POAC'25
St. John's, Newfoundland and Labrador, Canada

Proceedings of the 28th International Conference on Port and Ocean Engineering under Arctic Conditions
Jul 13-17, 2025
St. John's, Newfoundland and Labrador
Canada

Numerical simulation of continuous brittle crushing ice load based on Weierstrass-Mandelbrot function

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ABSTRACT

Ice-induced vibrations present critical challenges for offshore structures in ice-covered regions. According to observations, continuous brittle crushing (CBR) is the most prevalent failure mode during dynamic ice crushing in sea ice-structure interactions. As stipulated in ISO 19906, CBR ice loads can be represented through a prescribed load spectrum. However, this spectral approach neglects critical aspects of the load's temporal characteristics and its underlying physical interpretation. Moreover, it limits the ability to comprehensively evaluate the adequacy of the resulting loading scenarios. To better characterize the time-domain behavior of CBR ice loads, this study introduces a fractal-based analytical framework. First, the time-varying mean component of the CBR ice load is removed using wavelet packet transform. Second, wavelet packet decomposition was used to extract the fluctuation component of ice load within the target frequency range based on db4 wavelet basis. Then, the time-domain characteristics of the ice load fluctuation in this frequency range are then characterized using the Weierstrass-Mandelbrot function. Finally, the power spectral density spectrum and other statistical parameters of the generated load are compared with those of the original load.The research results show that this method can effectively characterize the time history curve of CBR ice load within a specific frequency range and reflect its time domain characteristics.

KEY WORDS: Ice load; Ice-induced vibration; Ice-structure interaction; Continuous brittle crushing

1 INTRODUCTION

Offshore structures operating in ice-covered waters face persistent challenges from ice-induced vibrations (IIV). The crushing of ice against structures generates complex loads that vary with structural motion, often triggering non-linear responses. Historical incidents from Cook Inlet (Blenkarn, 1970) and Bohai Bay (Yue and Bi, 2000) have demonstrated how these dynamic ice loads create critical conditions for multi-leg jacket structures, affecting both ultimate strength and fatigue performance (Gold and Williams, 1966; Määttänen, 1975).In addition to early observations of wide structures, slender installations like lighthouses and channel markers in the Baltic Sea have also experienced notable vibration issues (Engelbrektson, 1977). Field measurements, particularly the extensive dataset from the Norströmsgrund lighthouse, capture these interactions across varying ice conditions (Jakobsen, et al., 2001; Nord, et al., 2018; Schwarz, 2001). Analyses by Bjerkås (Bjerkås, 2006; Bjerkås, et al., 2007) revealed complex time-frequency patterns in the load signals, highlighting how localized crushing events initiate structural oscillations.

Such field evidence has shaped current engineering guidelines. ISO 19906 (ISO, 2019) defines three regimes of IIV: intermittent crushing, frequency lock-in (FLI) and continuous brittle crushing. Among these, continuous brittle crushing (CBR), is the most prevalent form of ice-induced vibration, it is particularly critical due to its significant contribution to structural fatigue and potential failure (Duan and Gao, 1999; Yue and Bi, 2000).

The ice load have usually been analyzed separately in the time and frequency domains. In ISO 19906, Kärnä and Qu (Kärnä, et al., 2004) followed up the idea of ice loads as stationary random processes and proposed a new spectral method based on measurements from the Norströmsgrund lighthouse which was recommended for IIV analysis of structures.

However, these approaches typically rely on the idealized assumption that CBR ice loads are stationary and follow a Gaussian distribution (Kärnä, et al., 2004). Field data, however, consistently contradicts this, revealing prominent non-stationary and non-Gaussian characteristics; for instance, research by Qu (Qu, 2006) has shown that CBR ice loads are not always Gaussian, and are often better described by an Gamma distribution. These discrepancies can lead to significant inaccuracies when modeling real ice load behavior.

