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DEVELOPMENT OF ICE FAILURE AND YIELD CRITERIA FOR ACCIDENTAL LIMIT STATE DESIGN

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ABSTRACT

Ships and offshore structures operating in Arctic waters are typically designed for ultimate limit states based on ice actions with annual frequencies of occurrence in the range of 10^{-2}. However, it is also often required to assess structural resistance to more rare ice actions (with an annual probability of 10^{-4}) such as iceberg collisions and impacts. In these cases, the structure may undergo severe deformation while maintaining the necessary global stability and cargo containment. These structures are designed using the Accidental Limit State (ALS) method, which requires integrated nonlinear finite element analysis of both the impacting ice object and the structure itself. An implementation of such analysis calls for advanced continuum mechanical models of the ice material (including ice fracture). This paper reviews the state-of-the-art in ice material models that can be used for iceberg impact simulations. Emphasis is placed on the manageability of the model, the need for calibration and the uncertainties associated with fitting failure criteria to laboratory strength tests. The paper highlights the problem of ice data interpretation and describes how the current understanding of ice fracture facilitates a realistic ALS approach for ships and offshore structures.

INTRODUCTION

Attention to the Arctic region is increasing as never before. With the development of the Arctic regions, the increase of marine transportation in the North and continued climate change, a substantial increase is expected in accidental situations with ships and offshore installations such as collisions and ship groundings. For ships and offshore installations operating in the Arctic, the possibility of accidental collisions of ships or offshore structures with ice masses such as bergy bits or small icebergs should not be neglected. In such scenarios, there is a need to assess at what level of ice action the structure may undergo severe deformations while still maintaining its integrity with respect to global stability. The structure can then be designed using the Accidental Limit State (ALS) method, in which an adequate design is achieved through integrated nonlinear finite element analysis of the ice object as well as the structure. The use of integrated finite element analysis requires a robust iceberg material model (including ice fracture) that can be validated. To design structures resistant to ice impact (with an annual probability of occurrence of $\sim 10^{-4}$), a constitutive model of ice must be incorporated into the finite element code used to simulate a collision between the structure and an ice mass. This constitutive ice model can then be used for the structural analysis. Vinogradov (1987) noted that "no single constitutive model currently in use is able to represent the response of ice in any practical situation".

In the last decade, a great deal of attention has been paid to the development of empirical and phenomenological models describing the behaviour of sea ice, ice rubble and freshwater columnar and granular ice during ice-structure interactions. However, few data are available on the mechanical behaviour of icebergs during collisions with ships and offshore installations. Despite recent development of a number of finite element models of iceberg-structure interactions, only a very few of these (Gagnon, 2007; Lee et al., 2009 and Liu et al., 2010) take into account realistic geometries or the physical/mechanical properties of iceberg ice (e.g., ice strength, temperature variations within the iceberg, or local and global iceberg shape at the point of contact). Lee et al. (2009) and Liu et al. (2010) showed that the consequences of iceberg collisions with floating structures can be predicted using plasticity-based models for both the ice and the structure utilising nonlinear finite element methods. In finite element simulations by Gagnon (2007), an analogy of a foam material model was used to represent the behaviour of iceberg ice during a collision where a ship-shaped structure remains intact after the collision. Mravak et al. (2009) noted the need for a suitable ice-crushing material model to improve upon existing mathematical models and so simulate a real-life collision with an iceberg. Liu et al. (2010) pointed out that there is lack of experimental data related to iceberg-ice material models. In addition, collision scenarios are required to model postfracture behaviour, such as reassembling the broken ice pieces. Often in finite element analysis, element elimination (i.e., the erosion of elements) is used to simulate ice fracture (Liu et al., 2010). An efficient utilisation of ice models gives high priority to understanding the performance limits of the different models. This study focuses on the constitutive behaviour of ice prior to fracture. In this work, attention is paid to the state-of-the-art constitutive ice models (mainly yield/failure envelopes) that can potentially represent the behaviour of iceberg ice during an impact event, during which local crushing is the dominant process.

