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A Probabilistic Model for Estimating Ship Performance in Ice Conditions

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ABSTRACT

During the winter months, ice conditions can significantly impact ship performance, posing challenges for efficient navigation. Traditionally, deterministic models, such as semi-empirical approaches, have been used to estimate ship performance in ice, resulting in non-linear dependencies between ship's speed (v) and ice thickness (h), i.e. so-called h-v curve. However, these models often fall short in capturing the inherent variability and uncertainty associated with complex ice conditions. Given the complex and unpredictable nature of ice, there is a motivation to adopt probabilistic modeling techniques that can account for these uncertainties. As a preliminary exploration, this study models ship transit speed in ice using the h-v curve guided Gaussian Process Regression (HGP), providing a probabilistic relationship between ice conditions and ship speed. The proposed model is applied to independent navigation trips of merchant ships, and the outputs include the mean transit speed and the uncertainty of each estimation. Mean absolute error and the root mean squared error are used to evaluate the effectiveness of this approach. The output of the HGP is compared with that of the standard Gaussian Process model and the recorded ship speed, highlighting the need for combining the physics-based guidance with the data-driven insights for better supporting ship performance analysis in ice. The proposed approach could be further developed e.g., by refining the integration of physics-based guidance with data-driven approach, incorporating additional variables to represent ship maneuvers, and integrating observed ice data to mitigate inherent data uncertainties.

KEY WORDS: Ice-going ship; Ship performance estimation; Probabilistic model; Ice conditions; Winter navigation

INTRODUCTION

Ship attainable speed in ice-covered waters is influenced by a complex interplay of factors, including ice conditions, ship characteristics, and operational strategies. Accurately predicting ship speed in varying ice conditions is essential for optimizing navigation efficiency and supporting icebreaker assistance planning.

A considerable number of studies have focused on investigating ship performance (attainable speed) in ice. There are deterministic, probabilistic, and machine learning approaches to predict the ship speed in ice. The deterministic approach models ship speed (v) as a function of level ice thickness (h), forming an h-v curve. This curve is defined by establishing an equilibrium between the ice resistance and propeller net thrust. It relies on the understanding of fundamental principles, including ship hull-ice interaction, ice icebreaking process, bending theory, crushing procedure, and ice properties (e.g., Lindqvist et al., 1989; Su et al., 2010; Külaots et al., 2013). Although the h-v curve is based on simplified assumptions about ice resistance and ship-ice interaction, it remains a widely used method due to its foundation in physical principles and its ability to theoretically explain how ship speed varies with ice thickness. Kulkarni et al. (2024) derived h-v curves for candidate ships by systematically matching them with reference ships that had known h-v curves to simulate the traffic in the Baltic Sea. The findings indicate that while the h-v curve effectively captures the general performance trends, its accuracy varies from case to case, highlighting the need for improvement in the approach used to represent ship speed in ice.

There has been emerging interest in probabilistic approaches and machine learning models for predicting ship speed under different ice conditions. These approaches can learn from recorded ship performance data, capturing complex patterns that may not be explicitly accounted for in physics-based models (e.g., Montewka et al., 2015; Li et al., 2017; Rao et a., 2021; Tarovik et al., 2024). Machine learning models primarily provide the predicted deterministic speed by assessing the input factors, while probabilistic models can provide the probability of a certain speed under given ice conditions, accounting for uncertainties across different ice conditions. However, data-driven approaches are highly dependent on data availability and may lack physical consistency, particularly in data-sparse regions (Shen et al., 2019). Montewka et al. (2019) proposed a hybrid model consisting of semi-empirical model and data-driven model to predict the ship besetment probability. Two sub models are established and used in parallel to compensate each other, covering a wider range of ice conditions and operational scenarios. While this parallel approach leverages the strengths of both models, an alternative is to integrate physics-based constraints directly within a data-driven approach to enhance predictive consistency. This study serves as a preliminary exploration of such an integration for ship speed prediction, capturing its distribution and uncertainties across different ice conditions.

