POAC'11
Montréal, Canada

Proceedings of the 21st International Conference on
Port and Ocean Engineering under Arctic Conditions
July 10-14, 2011
Montréal, Canada

POAC11-025

ICE INDUCED VIBRATION IMPLEMENTATION OF MÄÄTTÄNEN MODEL AND DEVELOPMENT OF DESIGN SUPPLEMENTS†

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ABSTRACT

Ice induced vibration (IIV) can be a serious design condition for structures located in arctic waters. In some conditions, ice-induced vibrations can cause discomfort to the crew onboard, create unsuitable conditions for operations and even cause damage to the structure. Currently, limited guidelines are available to design structures for IIV. Consideration is given here to understanding the phenomenon of steady-state IIV (SS-IIV) and to developing design aids for structures facing this condition. The paper begins with the Määttänen model, which can predict the onset (susceptibility) of SS-IIV based on ice crushing properties and structural period. This is next used to simulate the response of an arctic lighthouse for a set of given ice conditions, and detect IIV occurrence. A closed-form stability-based solution using Eigen value analysis is then developed to classify equilibrium as either: (a) stable vibration, i.e. damped IIV (D-IIV) in which structure vibration decays to a static deformed shape; or (b) unstable, i.e. SS-IIV, in which sustained vibration occurs for a given ice condition. Bifurcation theory is used to determine the boundary separating the two states, SS-IIV and D-IIV. This criterion is shown capable of not only providing insight into IIV response but also of generating design guidance for this condition in an early stage of the design process when only general design parameters are known.

BACKGROUND

Steady state ice induced vibration (SS-IIV) refers to the condition when a structure exhibits sustained vibration response caused by an ice floe with constant attributes. The phenomenon of SS-IIV has been recognized since the 1960s and researchers have proposed different models for ice-structure interaction.

One of the earliest studies reporting SS-IIV on a platform structure is from Peyton (1968). In this report, observations were made on a platform structure in ice-infested waters of Cook Inlet, Alaska. Peyton observed that the most dominant mode of ice failure against a vertically sided structure is crushing. Failure of ice by crushing requires a large amount of force; for the platform under observation this force was reported to be 15 million pounds (66.7 MN). He also reported

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† *Paper is based on work during a summer internship at ExxonMobil Upstream Research Company*

that under certain circumstances the force can cycle from zero to a maximum value with a frequency of one cycle per second. In the second part of the study (Peyton, 1968), ice properties were studied, and, by conducting laboratory tests on ice sheets, it was inferred that ice behaves like a viscoelastic material when failed in crushing mode. In this study, a characteristic failure frequency of ice was proposed, but this was later negated by other researchers who provided contradicting evidence.

Blenkarn (1970) presented the ice force measurements made on test piles in Cook Inlet, Alaska from 1963 to 1969. In this study the concept of negative damping was highlighted and explanation for negating the characteristic failure frequency was presented. Ice sheet velocity, structural flexibility and loading rate dependent ice sheet crushing strength were recognized as significant governing factors in IIV.

The concepts presented by Peyton and Blenkarn laid the groundwork for further research in the field of ice induced vibration. A comprehensive literature review was conducted by ExxonMobil Upstream Research Company to compare models proposed by Matlock (1969), Määttänen (1978), Sodhi (1994) and Kärnä (1992). Matlock assumed constant ice failure strength and did not incorporate the rate dependent behavior of ice. Kärnä's model simulated the detailed interaction between structure, soil and ice but requires a number of specific parameters for both the ice characterization and the structural model. Määttänen's model, while considering rate dependent ice crushing strength, requires relatively few parameters for both ice and structural modeling. In this paper, consideration has been given to Määttänen model.