YIELD/FAILURE CRITERIA

The literature often describes a semi-empirical failure surface. This surface may be treated as a yield surface (Liu et al., 2010). Some yield/failure criteria that have been suggested in the past for ice are now only of historical interest. These proposed yield functions were found to conflict with later experiments in predicting the influence of hydrostatic stress and possible phase changes due to high hydrostatic pressure and temperature. The circumstances of ice yield/failure surfaces are thus unsatisfactory. To investigate this situation, an overview of these past developments will be presented.

Historical overview

Figure 1 summarises past studies focusing on yield/failure surfaces that were used intensively in ice research. These past developments can be divided into four major groups or generations. Many of the strength criteria are well known, so references are only listed for the lesser-known works.

The *first-generation criteria* (1GC), Figure 1, are represented by the simple yield/failure surfaces of Tresca and von Mises for isotropic materials and the Hill criterion for anisotropic materials. In 1GC, the yield strength of ice (failure stress) is not influenced by hydrostatic pressure, i.e., there is no difference between the tensile and compressive yield strengths. To describe yield/failure stress in an isotropic material, only one material property is needed. According to Schulson & Duval (2009), the ductile strengths of plastic, isotropic granular ice and orthotropic columnar ice are governed by the von Mises and Hill criteria, respectively.

The *second-generation criteria* (2GC) are modifications of the 1GC forms and incorporate a linear dependence on hydrostatic stress. For a fully isotropic material, the Hoffman criterion reduces to the Drucker-Prager form. Here, to describe the yield/failure strength of an isotropic material, two material properties are required. According to Fish & Zaretsky (1997), the Drucker-Prager yield criterion is valid for ice under low hydrostatic pressure ($p \ll 49$ MPa for ice at -11ºC and a strain rate of 0.005 s^{-1}).

The *third-generation criteria* (3GC) are represented by the generalisation of the von Mises criterion by Yang (1980) and the elliptical failure surface of Tsai & Wu (1971). 3GC have quadratic stress terms. These criteria can describe materials that show finite hydrostatic stress. Here, three material properties must be provided to describe yield/failure strength.