This study adopts a h-v curve guided Gaussian Process Regression, abbreviated as HGP, to model the ship speed across different ice conditions. The h-v curve is derived from the systematic ship performance modelling approach, which is proposed in our previous study (Kulkarni et al., 2024). While it offers the prior knowledge of ship performance in ice, real-world speed measurements often deviate from the predicted h-v relationship due to unmodeled effects, such as ice property variations and complex ship-ice interactions. To account for the variations, HGP is applied to the residuals, which represent the differences between observed ship speed and the predictions from the h-v curve. Afterwards, the final prediction is the combination of the residuals predicted by HGP and the derived speed from h-v curve. As a preliminary study, the findings can provide insights into the potential of hybrid modeling for ship performance prediction in ice. Future improvements could focus on refining the integration of physics-based guidance with data-driven approach, incorporating additional variables to better capture ship maneuvers, and leveraging observed ice data to reduce inherent uncertainties.

METHODOLOGY

This study aims to adopt a GP model guided by h-v curve (HGP) to model the ship speed under varying ice conditions. The methodology consists of two stages: (1) establishing a semi-empirical ship performance model (h-v curve) to provide a baseline speed estimate and quantify speed residuals, which are the differences between recorded ship speeds and the model estimates, and (2) training the HGP to learn the residual variations and improve speed predictions. The output of the HGP will be compared against the recorded speeds and a standard GP model output, offering insights into the effectiveness of integrating physical constraints into data-driven models for ship speed prediction in ice-covered waters.

Semi-empirical ship performance model

To establish a physics-based baseline for ship speed prediction, this study adopts the method proposed by Kulkarni (2024) for deriving the h-v curve. The curve accounts for the effects of equivalent ice thickness in determining ship performance. Only the relevant information is presented in this study. The detailed information about the method can be found in Kulkarni (2024).

Deriving the h-v curve for candidate ships in ice is challenging due to the unavailability of hull angles for ice resistance estimation and the lack of information on actual power usage, which affects attainable speed. To address this, a systematic similarity-matching approach is used, where the speed performance of a candidate ship is estimated by comparing it with a database of reference ships (Kulkarni et al., 2024). This database consists of ships with known h-v curves obtained through theoretical calculations, model-scale experiments, or full-scale measurements. The matching process considers key ship characteristics that influence ice resistance and propulsion performance, including ice class, ship type, power to deadweight ratio, and attainable open water speed. These parameters help identify ships with comparable operational behavior in ice-covered waters. By identifying a reference ship with similar characteristics, the corresponding h-v curve is matched to the candidate ship, assuming full power operation.

The h-v curve represents the ship's ability to break through level ice. However, the real-world ice conditions are far more complex. Ice fields vary in concentration and floe size, with some areas containing ice ridges during the deformation process. The ice can also be deformed under pressure, forming ridges. Equivalent ice thickness is a convenient way to represent complex ice fields into a single level-ice thickness. The equivalent ice thickness is computed as shown in Eq. (1) according to Kulkarni et al. (2024).

$$h_{eq} = c(h_{level} + 0.082h_{sail}^2)(1 - \exp(-\frac{d}{100})) \quad (1)$$

Where c denotes ice concentration. h_{level} represents the level ice thickness, and h_{sail} represents the sail height. d denotes the ice floe diameter in meters. $(1 - \exp\left(-\frac{d}{100}\right))$ presents a correction coefficient as a function of ice floe size to be included in the equivalent ice concept. The magnitudes of the coefficient are tabulated in Table 1. When the floe size equals 500m, the coefficient reaches 0.993, while the resistance is almost the same as the level ice (Kulkarni et al., 2024).