MÄÄTTÄNEN MODEL

Ice-structure interaction is a complex phenomenon involving feedback from ice-structure interaction. To make the model mathematically tangible, the following assumptions are helpful (Määttänen, 1978):

(a) Using average ice crushing strength since crushing area is much larger than crystal size;

(b) Ignoring viscoelastic behavior of ice since IIV frequency is high enough to neglect rate dependent properties;

(c) Ignoring elastic deformation of ice relative to scale of displacement in the structure;

(d) Using load rate dependent crushing strength for ice; and

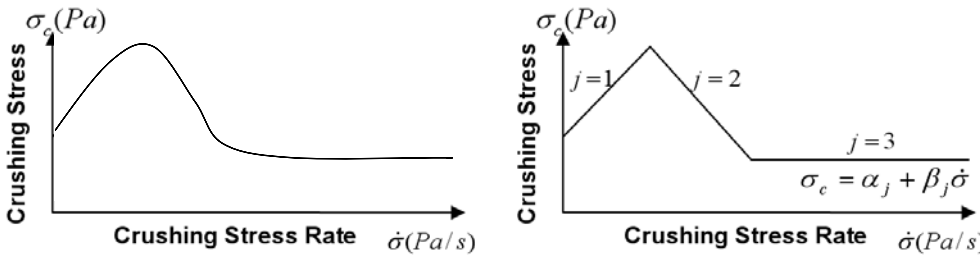
(e) Use of a Rayleigh damping coefficient to model structural damping.

The formulation presented below relates the ice mechanics equation to the structure force at the waterline. The ice mechanics is explained in form of rate dependent crushing strength of ice (Figure 1), as proposed by Peyton (1968). The Peyton's graph used in this study is an idealization of the referenced graph using straight lines to represent different segments of the actual plot. This idealization makes it easier to use while computing related values in a simulation. In this description α_j, β_j are the intercept and slope of the j^{th} segment of Peyton's curve.

The ice crushing stress rate ($\dot{\sigma}$) can be calculated using Blenkarn's equation, 1970:

$$\dot{\sigma} = (v_o - \dot{x}_i)\frac{4\sigma_c}{a\pi}\cos(\theta) \qquad (1)$$

in which $(v_o - \dot{x}_i)$ is the ice-structure relative velocity; *a* is the structure radius at the waterline; and θ is the angle measured from the structure centerline parallel to ice drift direction (Figure 2). When plotted on Peyton's graph, equation 1 looks like a straight line (Figure 3).



(a) (b)

Figure 1. Peyton's curve: (a) original; and (b) idealized.

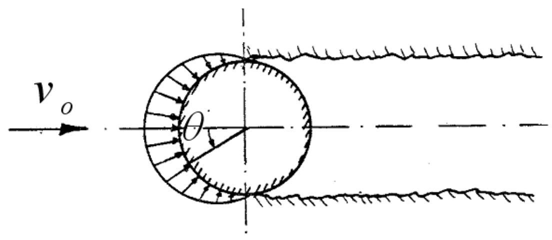


Figure 2. Ice pressure on a circular structure (Määttänen, 1978).

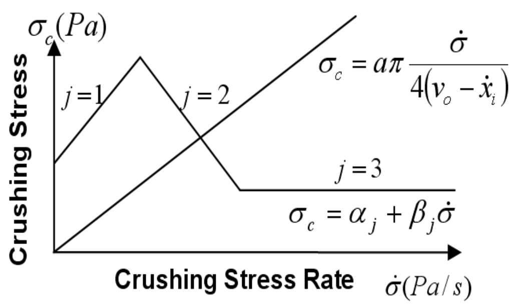


Figure 3. Solving Peyton's curve with equation proposed by Blenkarn.

By solving the two sets of equations from Peyton's curve and Blenkarn, explicit expressions for crushing stress (σ) and crushing stress rate ($\dot{\sigma}$) are obtained (Equations 2).

$$\dot{\sigma} = \frac{\alpha_j}{\frac{\pi a}{4(v_o - \dot{x}_i)\cos(\theta)} - \beta_j} \tag{2a}$$

$$\sigma_c = \frac{\pi a \alpha_j}{\pi a - 4\beta_j (v_o - \dot{x}_i)\cos(\theta)} \tag{2b}$$

Equation 2b provides the explicit expression for the crushing stress of ice, which is also the pressure applied by ice on the circular structure at the waterline. Thus, integrating the ice crushing stress along the structure waterline circumference will provide the ice-induced force, F, applied on the structure at its waterline node (Equation 3). Here θ is the angular position of the point on the cylindrical surface, a is the radius of the circular structure at the waterline and h is the thickness of ice.