To address these limitations, this study proposes a new modeling framework based on fractal theory. By recognizing the inherent fractal patterns in CBR ice loads, we apply wavelet packet decomposition to separate fluctuation components from time-varying components in the load signals. The Weierstrass-Mandelbrot function (Berry, et al., 1980) then characterizes these fluctuations' self-similar nature across different scales. Through comparisons with field measurements, this approach demonstrates its potential for improved accuracy in capturing both non-stationary behavior and non-Gaussian features of actual ice-structure interactions.

2 METHODS

2.1 Wavelet Packet Decomposition

Wavelet packet decomposition extends traditional wavelet analysis by providing enhanced frequency resolution. Unlike standard wavelet transforms that only split low frequencies, this method decomposes both low and high frequency bands at each level. The decomposition uses paired filters $h_0[k]$ and $h_1[k]$, generating coefficients through:

$$S_{j,\,\mathrm{low}}[m] = \sum_{k} S_{j-1}[k] \cdot h_0[2m-k] \tag{1}$$

$$S_{j,\mathrm{high}}[m] = \sum_{k} S_{j-1}[k] \cdot h_1[2m-k] \tag{2}$$

This process repeats until reaching the target level. The resulting tree structure captures local time-frequency features particularly relevant for ice load analysis. The implementation in MATLAB utilizes the wpdec function from the Wavelet Toolbox, where the decomposition level and wavelet basis are predefined. In this study, the db10 wavelet basis is used to extracted the trend, also known as the time-varying mean components below 0.16 Hz. Then, the components ranging from 0.16 to 8 Hz are extracted from the reconstructed components above 0.16 Hz and are referred to as the fluctuation components.

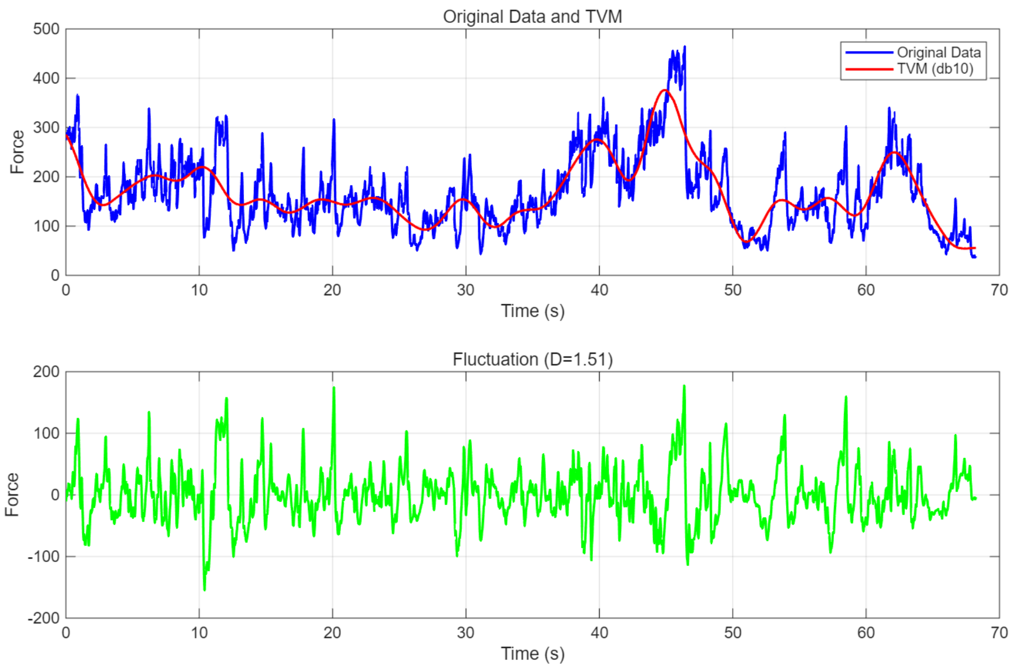

Figure 1. Comparison of the extracted fluctuation component and the time-varying mean component