Once the h-v curve is matched and the equivalent ice thickness is computed, the ship speed prediction follows Eq (2):

$$v_h = f(h_{eq}, \theta) \quad (2)$$

Where v_h represents ship speed calculated based on the matched h-v curve. θ represents parameters (e.g., empirical coefficients) in the matched h-v curve. This serves as the baseline speed for the HGP development.

Table 1. The coefficient tabulated as a function of d

$d(m)$	$1 - \exp(-\frac{d}{100})$
5	0.049
15	0.139
35	0295
75	0.528
150	0.777
300	0.950
500	0.993

Gaussian process regression guided by h-v curve(HGP)

To refine the baseline ship speed prediction obtained from the h-v curve, GP regression is applied to learn and model the residuals between the recorded speeds and the baseline estimates. The residuals are defined as Eq (3).

$$v_{gp} = v_{re} - v_h \tag{3}$$

Where v_{re} represents the recorded ship speeds, and v_h is the baseline speed derived from the matched h-v curve. By modeling the residuals, the HGP aims to capture additional variations in ship speed.

The residuals are modeled using the GP regression, where the relationship between the equivalent ice thickness and speed residuals is learned through a non-parametric probabilistic model. The GP is defined as Eq (4):

$$v_{gp} \sim GP(m(X), k(X, X^{'})) \tag{4}$$

where $m(X)$ is the mean function, X and X' represent two different input points from the dataset. $k(X, X')$ is the kernel function. The GP model employs rational quadratic kernel combined with a white noise kernel to define the covariance structure and balance flexibility with robustness. Thus, $k(X, X')$ can be written as Eq (5).

$$k(X, X') = (1 + \frac{X - X'}{2\alpha l^2})^{-\alpha} + \sigma^2 \delta(X, X') \tag{5}$$

Where the first term represents the rational quadratic kernel, allowing variations in smoothness across different scales. The parameter l determines how far points influence each other, while α controls the weighting of large-scale variations. In the second term, σ^2 represents the noise variance, and $\delta(X, X')$ ensures independent noise for different observations. The values of these parameters are set based on characteristics of the dataset to provide a reasonable starting point for the fitting process.

The GP model is trained on the speed residuals using standardized inputs and outputs. Standardization ensures numerical stability and improves fitting efficiency. The model learns the residual patterns based on ice conditions and refines the baseline speed predictions. The final HGP output (v_{final}) is obtained by combining the learned residuals with the baseline speed, as expressed in Eq. (6).

$$\hat{v}_{final} = v_h + \hat{v}_{gp} \tag{6}$$

Where $\hat{v}_{gp}$ is the predicted residual. The final output $\hat{v}_{final}$ provides a distribution of possible speeds for given ice conditions and corresponding uncertainty ranges.

The performance of the HGP is evaluated using mean absolute error (MAE) and the root mean squared error (RMSE) metrics to assess speed estimation performance, as defined in Eqs. (7)-(8), where N represents the total number of observations. Finally, the speed distributions generated by the HGP will be compared against recorded ship speeds and those generated by the standard GP (which operates without h-v curve guidance), offering insights into the impact of integrating physical constraints into data-driven predictions.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|v_{re} - \hat{v}_{final}\right| \tag{7}$$

$$MAE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(v_{re} - \hat{v}_{final}\right)^2} \tag{8}$$