$$F = 2\int_0^{\pi/2} \sigma_c \cdot a\cos(\theta)\cdot h d\theta = 2ah\int_0^{\pi/2} \frac{\pi a \alpha_j \cos(\theta)}{\pi a - 4\beta_j (v_o - \dot{x}_i)\cos(\theta)} d\theta \qquad (3)$$

The force calculated in equation 3 can now be applied to the waterline node in the equation of motion defining the whole structural response, as shown in equation 4:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \begin{Bmatrix} 0 \\ \vdots \\ F \\ \vdots \\ 0 \end{Bmatrix} = \tilde{F} \qquad (4)$$

in which [M] is the structure mass matrix; [C] is structure damping matrix; [K] is the stiffness matrix; $\tilde{F}$ is the force vector; x is the vector of structure displacements and rotations; and a dot superscript denotes one differential with respect to time.

NUMERICAL SOLUTIONS USING A LIGHTHOUSE STRUCTURE

To investigate the nuances of Määttänen's description of ice force on vertical structures, a lumped mass representation of Norströmsgrund lighthouse (Engelbrektson, 1983) has been used. First, a lumped mass representation of the structure was developed as shown in Figure 4, where the stick model of the structure is superimposed on the detailed drawing of the lighthouse. The idealized model has 15 nodes, with each node having a rotational and translational degree of freedom. The foundation for the structure was approximated using a translational and a rotational spring. The numerical model was calibrated by matching the modal frequency of the system to reported frequencies of vibration. A viscous damping of 2% was assumed.

To simulate the response due to a passing ice floe, definition of ice is needed in the form of ice floe thickness, ice floe drift velocity and values defining the shape of Peyton's graph. Ice floe thickness and drift velocity is defined for each simulation as a function of time, while the Peyton's curve parameters are taken from Määttänen, 1978 (Figure 5).

A Graphic User Interface (GUI) has been designed to analyze the results of the MATLAB tool (Figures 6 and 7). The GUI can also be animated in time so that the position in Peyton's curve can be observed at different instances of time.

Numerous combinations of ice thickness and drift velocity can be analyzed using the GUI tool:

(a) Response to a floe with constant ice thickness drifting at constant speed;

(b) Response to a floe with variable ice thickness drifting at variable speed; or

(c) Response to a recorded ice thickness and drift velocity profiles.

Two examples are presented in the following to demonstrate the program's capability. Figure 6 shows the response to a profile of ice thickness and ice drift velocity. The profile has been created by introducing step changes in thickness and speed. Figure 7 displays the response for ice thickness and drift velocity profile recorded in the field.

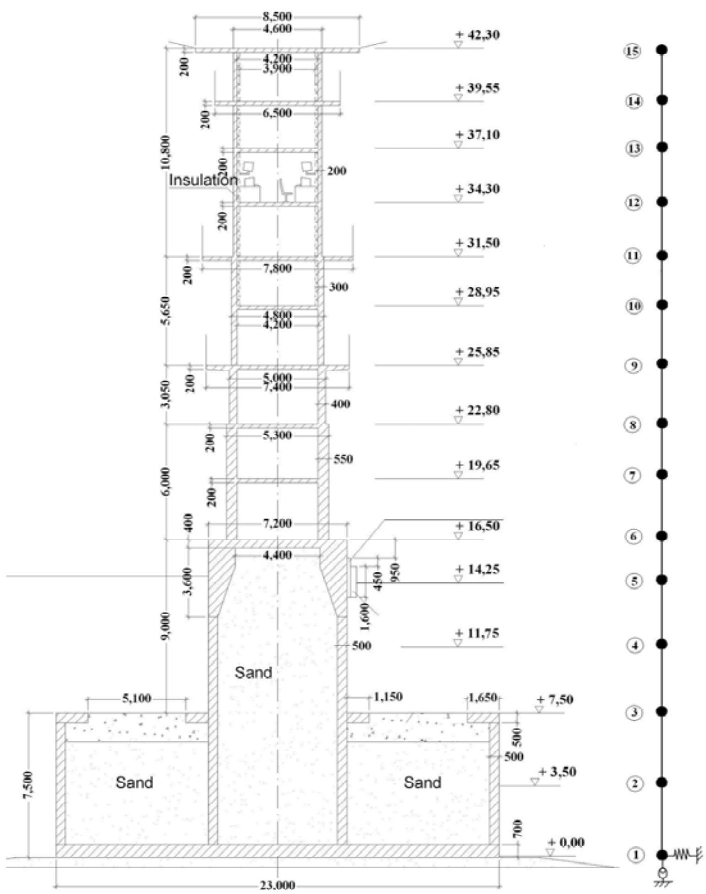


Figure 4 - Norströmsgrund Lighthouse and its simplified model (Engelbrektson, 1983)