2.2 Fractal Analysis

2.1.1 Fractal characteristics of continuous brittle crushing ice loads

Previous studies have identified fractal behavior in ice crushing processes. For example, Palmer and Sanderson (Palmer and Sanderson, 1991), Bhat (Bhat, 1990), and Xu (Xu, 2005) observed scale-invariant features in the mechanical fragmentation of ice during crushing events. However, these investigations primarily focused on the fractal nature of the crushing process itself, rather than the resulting ice force time histories. Field measurements of continuous brittle crushing (CBR) ice loads from Norströmsgrund lighthouse (March 3, 2000) show typical fractal characteristics. Qu (Qu, 2006) reported that the time history of CBR ice force demonstrates self-similarity: fluctuations observed over shorter intervals (T_s) resemble those over longer durations (T_m). Magnify the local area of ice load, we will find that it is closer to the overall form. This means that the CBR ice load exhibits the fractal characteristics commonly discussed in mathematics.

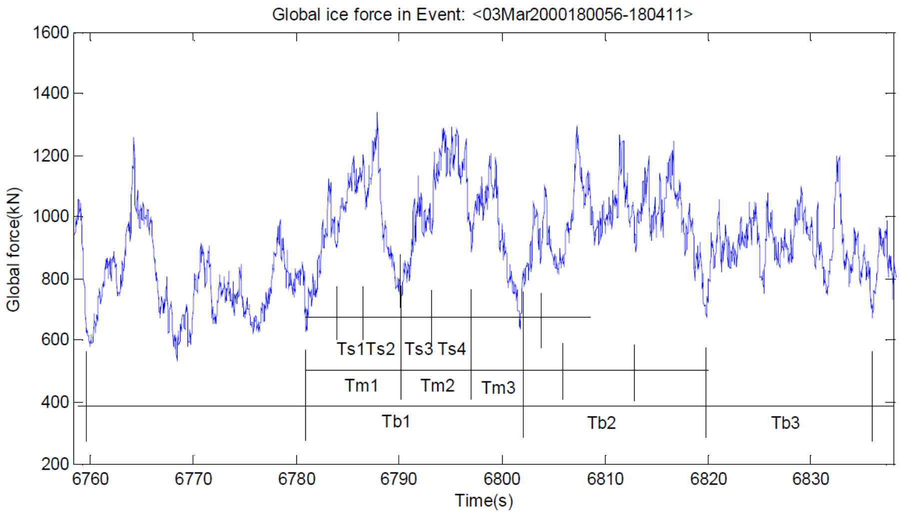

Figure 2. Field measurements of CBR ice load from Norströmsgrund lighthouse (March 3, 2000)

2.1.2 Weierstrass-Mandelbrot function

A common application of fractal theory is in the simulation of rough surfaces, including the fractured surface of rock and and corroded steel structures (Barabási and Stanley, 1995). In this study, the analysis of CBR ice load signals—revealed geometric patterns similar to those produced by the Weierstrass–Mandelbrot (W–M) function. This function is widely used to describe self-affine and scale-invariant features. In this study adopts the stochastic W–M function to simulate the fluctuating component of the ice load.

$$W(t) = \sum_{n=-\infty}^{\infty} \frac{(1-e^{i\gamma^n t})e^{i\varphi_n}}{\gamma^{(2-D)n}}, \tag{3}$$

Formula (3) shows the original form of the W-M function.

$$W(t) = \sum_{n_{min}}^{n_{max}} \frac{\cos\phi - \cos(\gamma^n t + \phi)}{\gamma^{(2-D)\cdot n}} \tag{4}$$

Formula (4) shows the stochastic W–M function, and in this study, n takes a finite value and is calculated as follows:

$$f = \gamma^n / 2\pi \tag{5}$$

f range in this study is 0.16 to 8.00, and the unit is HZ. The fractal roughness parameter γ is suggested to be 1.08 in order to provide dense spectral information.(Berry, et al., 1980). D is the fractal dimension ranging from 1 to 2, calculated by the structure function method in this study. φ *i*s a random phase uniformly distributed on $[0,2\pi]$.