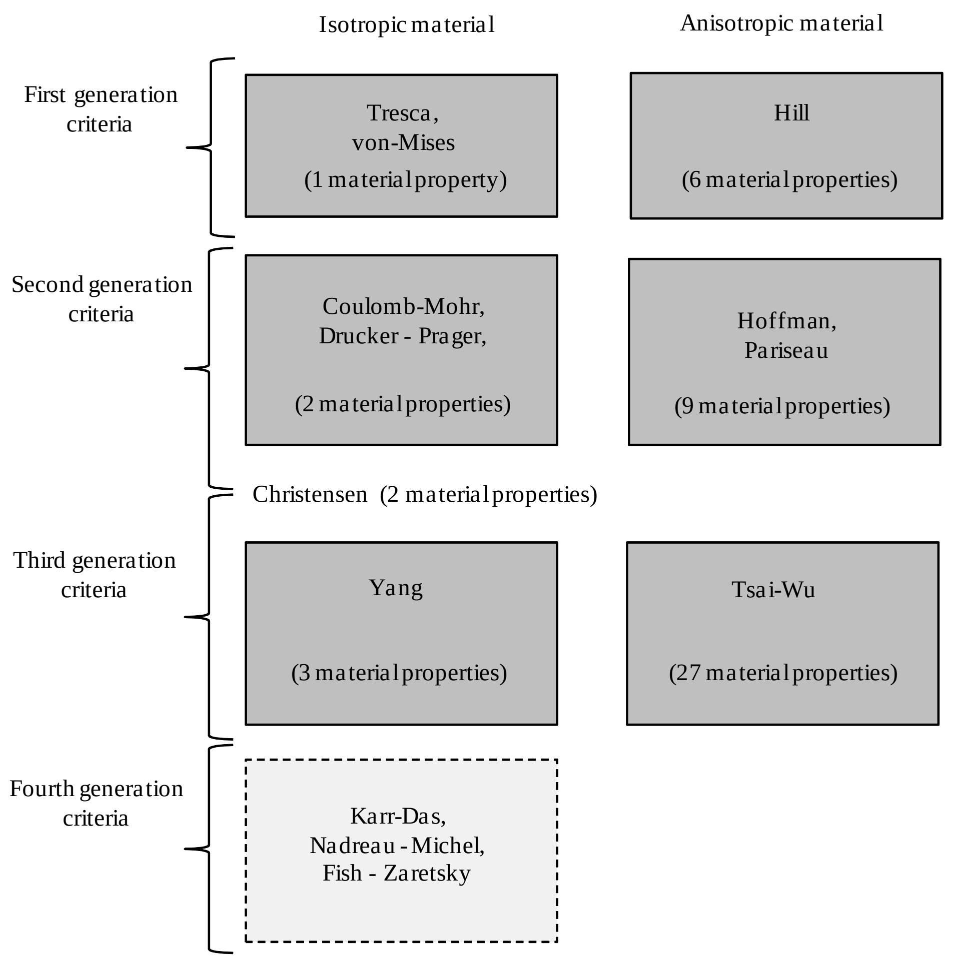

Figure 1. A schematic generalization of the ice yield/failure criteria.

The Christensen (1997) criterion is less known in ice science. This criterion lies on the boundary between 2GC and 3GC. It is a two-parameter yield/failure criterion developed for isotropic materials. Expressed in terms of uniaxial strength values and principal stresses (σ_1, σ_2 and σ_3), the Christensen yield criterion is written as:

$$f(\sigma)=\left(\frac{1}{R_t}-\frac{1}{R_c}\right)(\sigma_1+\sigma_2+\sigma_3)+\frac{1}{R_t}\frac{1}{R_c}\left\{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right]\right\}-1 \quad (1)$$

where R_t and R_c are the uniaxial tensile and compressive strengths, respectively. Later, it will be shown that the Christensen criterion can be used to describe the ductile strength of multiyear ice.

The *fourth-generation criteria* (4GC) are a combination of the 1GC and 3GC forms (Karr & Das, 1983), resulting in teardrop curves having cubic or higher stress terms (Nadreau & Michel, 1986; Fish & Zaretsky, 1997). These criteria can also describe materials that show finite hydrostatic stress. The advantages of using 4GC for the problems of ice impact will be discussed later. Three or more material properties are needed for such criteria.

Recently, Gagnon (2011) suggested a numerical model for ice crushing that could be used in an ice-structure impact scenario. The ice model is phenomenological and represented by a plasticity-based foam model. Gagnon (2011) noted that some model parameters (e.g., yield stress versus volumetric strain curves) do not realistically represent ice properties. Therefore, discussion regarding the physical plausibility of the material constants and manageability of their calibration is not relevant here, and the Gagnon model was omitted from further consideration.

Applicability of criteria

This section focuses only on the yield/failure criteria that can potentially be used in the ALS design procedure. In numerical simulations of iceberg impacts, it is common to consider ice as a polycrystalline mass and treat it as a fully isotropic material. In the present work, only fully isotropic forms of the strength criteria were evaluated. Accordingly, the yield/failure surface was expressed in terms of stress invariants.

Experimental studies of freshwater ice by Jones (1978), multiyear ice by Riska & Frederking (1987) and iceberg ice by Gagnon & Gammon (1995) showed that ice strength has a nonlinear dependence on hydrostatic pressure. The uniaxial strength of freshwater ice in tension is 2–10 times smaller than in compression (recalculated from data in Ashton, 1986). In addition, Mellor (1980) suggested that phase changes due to hydrostatic pressure should also be taken into account. Therefore, the 3GC or 4GC criteria are the most suitable for our analysis.

The expressions in Table 1 have been used by a number of researchers (Reinicke & Remer, 1978; Riska & Frederking, 1987; Liukkonen & Kivimaa, 1991; Kierkegaard, 1993; Horrigmoe et al., 1994; Løset & Kvamsdal, 1994; Liu et al., 2010) to describe ice behaviour (including polycrystalline ice, multiyear ice and iceberg ice) in different situations. It can be shown that criteria № 2–4 in Table 1 are mathematically the same as № 1. Hence, the expressions in № 1–4 can be seen as an isotropic case of the Tsai-Wu surface (Riska & Frederking, 1987; Liu et al., 2010), as a three-parameter function (Reinicke & Remer, 1978), or as a generalisation of the von Mises criterion proposed by Yang (1980). The 4GC criteria for ice are given in Table 2. Although application of these criteria to problems of accidental ice actions was not found in the literature, we believe that some of the criteria listed in Table 2 can be implemented in ALS design.