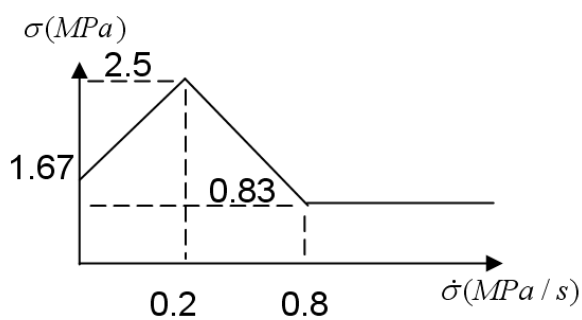


Figure 5 - Peyton’s curve used in this numerical example.

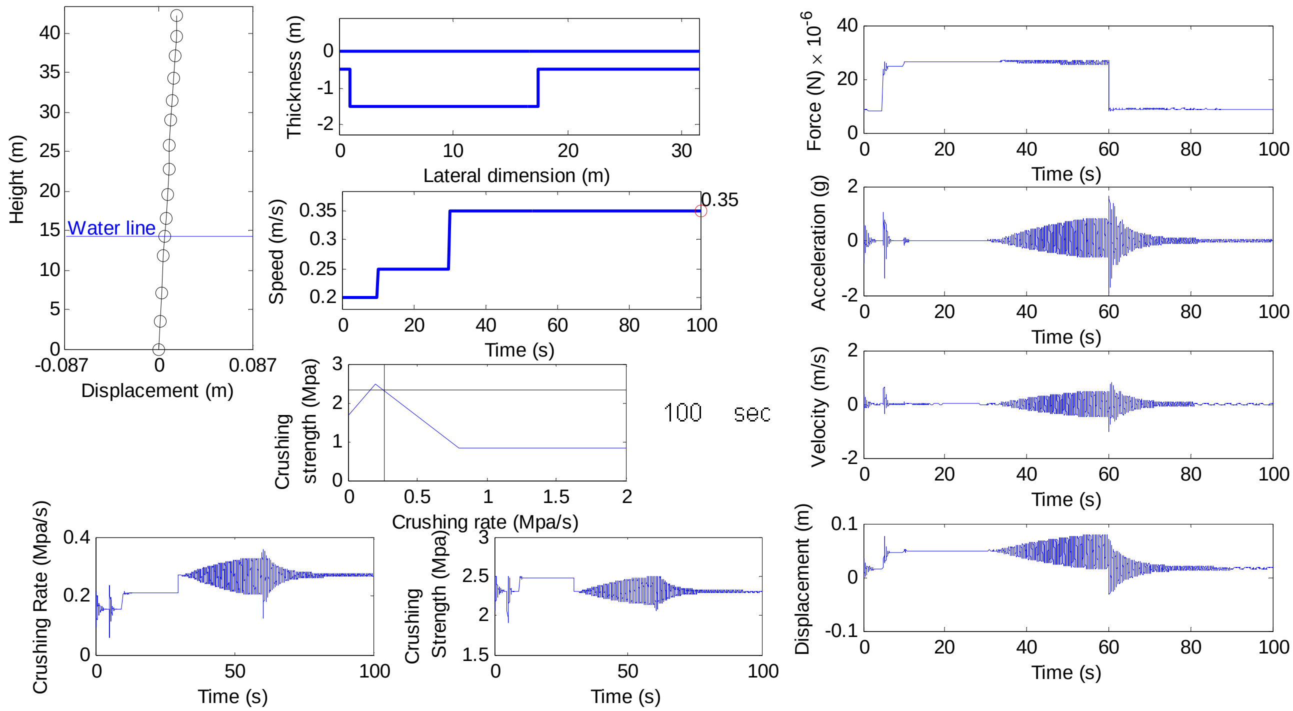


Figure 6 - Response for a variable ice thickness and ice drift velocity profile.