In a finite range of n values, the W-M function consists of a finite number of discrete points, and its power spectrum density is also discrete in a finite frequency range. To obtain a continuous spectrum, the discrete power spectrum needs to be averaged. The power spectral density of the continuous W-M function can be approximately expressed as follows:

$$S_R(f) = A^2 \hat{S}(f) = A^2 \cdot \frac{\pi}{2\ln(\gamma)}(2\pi f)^{-5+2D} \quad (6)$$

where A is the amplitude coefficient, which can be obtained by calculation.

3 ANALYSIS AND RESULTS

3.1 Continuous Brittle Crushing Ice Load Analysis

The ice load data used in this study were collected at the Norströmsgrund lighthouse on March 3, 2000. Details regarding the load panel parameters, including the load panel geometry and inclination angles, can be found in the documentation of the LOLEIF project and related researches (Jochmann and Schwarz, 2000). In this analysis, data recorded between 17:56 and 20:34 were selected, during which nine distinct loading events were identified. Each event includes measurements from nine individual channels of load channels.

To enable comparison with the spectral method recommended in ISO 19906, the analysis adopts a fixed data length of 2048 points per segment. This choice is consistent with the segment length used by Kärnä et al (Kärnä, et al., 2004). in their earlier studies on CBR ice load modeling. The data processing was carried out in the following steps:

- Segmentation of Ice Load Time Series

The recorded ice load signals were divided into fixed-length segments, each containing 2048 data points. Segments shorter than this threshold were exclude.

- Stationarity Testing

The segmented time series were subjected to two complementary statistical tests to assess their stationarity properties: the Augmented Dickey-Fuller (ADF) test and the Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test. Count how many data pass each inspection method

- Wavelet Packet Decomposition (WPD)

Non-stationary segments were processed using wavelet packet decomposition. The initial decomposition employed the Daubechies 10 (db10) wavelet basis to extract low-frequency components below 0.16 Hz which are also interpreted as trend or time-varying mean component. The remaining reconstructed signal component was then further decomposed using the Daubechies 4 (db4) wavelet basis to extract the fluctuation components between 0.16 Hz and 8.0 Hz, which is the frequency range of interest in analyzing.

- Fractal Dimension via Structure Function

The extracted fluctuation components were analyzed using the structure function method to estimate their fractal dimension D.

- Spectral Characterization and Fractal Scaling

A log–log plot of the power spectral density (PSD) of the fluctuation component was constructed. The linear portion of the PSD in the log–log domain was fitted by using the estimated fractal dimension to determine its slope. This slope was based on the theoretical relationship between spectral exponent and fractal geometry (This method is also called power spectral density method) as below:

$$D = \frac{k_D + 5}{2} \tag{7}$$

k_D is the PSD fitted slope.

- Amplitude Calibration of W–M Function

To simulate the ice load fluctuation using the Weierstrass–Mandelbrot (W–M) function, an amplitude scaling factor A was determined. This was done by minimizing the mean square error (MSE) between the theoretical PSD of the extracted fluctuation and that of the W–M function over the target frequency range in log–log coordinates.

- Generation of Fractal Load Component

Using the derived parameters—including amplitude A, frequency scaling factor γ, and fractal dimension D , synthetic fluctuation time series was generated via the stochastic W–M function. The generated signal was then mean-adjusted to remove residual offset and ensure zero-mean conditions for further analysis.

- Time and Frequency Domain Comparison

The original and simulated fluctuation components were compared in both time and frequency domains. Statistical measurements such as mean, maximum, minimum, and variance were computed for both signals. In addition, PSD plots were used to validate the fidelity of spectral characteristics between the measured and simulated data.

3.2 Analysis Results

This study mainly processes and simulates 9 events measured on the Norströmsgrund lighthouse and each measurement covers 9 channels. After segmentation, 261 segments were obtained, with 2048 data points. All 2048-point data segments extracted from the nine loading events were subjected to stationarity testing using both the ADF test and the KPSS test. Both tests were conducted at a significance level of 0.05. The results indicate that only 121 segments passed the ADF test, while none of the segments satisfied the stationarity criterion under the KPSS test. This discrepancy suggests that a substantial portion of the data exhibits non-stationary behavior. This result challenges the assumption of stationarity often adopted in earlier studies.