Table1. Third generation criteria for ice.

№	Yield/failure Criterion	Description
1	$f(\sigma_{ij}) = aJ_2 + bI_1 + cI_1^2 - 1$	This criterion was used by Reinicke & Remer (1978). I_1 is the first invariant of stress tensor, J_2 is the second invariant of the deviatoric stress tensor, *a*, *b*, *c* describe the material properties. Reinicke & Remer (1978) used the experimental data in Jones (1978) to determine the coefficients of the yield function.

2	$f(\sigma_{ij}) = F_{11}I_1 + \frac{1}{3}(G_{1111} + 2G_{1122})I_1^2 + 4G_{1212}J_2 - 1$ $G_{1212} = \frac{1}{2}(G_{1111} - G_{1122})$	This criterion was used by Riska & Frederking (1987). F_{11}, G_{1111}, G_{1122} and G_{1212} are material constants derived from uniaxial, biaxial and triaxial tests.
3	$f(\sigma_{ij}) = a_1 I_1 + F_{ij}\sigma_i\sigma_j - 1$ $a_1 = \frac{1}{R_c} - \frac{1}{R_t}$ $F_{11} = F_{22} = F_{33} = \frac{1}{R_c}\frac{1}{R_t}$ $F_{44} = F_{55} = F_{66} = \frac{1}{R_4^2}$ $F_{12} = F_{13} = F_{23} = 2\left(\frac{1}{R_c}\frac{1}{R_t} - \frac{2}{R_4^2}\right)$	This criterion was proposed by Vershinin et al. (2005) for ice with a randomly oriented crystal structure and for strain rates greater than 10^{-3} s^{-1}. R_4 is the shear strength of ice. The strength terms a_1 and F_{ii} are determined directly from the uniaxial strengths of ice. The remaining parameter $F_{12} = F_{13} = F_{14}$ is the interactive strength term, which must be determined experimentally using combined stress tests. Vershinin et al. (2005) suggest that the interactive term can be specified as a function of the ice strengths, eliminating the need for complex combined stress testing.
4	$f(q,p) = \left(\frac{q}{q_{\max}}\right)^2 + \left(\frac{p-\lambda}{P_c}\right)^2 - 1$ $q = \sqrt{3J_2}$ $p = I_1/3$	The semi-empirical failure criterion was suggested by Derradji-Aouat (2003) for iceberg and freshwater ice, where the stress state $\sigma_2 = \sigma_3$. q is the equivalent stress, p is the hydrostatic pressure, and λ, P_c and $q_{\max}$ are parameters of the failure envelope. For freshwater ice and iceberg ice, the parameters were derived from triaxial experiments (with uniform confinement) by Derradji-Aouat (2003).

i,j=1,2,...6

Table 2. Fourth generation criteria for ice.

№	Yield/failure Criterion	Description
1	$\begin{cases} f(\sigma_{ij}) = 2J_2 + \bar{\alpha}(I_1 + \bar{\beta})^2 - Y^2 \;\; for\; I_1 > -\bar{\beta} \\ f(\sigma_{ij}) = 2J_2 - Y^2 \;\; for\; I_1 \le -\bar{\beta} \end{cases}$	Modified von Mises criterion proposed by Karr & Das (1983) combining the surfaces of von Mises and Yang. $\bar{\alpha}$, $\bar{\beta}$, Y are material constants. The constant $\bar{\beta}$ reflects the level of hydrostatic pressure. The modified Drucker-Prager criterion can be obtained by replacing the Yang surface with the Drucker-Prager function (Karr & Das, 1983). The criterion suggested by Schulson and Duval (2009) can be seen as an improved version of the criterion in