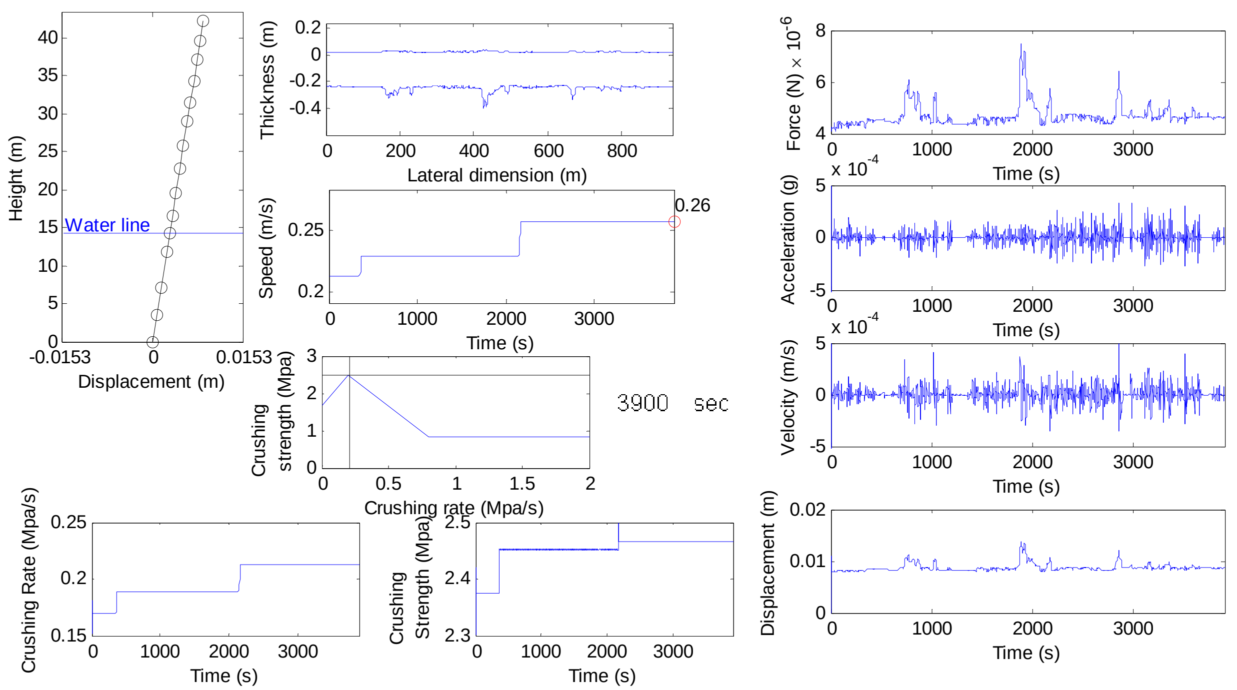


Figure 7 - Response for a recorded thickness and ice drift velocity profile.

OCCURRENCE OF SS-IIV

Some structure-ice conditions lead to SS-IIV and in others to D-IIV in which vibrations decay. A sensitivity analysis was first carried out to investigate the occurrence or lack of SS-IIV. To conduct it in an automated fashion, though, it was necessary to define a criterion to identify SS-IIV while keeping the simulation time reasonable.

SS-IIV Identification Criteria

In this study, we investigated two types of criteria, both based on the fact that system acceleration goes to zero as time goes to infinity in case of a damped IIV, D-IIV:

(a) Compare max acceleration in the vicinity of a prescribed time t_i to a threshold value (Figure 8). The approximation will converge to better results as time length is increased.