The fractal dimension D of each segment was estimated using the structure function method. The statistical distribution of the resulting fractal dimensions reveals a minimum value of 1.34 and a maximum of 1.71, with a mean value of approximately 1.51. This average aligns closely with the value reported by Qu (Qu, 2006).

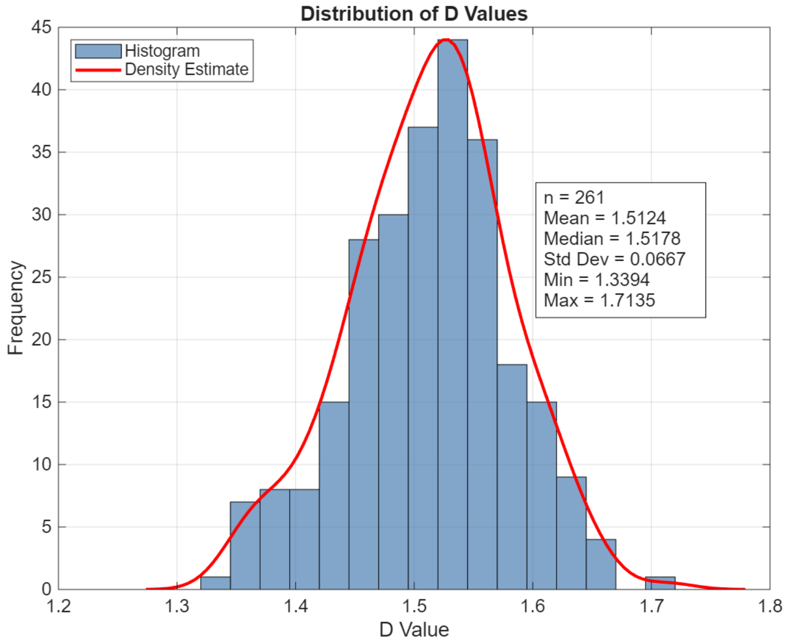

Figure 3. Fractal dimension distribution of extracted fluctuation components

By comparing and analyzing the fluctuation component generated based on the W-M function with the actual extracted fluctuation component, it is found that the time domain curve shapes of the two are highly similar. It can be found that the variance and maximum value of the generated fluctuation term are larger than those of the extracted fluctuation term in Figure 5.