		Karr & Das (1983).
2	$f(\sigma_{ij}) = AI_1^3 + BI_1^2 + CI_1 + DJ_2 - 1$ *or* $f(\sigma_{ij}) = \frac{\tan\theta}{\sqrt{P^* - T^*}}(P^* - \frac{I_1}{3})\sqrt{\frac{I_1}{3} - T^*\sqrt{3}} - \sqrt{J_2}$	Based on the experiments of Jones (1982), Nadreau & Michel (1986) proposed a yield surface for polycrystalline isotropic ice. Kormann & Brown (1990) proposed the modified yield surface. The constants *A*, *B*, *C* and *D* are dependent on ice-melting pressure (P^*), the yield stress in biaxial tension (T^*) and the function tanθ given in Nadreau & Michel (1986).
3	$\tau_i(p,T) = (c_1 + b_1 p) - (c_1 + b_1 P^*)\left(\frac{p}{P^*}\right)^2$ $\tau_i = \sqrt{J_2}$	Fish & Zaretsky (1997) developed a temperature model that describes ice strength in a multiaxial stress state over a wide spectrum of negative temperatures. τ_i is the octahedral shear stress. The ice cohesion, $c_1(T)$, internal friction angle, $b_1(T)$, and $P^*(T)$ are functions of temperature *T*. The model has been verified by Fish & Zaretsky (1997) using test data on the strength of iceberg ice and laboratory-made polycrystalline freshwater ice under triaxial compression ($\sigma_2 = \sigma_3$) at strain rates ranging between 10^{-3} and 10^{-5} s^{-1} over the temperature range between -1ºC and -40ºC.

i,*j*=1,2,...6

Evaluation of yield/failure criteria

A single requirement was used to evaluate the failure functions: *Any three-dimensional failure surface must satisfy a one-dimensional loading condition.* This implies that if $\sigma_2 = \sigma_3 = 0$ and $\sigma_1 > 0$ ($\sigma_1 < 0$), then $\sigma_1 = R_c(R_t)$. This verification was performed for 3GC № 1, 2, and 4 in Table 1 and for 4GC № 3 in Table 2. Table 3 summarises the results of the calculations that performed using the different material parameters available in the literature. In addition, the ice-melting pressure (the ice strength under hydrostatic compression) is given for each set of ice parameters.

Based on Table 3, the failure curve suggested by Derradji-Aouat (2000) for iceberg ice and the failure surface with ice strength parameters in Fish & Zaretsky (1997) significantly overestimates the tensile strength of ice in the case of one-dimensional loading conditions. For ice at a given temperature, one can estimate the magnitude of the ice-melting pressure as $P^* = -T/0.074$, where T = ice temperature (ºC) (Fish & Zaretsky, 1997; Schulson & Duval, 2009). Barrette & Jordaan (2003) determined that at a temperature of -10 ºC, ice will melt if it is exposed to a pressure of ~110 MPa. Therefore, the values of P^* in Table 3 for cases № 1, 2 and 4 are too low. The remaining results given in Table 3 seem to be physically plausible, and we concluded that the failure criteria (including material parameters) given in Riska & Frederking (1987) (for a strain rate of 0.002 s^{-1}), Kierkegaard (1993) and Liukkonen & Kivimaa (1991) satisfy the one-dimensional loading case requirement but underestimate the ice-melting pressure. In turn, the failure criterion (including material parameters) suggested by Derradji-Aouat (2000) for ice at -10 ºC and strain rates >10^{-3} s^{-1} correctly predicts the magnitude of the ice-melting pressure but fails to predict the correct tensile strength. The criterion from Fish & Zaretsky (1997) has a similar behaviour.

Table 3. Ice properties that satisfy 3GC (1, 2 and 4) and 4GC (3): R_c (compressive strength), R_t (tensile strength), P^* (ice-melting pressure).

