed and determined below.

If the dynamic ice force during IISV is simplified as harmonic process, sophisticated structure dynamics provide the analytical solution for amplitude of structure's vibrating displacement:

$$A_{str} = \frac{F_d}{K} \frac{1}{\sqrt{(1-\beta^2)^2 + (2\xi\beta)^2}}$$

where F_d is the amplitude of dynamic ice force, K is the structure's stiffness at ice loading level, β is the ratio between frequency of dynamic ice force and lowest natural frequency of structure, ξ is damping ratio corresponding to the structure's lowest natural mode. In the case of

IISV, period of dynamic ice force is very close to structure's natural period, so that $\beta \approx 1$. As a result, structure's vibrating amplitude is:

$$A_{str} = \frac{F_d}{K}\frac{1}{2\xi}$$

The equation above could be rewritten as:

$$A_{str} = \frac{F_d}{2K\xi} = \frac{\lambda_d F_{peak}}{2K\xi} = \frac{\lambda_d (\alpha D h \sigma_c)}{2K\xi} \tag{10}$$

where F_{peak} is the peak ice force acting on cylinder structure, and the dimensionless coefficient λ_d is the ratio between F_d and F_{peak}. The peak ice force could be further expressed as $F_{peak} = \alpha D h \sigma_c$, in which D, h, σ_c and α are respectively structure diameter, ice thickness, uniaxial compressive strength of ice and coefficient representing the transition from uniaxial compression to plane stress loading state. Applying the relation in equation (3) and equation (10), the amplitude of structure's vibrating velocity is expressed as:

$$V_{str} = A_{str}\omega = A_{str}(2\pi f) = \frac{(\pi f D h \sigma_c)(\alpha\lambda_d)}{K\xi} \tag{11}$$

where f is the vibration frequency and also the natural frequency of structure. In equation (11), the dimensionless coefficient α is used to calculate peak ice force as denoted in the bracket of equation (10), and the value $\alpha = 0.8$ is used in this analysis (more detailed discussion on this coefficient can be found in textbooks like Sanderson, 1988). An approximate range $\lambda_d = 0.05 \sim 0.15$ could be proposed based on ice force data from full scale measurement (Yue et al, 2001, Kärnä, 2007). Therefore, the coefficient in the bracket of equation (11) is:

$$\alpha\lambda_d = 0.04 \sim 0.12 \tag{12}$$

Applying the equation (5), (8), (11) and value of equation (9) and (12), a criterion of IISV taking place could be developed. The only parameter undetermined is α_L in equation (8), which is analyzed as follows: As mentioned above, the coefficient α_L represents the ratio between length of stress field L shown in Fig.5 and structure diameter D. Michel (1977) conducted a series of experiments and indicate that the equivalent length of stress field is about four times structure diameter, so that the 1st possible formula for equivalent strain rate is:

$$(\alpha_L)_1 = 4, \quad (\dot{\varepsilon}_e)_1 = \frac{V_r}{4D} \qquad \text{(Michel, 1977)}$$

Because the duration of IISV is very short (normally less than 0.5s), there is not enough time for viscous deformation arising, as a result, the stress field in ice sheet could be considered as elastic approximately. If a two dimensional linear elastic FE analysis is made referring to Fig.5, the elastic compressive stress along X axis vanish to almost zero at point B, where the length L is about 15 times D, i.e. the 2nd alternative value for coefficient is $(\alpha_L)_2 = 15$. However, it should be noted that L is the total length of the stress field, and compressive stress descends very quickly along X axis, therefore, strain rate calculated by $(\alpha_L)_2 = 15$ is too small for equivalent or "average" strain rate in ice sheet. In present analysis, a medium value $\alpha_L = 10$ is adopted between $(\alpha_L)_1$ and $(\alpha_L)_2$, applying the value of equation (9), the inequation (8) becomes:

$$10^{-3} D \le V_r \le 10^{-1} D \tag{13}$$

Accordingly, the criterion for IISV could be calculated in the following flow chart:

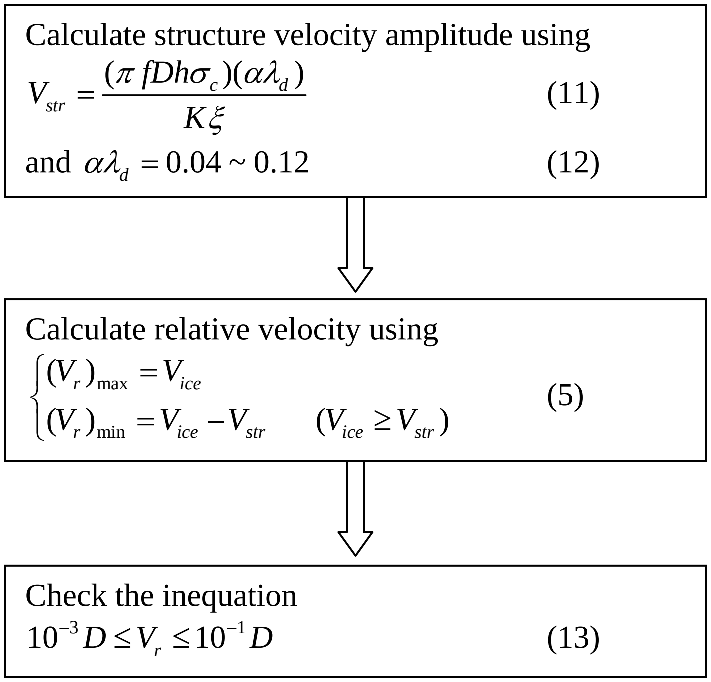


Fig.6 Flow chart for calculating the criterion of IISV

4. CASE STUDY

In order to validate the applicability of the proposed criterion, a case study is presented herein. The monopod platform named JZ9-3 MDP-1 in Bohai Sea has experienced lots of events of IISV, and Fig.7 is the FE model of the structure.

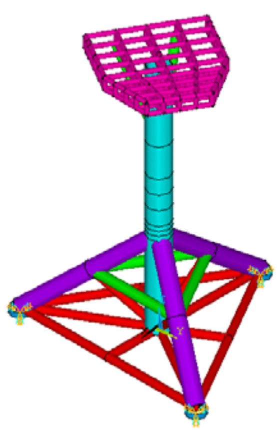


Fig.7 FE model of the structure JZ9-3 MDP-1

The parameters requested by the first box in the flow chart (Fig.6) are listed in Table.1, including structure's characteristics and ice's properties.

Table.1 Parameters requested for calculating V_{str} in equation (11)

Parameters	Values	Note
K (N/m)	1.4e8	Structure's stiffness at ice loading level
ξ	0.05	Damping ratio
f (Hz)	2.32	Lowest natural frequency
D (m)	1.5	Structure diameter
h (m)	0.2	Ice thickness
σ_c (MPa)	2.0	Uniaxial compressive strength of ice

Substituting the values in Table.1 into equations (11) and (12), the lower and upper bound of structure velocity amplitude are obtained:

$$(V_{str})_{\min} = 0.025m/s \qquad (V_{str})_{\max} = 0.074m/s \tag{14}$$

Substituting the structure diameter in Table.1, equation (13) becomes:

$$1.5\times10^{-3} \le V_r \le 0.15 \tag{15}$$

If the structure velocity amplitude takes a low value $V_{str} = 0.03m/s$, ice velocity has to be higher than 0.03m/s and $(V_r)_{\max} = V_{ice}$ and $(V_r)_{\min} = V_{ice} - V_{str}$ must fall into the range formulated by inequation (15). The ice velocity range satisfying this request is $V_{ice} = 0.032 \sim 0.15m/s$. Similarly, if a higher structure velocity amplitude value $V_{str} = 0.05m/s$ is used, the ice velocity range satisfying inequation (15) is $V_{ice} = 0.052 \sim 0.15m/s$.

A couple of IISV events were recorded during the full scale measurement on JZ9-3 MDP-1 platform in the winter 2000-2001, in which the ice drifting velocities are all below 0.15m/s and higher than 0.02m/s when ice thickness is about 20cm. Accurate ice velocity 4.4cm/s and thickness 22cm are obtained in a typical IISV events on 13/02/2001. It can be seen that the measured ice velocities fall into the predicted ice velocity range derived above, and the approach shown by Fig.6 is an applicable criterion for IISV.

5. CONCLUSIONS AND DISCUSSIONS

A criterion is derived for ice induced self excited vibration (IISV), which is expressed as the relation among structure's parameters and ice properties. A real fixed structure is simplified into a single-degree-of-freedom system at ice loading level, and the parameters affecting IISV include equivalent stiffness, mass, damping ratio, diameter etc. Ice's parameters are crucial for triggering IISV, and the most important parameters are thickness, moving speed and compressive strength. The theoretical basis for this criterion is that ice sheet near the ice-structure interface undergoes ductile-brittle transition compressive loading, and then the vibrating cycle of ice load is dominated by the structure vibration.

However, it should be noted that because ice properties are quite scattered and difficult to predict, there are still lots of uncertainties in the criterion analyzed in this paper, which must be refined in further study. On the other hand, more precise test data must be applied to verify the criterion, and more academic and practical research works are necessary for IISV.

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