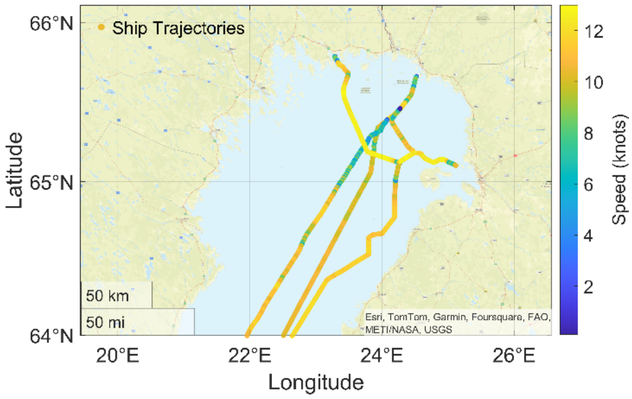
DATA SOURCES

There are three primary data sources to represent the ship performance in ice, including traffic data, ice data, and ship information. The first source, traffic data, is derived from Automatic Identification System (AIS) data provided by the Finnish Transport Infrastructure Agency. It is used to present traffic scenarios, including geographical locations and recorded ship speed. The second data source comes from the Helsinki Multi-category sea-ice model (HELMI), including thickness and concentration of level ice, ridged ice, and rafted ice. The ridged ice thickness is assumed to represent the total thickness, including both the sail and keel. The keel-to-sail ratio is assumed to be 4.85 (Kuuliala et al., 2017; Strub-Klein & Sudom, 2012). Therefore, the sail height is calculated by dividing the ridged ice thickness by 5.85. According to Lensu et al. (2013), the HELMI model has been validated against observational data in many projects for a decade, demonstrating its effectiveness for its intended applications. The third data source is IBNet, a system jointly managed by the FTIA and the Swedish Maritime Administration to coordinate icebreaking operations (BIM, 2020). This system provides ship information, including ice class, ship type, deadweight, breadth, power, and open water speed.

The candidate ships used in this study are general cargo, with their specifications detailed in Table 2. The analysis includes four trips conducted in February 2018, as shown in Table 2. The corresponding figure visually represents these trips, with a color bar indicating recorded ship speeds, where yellow indicates higher speeds and blue indicates lower speeds.

Table 2. Ship information and sample trajectories

	Candidate ships		
	Ship I	Ship II	ship III
Ship type	General cargo	General cargo	General cargo
Ice class	IA	IA	IA
DWT (ton)	6000	8664	6053
Length (m)	110.78	132.20	113.76
Breath (m)	14.00	15.87	14.40
Power (kw)	2640	3960	3000
Open water speed (knot)	13	14	13



Assessing the ship speed in ice-covered waters requires integrating ice data with dynamic ship traffic information. HELMI stores data in a three-dimensional NetCDF format, with variables organized on a fixed grid at a resolution of 1 nautical mile by 1 nautical mile, updated hourly (Haapala et al., 2005). In contrast, AIS data is updated at intervals ranging from a few seconds to six minutes (Liu at al., 2022). To integrate ice data with dynamic traffic data, we assign ice conditions to each point along a ship's route based on the nearest temporal and spatial data points. The detailed information about the integration method can be found in Liu et al, (2024). This approach ensures a location accuracy of approximately one grid cell or better (Lensu and Goerlandt, 2019).

Figure 1 illustrates the recorded ship speed and corresponding equivalent ice thickness over time for four trips. Ship speed is represented in blue on the left y-axis, while equivalent ice thickness is shown in red on the right y-axis. The data reveals that the speed fluctuations align with changes in ice conditions, with notable speed reductions as ice thickness increases. In the first two trips, more significant variations in speed and ice conditions are observed, whereas the latter two trips feature relatively stable speeds and thinner ice. The following analysis is conducted based on this presented dataset.