№	Authors	R_c, MPa	R_t, MPa	P^*, MPa
1	Riska and Frederking (1987) [1],-2 ºC (-10 ºC)	4.4 (8.1)	0.9 (0.9)	7 (11)
2	Kierkegaard[2] (1993)	9.0	0.8	53
3	Derradji-Aouat[2,3] (2000), -10 ºC	9.3	7.3	100
4	Liukkonen and Kivimaa [4] (1991)	8.8	0.9	9
5	Fish and Zaretsky (1997)[5], -11 ºC	14.5	12.9	148.65

[1]for a strain rate of 0.002 s^{-1}; [2]via Liu et al. (2010); [3]for strain rates >10^{-3} s^{-1}; [4]via Kierkegaard (1993);
[5]for a strain rate of 0.005 s^{-1}

We next compared the failure surface for ice (including material parameters) suggested by Riska & Frederking (1987) and the Christensen failure criterion (1) for ductile isotropic materials. The condition of equivalence for those envelopes (i.e., the surfaces are the same when expressed in terms of principal values) can be derived as follows:

$$F_{11} = \left(\frac{1}{R_t} - \frac{1}{R_c} \right) \tag{2}$$

$$\frac{2}{3}(G_{1111} - G_{1122}) = \frac{1}{R_t}\frac{1}{R_c} \tag{3}$$

$$G_{1111} = -2G_{1122} \tag{4}$$

After examination of the parameters G_{1111} and G_{1122} in Riska & Frederking (1987) for different strain rates and temperatures, it was found that Condition (4) is fulfilled for the triaxial tests with multiyear ice at a strain rate of 0.0002 s^{-1}. Therefore, it is possible to use (2) and (3) to obtain values of R_c and R_t by solving a system of equations.

R_c = 2.7 MPa and R_t = 0.3 MPa for multiyear ice at (-2 ºC)

R_c = 3.8 MPa and R_t = 0.9 MPa for multiyear ice at (-10 ºC)

Riska & Frederking (1987) reported the results of experiments on uniaxial compression of ice. For ice at a strain rate of 0.0002 s^{-1}, R_c ~ 2.5 MPa (-2 ºC) and R_c ~ 3.85 MPa (-10 ºC). The value of R_t = 0.9 MPa is in a good agreement with that of Petrovic (2003), whereas R_t = 0.3 MPa seems to be low in comparison with the data reported by Gold (1977), Ashton (1986) and Petrovic (2003). Schulson & Duval (2009) reported values of R_t = 0.2–0.3 MPa at -2.5 ºC for saltwater ice. For second-year and multiyear sea ice, Timco & Weeks (2010) gave the values of R_t = 0.5–1.5 MPa. Petrovic (2003) notes that the compressive strength of ice increases with decreasing temperature and that ice tensile strength is relatively insensitive to temperature variation. According to Schulson & Duval (2009), the strength increases slightly with decreasing temperature for freshwater ice. Saltwater ice exhibits greater thermal sensitivity (Weeks, 1962 via Schulson & Duval, 2009). Taking into account that the values in Riska & Frederking (1987) correspond to multiyear ice (with a salinity close to zero), it was concluded that the values of R_c and R_t (retrieved using Failure Function 2 (Table 1) and the Christensen failure function as an equivalent curve) are physically plausible and in good agreement with the observed experimental values and trends.

As the criteria in Table 1 are mathematically equivalent, it is possible to compare the ice parameters used in the scenario of the penetration of a structure into a large multi-year ice feature used by Riska & Frederking (1987) and the yield-surface parameters used by Liu et al. (2010) to study bergy bit–ship collisions with those in criterion № 3. This comparison can give physical meaning to the coefficients in 3GC № 1, 2 and 4 in Table 1. For ice at -10 ºC and a strain rate of 0.002 s^{-1}, the parameters used in Riska & Frederking (1987) correspond to the parameters in criterion № 3:

$$\begin{cases} \dfrac{1}{R_c}-\dfrac{1}{R_t}=F_{11}=0.988\ \text{MPa}^{-1} \\ \dfrac{1}{R_c}\dfrac{1}{R_t}=0.137\ \text{MPa}^{-2} \\ 2\left(\dfrac{1}{R_c}\dfrac{1}{R_t}-\dfrac{2}{R_4{}^2}\right)=-0.024\ \text{MPa}^{-2} \end{cases} \Rightarrow \begin{cases} R_c=8.1\ \text{MPa} \\ R_t=0.9\ \text{MPa} \\ R_4=3.7\ \text{MPa} \end{cases} \Rightarrow$$

The obtained values of R_c, R_t and R_4 are in good agreement with Ashton (1986) and with Timco & Weeks (2010).