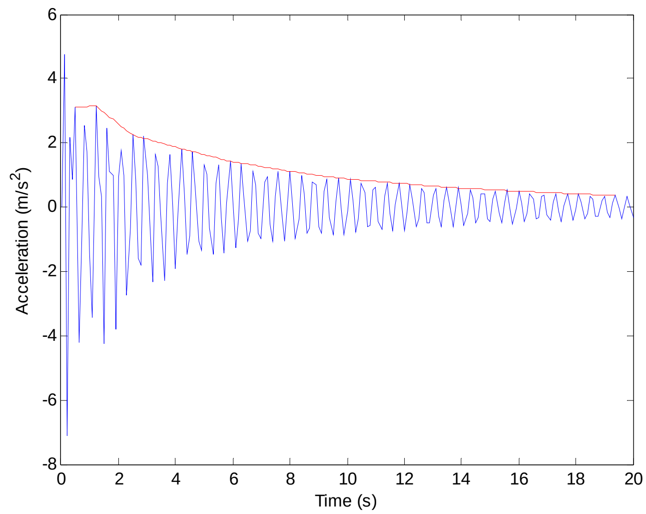


Figure 8 - Maximum acceleration at a prescribed time.

(b) Mathematically extrapolate the envelope of acceleration response and see if it decays, i.e. has a negative slope with time (Figure 9).

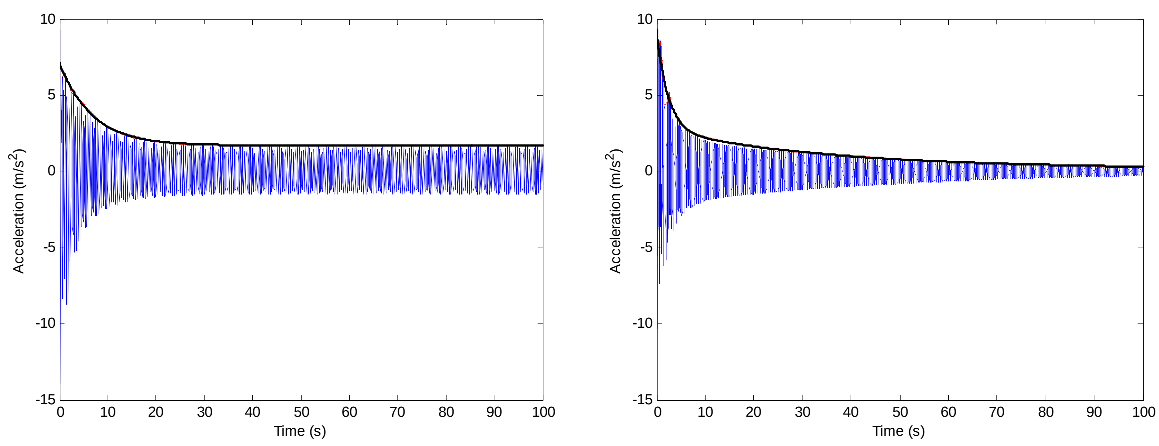


Figure 9 - Extrapolation of acceleration envelope.

Sensitivity Analysis

A sensitivity analysis was then conducted using the above mentioned criteria with respect to mass, stiffness and damping. Figure 10 shows the SS-IIV acceleration values as the mass matrix of the lighthouse model is multiplied by the x-axis factor of the figure. It is seen that an increase in mass of the structure reduces its vulnerability to SS-IIV; a reverse effect was observed with increasing stiffness. However, SS-IIV was observed to be most sensitive to structural damping (Figure 11), i.e. increasing structural damping reduces significantly the vulnerability to SS-IIV.

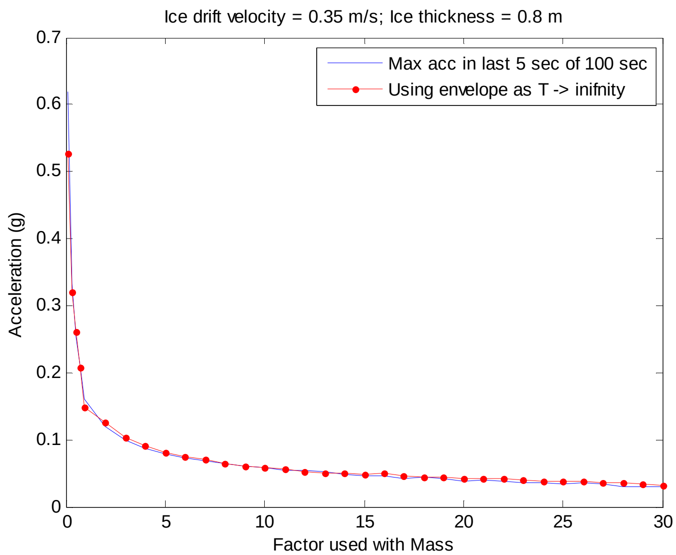


Figure 10 - Sensitivity of SS-IIV to mass of the structure.