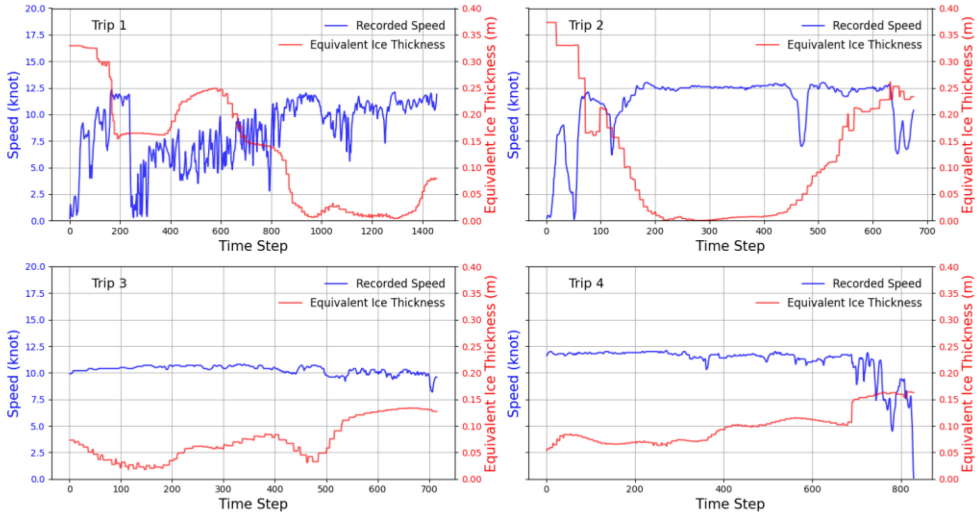


Figure 1. Speed and different ice thickness distributions

There are 3692 data points in the dataset. Since low-speed records may correspond to docking, waiting, or maneuvering near port rather than actual ice navigation, data points with recorded speeds lower than 0.1 knots are removed to ensure the analysis focuses on ship performance in ice-covered waters. After the filtering step, 3676 data points remain for further analysis. Given the variability in recorded ice thickness and speed, the data is subsequently binned based on equivalent ice thickness using a bin size of 0.005 m to reduce noise and facilitate model training. Within each bin, the mean and standard deviation of ship speed are calculated, providing a structured representation of speed variations under different ice conditions while retaining key statistical characteristics of the dataset.

RESULTS AND DISCUSSIONS

Speed distribution based on HGP

According to the methodology section, the h-v curve for candidate ships is derived by matching their characteristics with a reference ship for which h-v curves are available. Given that the candidate ships exhibit similar attributes in terms of ice class, ship type, power to deadweight ratio, and attainable open-water speed (see Table 2), they are assigned the same h-v curve. The obtained h-v curve is obtained under the assumption that ships operate at full power. Any changes in power usage would alter the h-v curve, affecting the predicted ship performance. However, the impact of power variations on ship speed in ice is beyond the scope of this study and is reserved for future work.

Figure 2 shows the recorded ship speeds across four trips, compared against the speeds estimated using the h-v curve. Each subplot corresponds to an individual trip, with recorded speeds represented by scattered markers and h-v curve predictions shown as dashed lines. The results indicate a general trend of decreasing speed with increasing ice thickness, as expected. However, deviations between the recorded speeds and the h-v curve predictions are observed, particularly in trips 1 and 2, where the recorded speeds exhibit greater variability. The matched h-v curve simplifies real-world conditions and does not fully capture complexities like ice properties and ship-ice interactions. Additionally, the ice data is obtained from a forecasting model rather than direct observations, introducing inherent uncertainties. As a result, reasonable deviations between the recorded speed and the h-v derived speed are expected. In the following section, we incorporate the h-v curve as a constraint in a data-driven approach to evaluate whether integrating physics-based constraints can improve the speed estimation under varying ice conditions. Specifically, the speed derived from the matched h-v curve serves as a baseline in the HGP, where the model is trained to learn the residuals between the recorded speed and the h-v derived speed. This approach allows the HGP to capture deviations which are not explicitly considered in the h-v curve, while retaining the underlying physical relationship between speed and ice thickness