For iceberg ice at -11 ºC and strain rates greater than 0.001 s^{-1}, comparing the data in Liu et al. (2010) with the strength parameters in criterion № 3 (Table 1) reveals that R_c = 9.2 MPa, R_t = 7.2 MPa and R_4= 9.6 MPa. For iceberg ice (at a strain rate of ~10^{-3} s^{-1} and a temperature of 8.0 ºC), Lachance & Michel (1987) reported a maximum value of R_c of approximately 10 MPa. Jones (2007) summarised that the average values of R_t for iceberg ice are in the range of 0.5 to 0.7 MPa. Experimental data for R_4 (iceberg ice) were not found; therefore, the values given in Ashton (1986) for freshwater ice may serve as a reference. In this case, the values of the shear and tensile strengths are significantly overestimated, whereas the value of R_c = 9.2 MPa agrees well with Lachance & Michel (1987). The difference between the calculated and experimental values for R_t and R_4 could be because the assumed interactive term in criterion № 3 (Table 1) is not exactly correct for iceberg ice.

Due to these shortcomings, the problem of assessing proper values for the coefficients of yield/failure criteria for ice masses mentioned by Løset & Kvamsdal (1994) still exists. Three general ways to establish those coefficients can be found in the literature:

Method 1 Experiments with uniaxial and triaxial loading. Schulson & Duval (2009) included an overview of these experimental methods.

Method 2 Experiments with uniaxial compression and tension of ice and additional experiments with pure shear.

Method 3 Experiments with uniaxial compression and failure/yield strength under hydrostatic compression and tension. This is typically a set of input data for the finite element software package ABAQUS. Here, one may assume that the failure/yield strength in hydrostatic tension is approximately 1–10% of the strength in hydrostatic compression. The values of ice strength under hydrostatic compression (or the ice-melting pressure) are suggested in Fish & Zaretsky (1997) as a function of temperature.

In addition to these methods, one could also perform experiments with biaxial loading of ice. These testing methods are compared in the following section. Horrigmoe et al. (1994) indicate that “uniaxial tension tests should always be used, and the confining pressure used in confined-compression tests should be relatively high”.

DISCUSSION

In this section, the different ice failure/yield criteria are discussed and recommendations for which criterion to use in ALS design are given. Special attention is paid to the possibility of calibrating the coefficients used in failure/yield criteria. Table 2 illustrates the main findings of the previous section.

Table 2: Ice failure/yield envelopes.

Ice failure/yield envelope (values of material parameters were included)	Physical plausibility
Riska & Frederking (1987)	good
Kierkegaard (1993) via Liu et al. (2010)	good
Liukkonen & Kivimaa (1991) via Kierkegaard (1993)	good
Derradji-Aouat (2003), 3GC №4	nonconservative[1]
Fish & Zaretsky (1997)	nonconservative[1]/conservative[2]
Karr & Das (1983)/Schulson & Duval (2009)	promising[*]
Vershinin et al. (2005)	good[*]

[1]problems where tension predominates; [2]problems with predominating compression; [*]uncertain due to lack of data in the reviewed literature.

A form of the Tsai-Wu failure envelope as found in Riska & Frederking (1987), Kierkegaard (1993) Liu et al. (2010), Liukkonen & Kivimaa (1991), Derradji-Aouat (2003) (3GC №4), Vershinin et al. (2005) and Løset & Kvamsdal (1994) can be split in two groups based on the values of the constants: Group I includes those criteria which have a relatively low value for the hydrostatic compressive strength (Riska & Frederking, 1987; Kierkegaard, 1993, Liukkonen & Kivimaa, 1991 and Løset & Kvamsdal, 1994). Group II includes those criteria that assume relatively high values for ice tensile strength (Liu et al., 2010 and Derradji-Aouat, 2003 (3GC №4)). The criterion proposed by Shulson & Duval (2009) seems promising, but more data are needed for the determination of parameters for such a criterion.