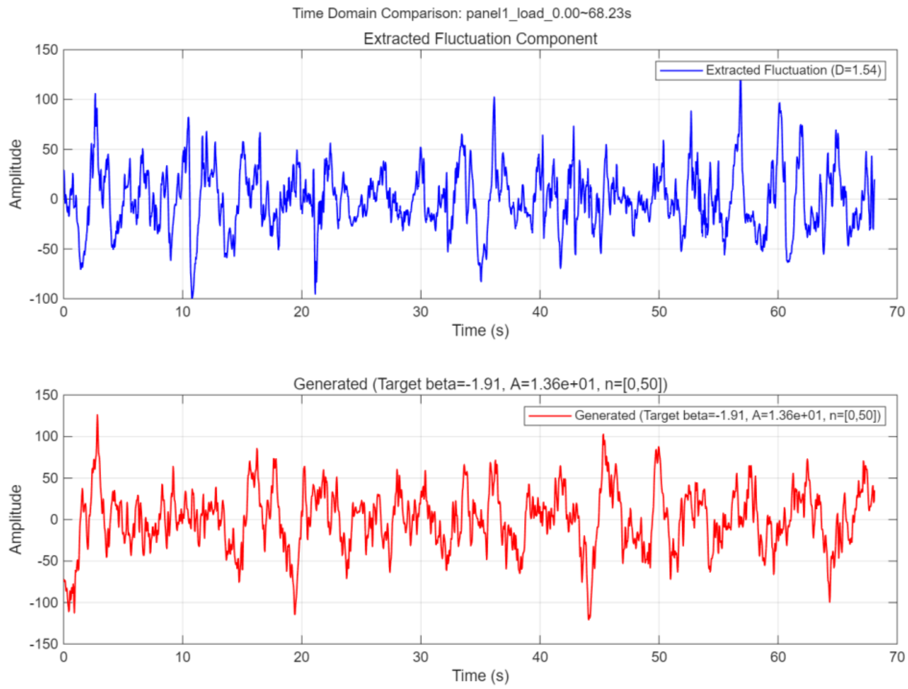

Figure 4. Comparison of the fluctuation component extracted from the measured data with the fluctuation component generated by the W-M function

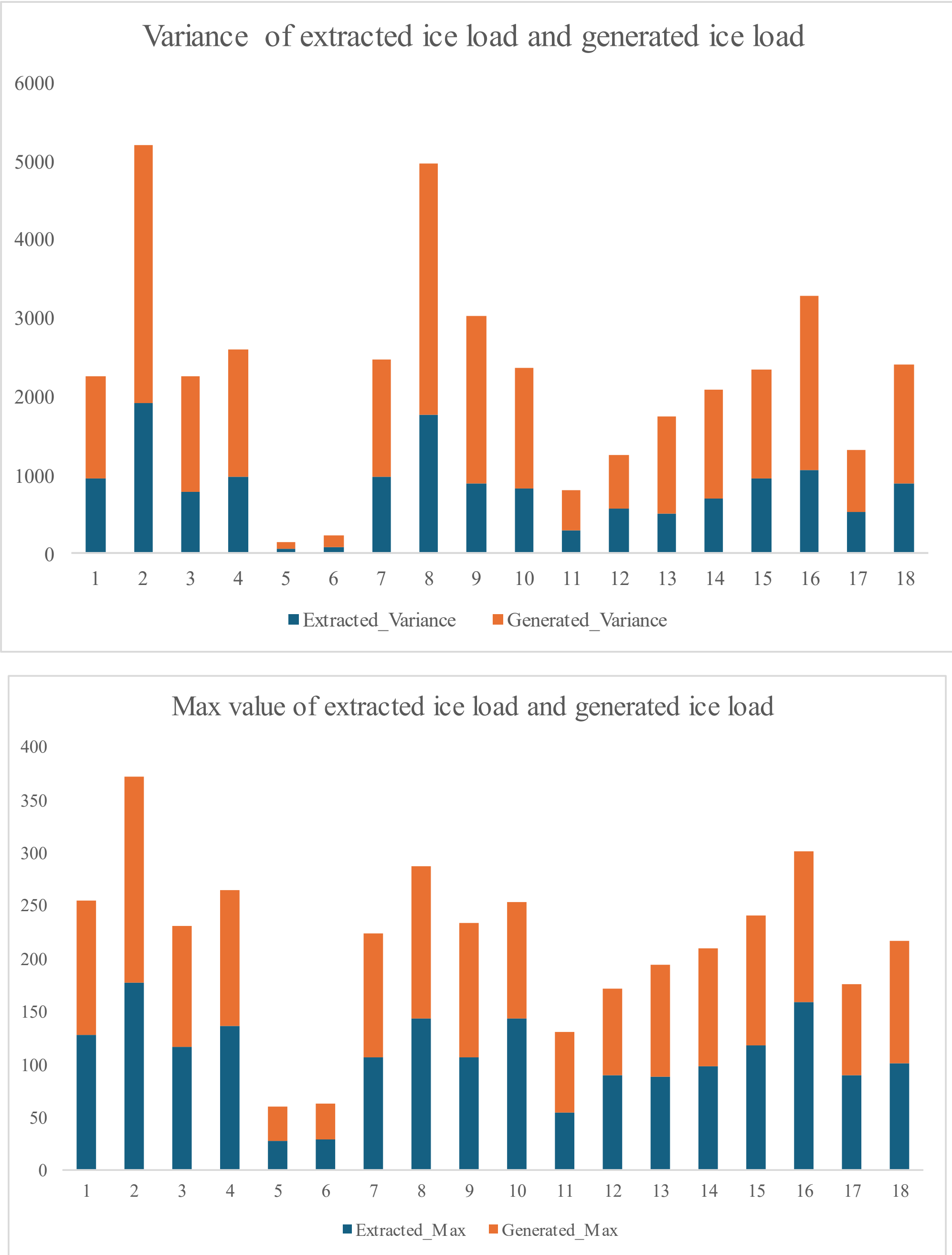

Figure 5. Statistical comparison of generated ice load fluctuation components and extracted ice load fluctuation components: a) variance; b) maximum value