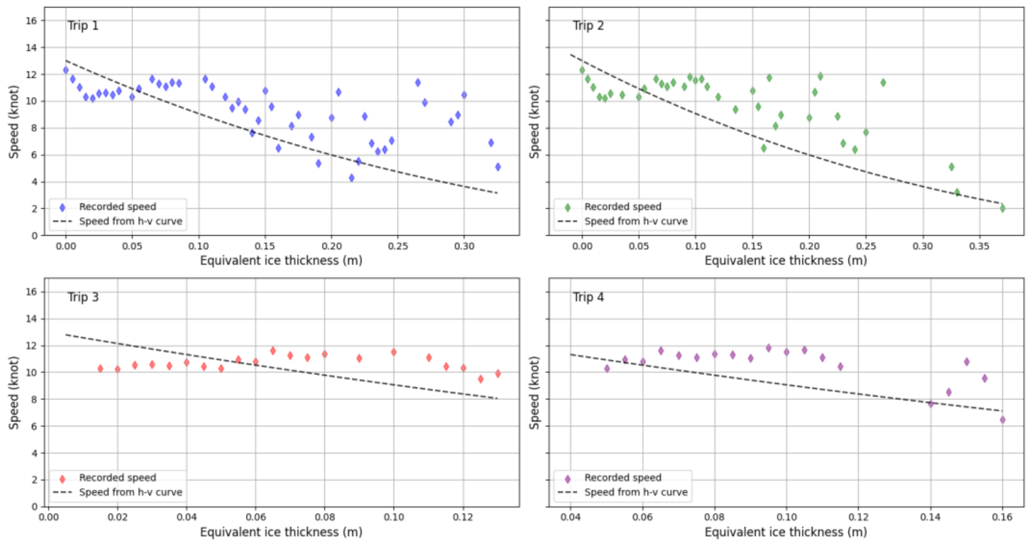


Figure 2. Speed derived from the matched h-v curve

Figure 3 presents the predicted ship speed using the HGP under varying ice conditions based on the full dataset. The recorded speed data is depicted as red dots, while the blue line represents the HGP predictions, with the shaded region indicating the 95% confidence interval. The model achieves a reasonable fit with an MAE of 0.921 knots and an RMSE of 1.426 knots. The HGP effectively captures the overall trend of decreasing speed with increasing ice thickness, demonstrating a close alignment with the recorded speed. The confidence interval remains relatively narrow across most regions, suggesting reasonable predictive reliability. However, despite the physics-based guidance, in regions with sparse data, such as ice thickness beyond 0.3 m, the model exhibits higher uncertainty, reflected in the widened confidence band. Notably, the predicted speed shows an increase when ice thickens from 0.2 m to 0.3 m, which deviates from the expected trend. This anomaly is further discussed in the following comparative analysis.

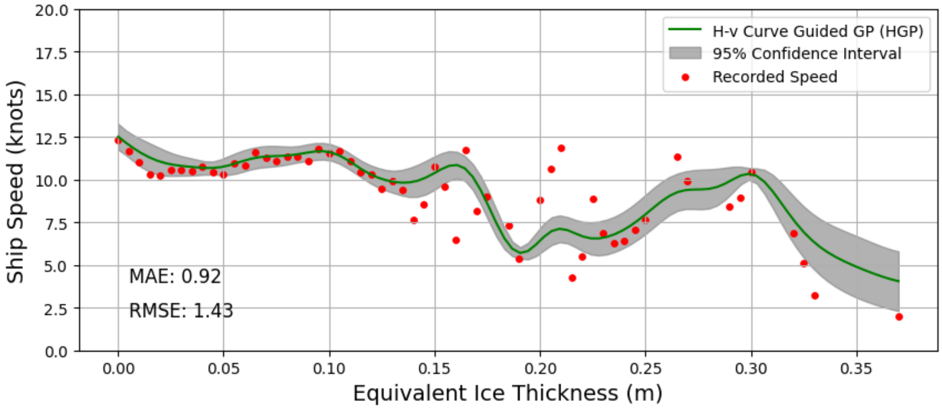


Figure 3. Speed prediction based on HGP based on the full dataset

Comparative results

This section presents the comparative results of the HGP, standard GP, and the recorded speeds. Figure 4 provides an overall comparison of the recorded speeds, HGP, standard GP, and h-v curve across available test data, while Figure 5 further breaks down the predictions for individual trips. This preliminary analysis aims to assess the predictive performance of each approach and identify their respective strengths and limitations in different ice conditions.