Figure 2 illustrates the Schulson-Duval failure surface in I_1–$J_2^{1/2}$ space. Under high hydrostatic pressure, the surface narrows to a point on the I_1 axis. The failure surface may contract or expand depending on the strain rate and temperature, as described by Schulson & Duval (2009).

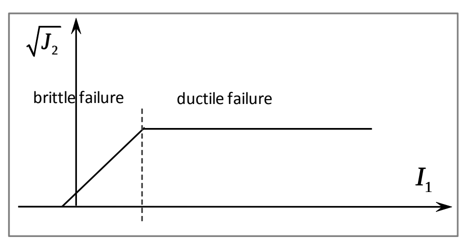

Figure 2. Schematic sketch of the ice failure surface.

In the previous section, three methods (Method 1-Method 3) for determining the parameters for the isotropic case of the Tsai-Wu surface were described. An attempt to use Method 3 leads to a less-conservative criterion than that derived from the triaxial set of experiments by Derradji-Aouat (2003). Additionally, for both the isotropic case of the Tsai-Wu surface and the Fish-Zaretsky criterion, it seems to be impossible to simultaneously satisfy both high values for the hydrostatic compressive strength (or ice-melting pressure) and reasonably low values for the uniaxial tensile and compressive strength. An attempt to decrease the uniaxial tensile strength to physically plausible values and at the same time maintain an ice-melting pressure of approximately 100 MPa leads to a substantial increase of the maximum allowed octahedral shear stress. Thus, it can be suggested that before using the criteria in Table 1 or Table 2 (№ 3), one might decide which part of the yield/failure curve is to be used in the future calculations and focus only on the calibration of either accurate values of ice tensile strength or ice-melting pressure.

The initial/residual internal stresses (prior to the impact event) in ice resulting from buoyancy or temperature variations are not known. Therefore, it is difficult to say anything about the initial stress state of the ice, nor the level of its confinement prior to impact. Some parts of the ice may be under confinement prior to impact and therefore brought into a plastic state. It is possible that a rapid supply of external energy to such a state could produce the pressure-melting phenomenon/recrystallisation mentioned in Melanson et al. (1999). In contrast, it could be argued that the multiaxial loading of ice occurs rapidly during impact conditions. According to Melanson et al. (1999), strain rates associated with such interactions in the field can approach 1 s^{-1}, and consequently ice would not have time to yield plastically and would instead fail in a brittle mode. In this case, the effect of the conversion of internal energy into heat or phase changes of the ice will be less important. Applying the first law of thermodynamics and neglecting the rate of work provided by heat flux, it could be assumed that the ice would fail by splitting rather than due to pressure melting.

Based on the previous discussion, freshwater ice, iceberg ice and multiyear ice may be subject either to the material parameters derived by Riska & Frederking (1987), where ice strength (yield stress) under hydrostatic compression is lower than the ice-melting pressure, or to the parameters given in Fish & Zaretsky (1997) and Derradji-Aouat (2003). For iceberg ice and freshwater ice, the parameters provided by Derradji-Aouat (2003) give relatively high tensile strengths. To improve this failure criterion, one might add a tensile cut-off pressure (see, e.g., Liu et al., 2010).

CONCLUSIONS

Several ice failure/yield criteria were compared and evaluated, with an emphasis on iceberg and freshwater ice, focusing on further incorporation of these criteria into the ALS design procedure. Seven different ice yield/failure criteria developed in the period of 1978 to 2010 were analysed and compared. The Christensen failure criterion (for an isotropic ductile material) agrees well with triaxial sets of experiments by Riska & Frederking (1987) for multiyear ice under a strain rate of approximately $2 \cdot 10^{-4}\ s^{-1}$.

Three methods that can be used for calibration of the ice failure criteria were listed. Each of these was evaluated with respect to the related uncertainties. For ALS analysis, it is recommended that the failure criterion described by Schulson & Duval (2009) can be used. To avoid the occurrence of a nonconservative failure envelope, we recommend the use of experiments that combine uniaxial, biaxial and triaxial loading rather than experiments combining uniaxial compression, tension and pure shear.

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