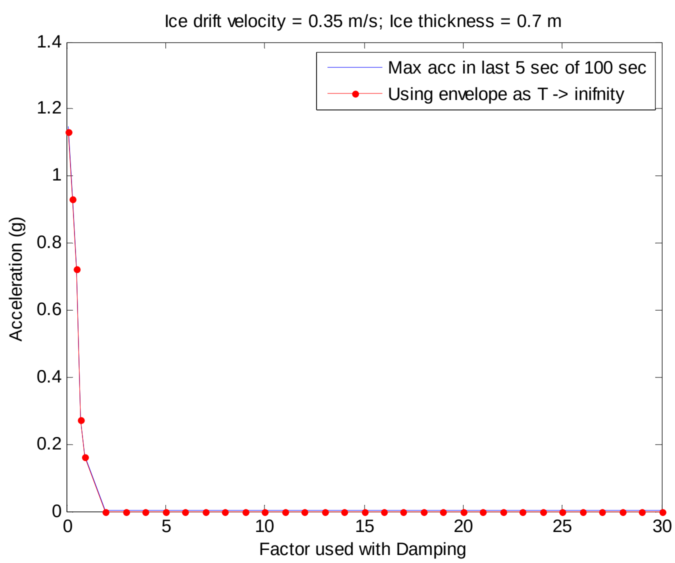


Figure 11 - Sensitivity of SS-IIV to damping of the structure.

The above qualitative trends are not enough to establish design guidance. In the next section, we aim at doing that by seeking an analytical expression to identify occurrence of SS-IIV, the parameters governing it and the sensitivity to changes in these parameters.

Closed-Form Stability Based Criteria for IIV

A damped IIV (D-IIV) response corresponds to a stable response which drives the system to the statically deformed shape, while the SS-IIV response corresponds to an unstable equilibrium position, resulting in a stable response with a limit cycle present around the equilibrium. From this perspective it is natural to analyze the stability of the system around the equilibrium point.

The response of a structure in the vicinity of its equilibrium is governed by predominant modes of structure obtained by linearizing the structure around the equilibrium. Stability of these predominant modes can be checked using the bifurcation theory. For the structure under study first mode is the most dominant mode while in SS-IIV response. Thus the equation of motion for the first mode can be represented as Equation 5:

$$\phi_1^T [M] \phi_1 \ddot{\eta}_1 + \phi_1^T [C] \phi_1 \dot{\eta}_1 + \phi_1^T [K] \phi_1 \eta_1 = \phi_1^T F \qquad (5)$$

Stability of this system which describes the behavior in the vicinity of the equilibrium point is determined by checking the Eigen value of this system. For a stable response, one can obtain:

$$\frac{2\varsigma_1 \omega_{n1}}{m_1} + \frac{\phi_{i1}^2}{m_1} \times 2ah \int_0^{\pi/2} \frac{4\pi a \alpha_j \beta_j \cos^2(\theta)}{\pi a - 4\beta_j (v_o) \cos(\theta)} d\theta > 0 \qquad (6)$$

The governing parameters are: ς_1 the modal damping, ω_{n1} the modal frequency and a the pile radius. Notice that these are in line with the parameters identified in the sensitivity analysis. Equation 6 serves as an analytical condition for defining the region where the structure will have a damped response. Figure 12 shows a contour plot of this analytical condition, the area in white represents the stable response region, shaded areas represent SS-IIV, and contour values represent the value of the left hand side of equation 6. Superimposed on the figure are simulation results: red dots represent SS-IIV occurrences while green dots correspond to those without. Notice the close match except for few points due to the boundary first mode approximation.