As shown in Figure 4, the HGP demonstrates improved predictive accuracy compared to both the standard GP and the h-v curve, achieving the lowest prediction errors (MAE: 0.92 knots, RMSE: 1.43 knots). The h-v curve, derived from semi-empirical equations, serves as a physics-based reference for ship speed under varying ice conditions. However, errors from h-v curve arise from simplified assumptions, such as simplified ship-ice interactions and the omission of ship maneuvering and propulsion variations. The standard GP, on the other hand, is purely data-driven and performs reasonably well where recorded data is dense. However, in regions with sparse data, it struggles to generalize effectively, leading to higher uncertainty, as indicated by the widening light blue confidence interval. When ice thickens from 0.3m to 0.35m, it falls short in capturing a decreasing trend of the speed. This highlights the challenge of relying solely on data-driven models in regions with limited observations, highlighting the advantage of incorporating physics-based constraints as in the HGP. The uncertainty range is notably narrower for the HGP compared to the standard GP, particularly in thicker ice conditions. By incorporating the h-v curve as a guiding constraint, the HGP exhibits improved alignment with recorded speeds, especially when the ice thickness approaches 0.35 m.

It is noticeable that both the HGP and standard GP predict an increase in speed when the ice thickness approaches 0.3 m, which deviates significantly from the speed predicted by the h-v curve. This discrepancy arises from the characteristics of the recorded data and the limitations of the available ice information. In practice, merchant ships can maintain relatively high speeds when navigating through open ice channels compared to level ice conditions. However, the ice data used in this study, derived from the HELMI model, does not explicitly account for the presence of existing ice channels or brash ice. As a result, the data-driven models learn correlations based only on the available input variables. This indicates a fundamental challenge for data-driven models, which are inherently constrained by the quality and comprehensiveness of their training data. To improve predictive accuracy, future research should focus on enhanced data acquisition, incorporating additional ice parameters such as brash ice and navigable ice channels to better represent real-world navigation conditions.

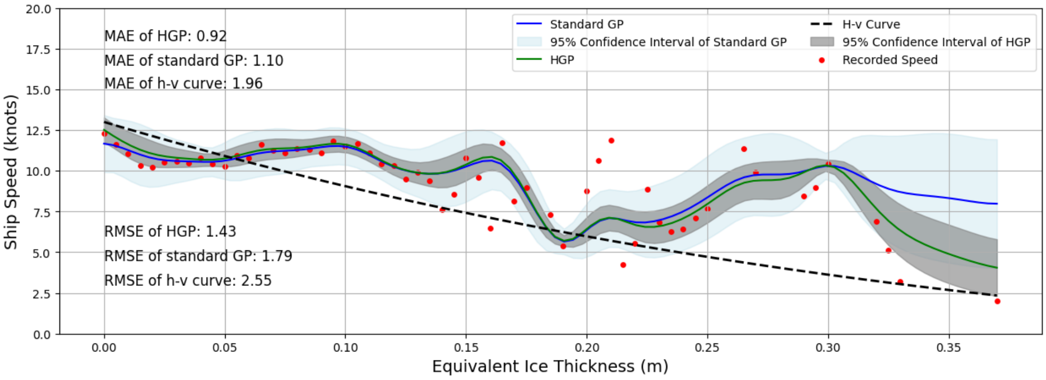


Figure 4. Comparison of recorded and predicted ship speeds based on the full dataset

Figure 5 further presents the predictions results of different approaches across individual trips. The HGP constantly demonstrate good alignment with recorded speeds. The results indicate that the effectiveness of each approach varies depending on ice thickness and data availability. In lighter ice conditions (trips 3 and 4), the standard GP achieves the lowest MAE and RMSE, as the recorded data is dense enough for the model to capture the underlying patterns effectively. However, as the ice thickens (trips 1 and 2), the HGP outperforms the standard GP, benefiting from the h-v curve guidance, which helps reduce uncertainty and improve prediction accuracy where data is sparse. The h-v curve alone shows the largest deviation from the recorded speed across all trips, reflecting the limitations of its simplified assumptions and the underlying uncertainties from the available dataset.

These results illustrate the strengths and limitations of different modeling approaches under varying ice conditions. While purely data-driven models rely on sufficient and diverse training data, physics-based constraints offer valuable guidance but can be limited by simplifications. The HGP leverages physics-based constraints to provide more stable predictions, particularly in thicker ice conditions where data is sparse. However, the effectiveness of the HGP model still depends on the accuracy of the guiding physics-based model, and discrepancies may arise if real-world conditions deviate from its assumptions. Further refinement of the constraints and the incorporation of additional physical variables could enhance the hybrid models, allowing them to better capture the varying ship performance in complex operational conditions.

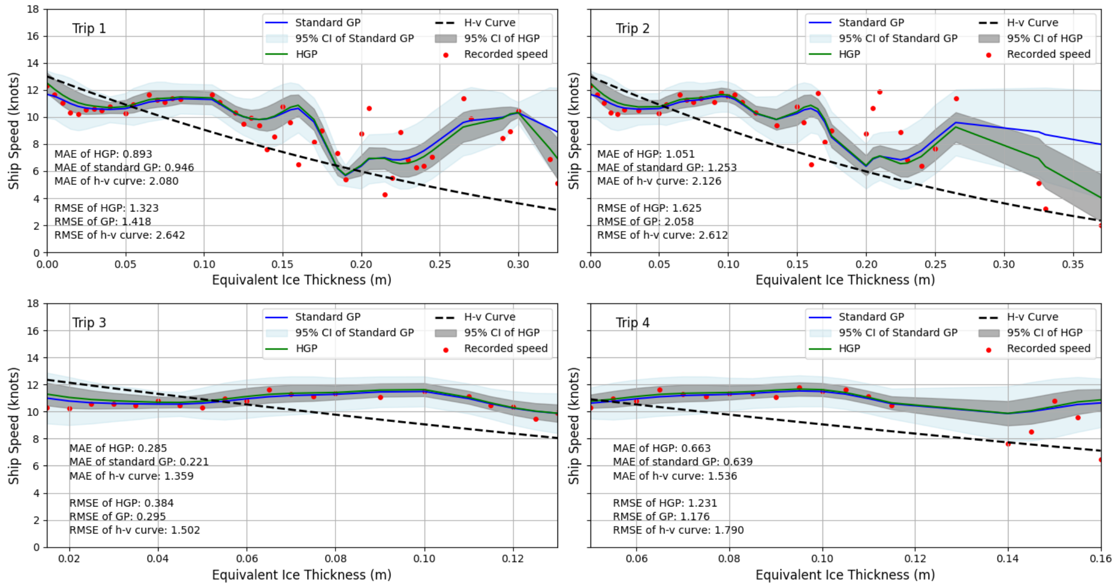


Figure 5. The comparison between the recorded and predicted speed for each trip

CONCLUSIONS AND FUTURE WORK

This study adopted the HGP to predict the ship speed under different ice conditions. The preliminary results indicate the incorporation of physical constraints can provide an opportunity to improve the representation of ship performance trends in ice. Nonetheless, the results remain subject to uncertainties, particularly due to the limited availability of the recorded data, forecasting-based ice data, and the assumptions of the physical constraints used. Future work needs to prioritize data acquisition to enhance the dataset comprehensiveness. Additionally, further research can explore refining the integration of physical constraints with data-driven models and adjusting these constraints using real-world data to improve the representation of ship performance in complex operational conditions.

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