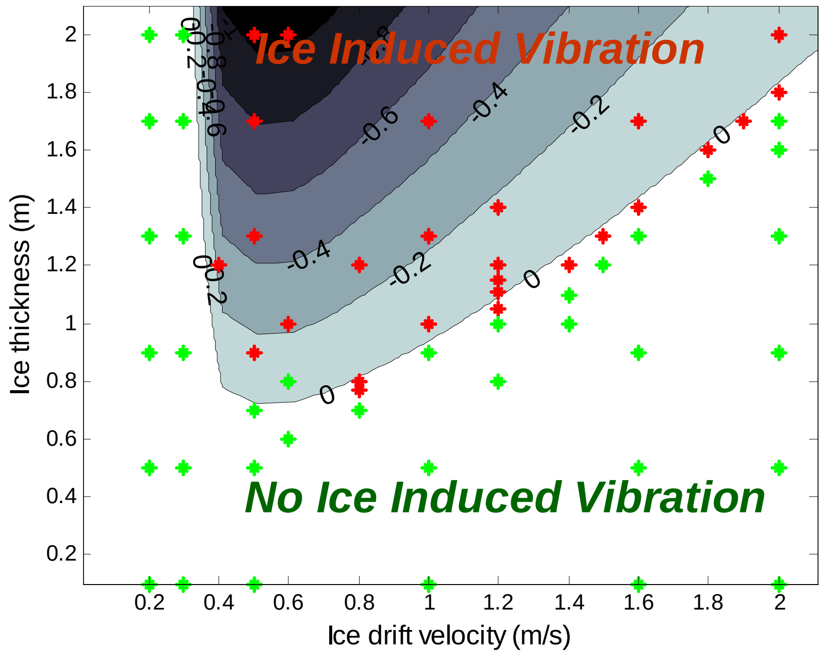


Figure 12 - Analytical boundary superimposed with simulation based results.

The influence of governing parameters on the boundary was also studied. For example, Figure 13 shows the effect of the first modal damping: increasing damping reduces SS-IIV susceptibility.

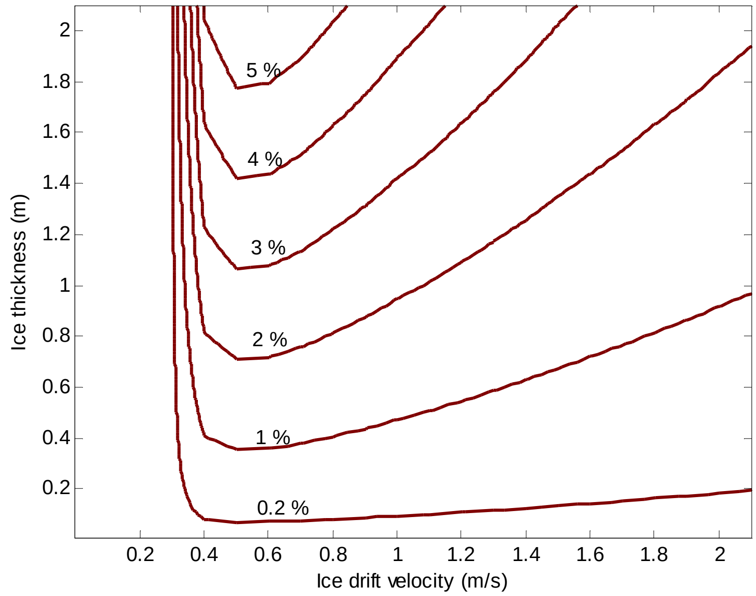


Figure 13 - Effect of modal damping on SS-IIV boundary.

CONCLUSIONS

Määttänen's model was used to study ice induced vibration of a lighthouse. The model can incorporate variable ice thickness and drift velocity profile, and enables analyzing the response of structures with cylindrical cross-sections at the waterline. We used the model to formulate a closed form stability criterion that can provide contour lines to define a boundary for ice-structure conditions conducive to SS-IIV. Parameters governing this boundary were used to guide structure design away from SS-IIV, and the sensitivity of this boundary to these parameters was investigated. Because the analysis parameters for both ice and structure are defined at a general, "high-level", this approach can be applied at an early stage of the design process.

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