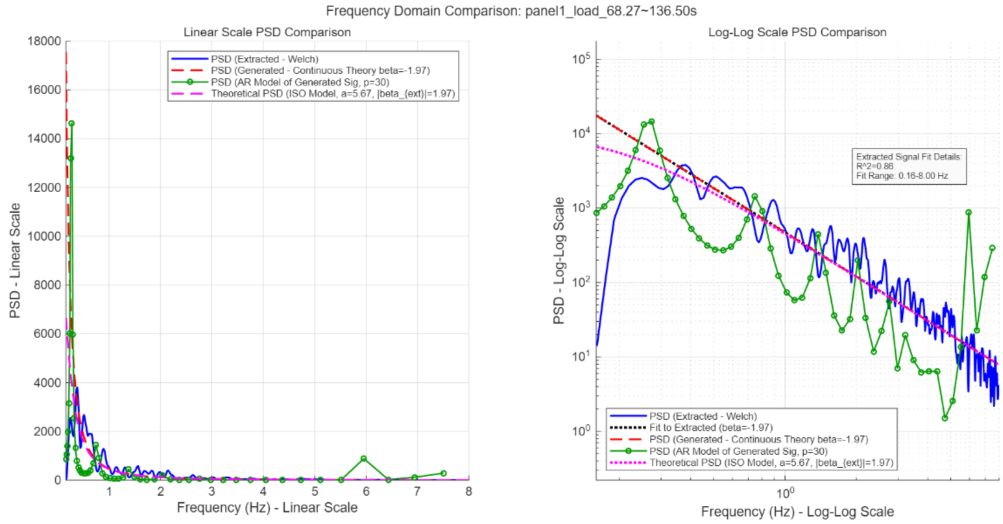

Figure 6. Comparison of the PSD spectrum of the fluctuation component extracted from the measurement data and the fluctuation component generated by the W-M function and the double logarithmic PSD spectrum.

To evaluate the spectral characteristics of the extracted ice load fluctuations and assess the performance of different modeling strategies, four power spectral density (PSD) curves were generated and compared in both linear coordinates and log–log scale plot.The blue curve represents the PSD of the extracted fluctuation component, calculated using the Welch method. This provides a smoothed estimate of the spectral content over the frequency range of interest. A black straight line fitted to this curve in the log–log domain captures its scaling behavior and allows for estimation of the spectral exponent, which is theoretically related to the signal's fractal dimension.

The magenta dashed line corresponds to the spectral model recommended in ISO 19906, which was originally derived by Qu (Qu, 2006). through regression analysis of detrended segments of the LOLEIF project data. The fitting parameters and functional form of this spectral model are consistent with those reported in their study.

The red curve represents the theoretical PSD of a continuous W–M function, derived from its analytical formulation in the frequency domain under specified fractal parameters. In contrast, the green curve corresponds to the PSD of a numerically generated W–M signal, where the function is truncated to a finite number of terms n.As a result of this truncation, the PSD is no longer continuous but exhibits a discrete spectrum. This discrete PSD was estimated using the autoregressive (AR) method, which provides a smooth approximation to the underlying spectral structure despite the finite resolution caused by the limited number of harmonics.

Comparative analysis reveals that both the theoretical and estimated PSDs of the W–M generated signal are consistently higher than those of the extracted fluctuation component, particularly in the low-frequency region (approaching 0 Hz). This discrepancy diminishes with increasing frequency, where the PSD values of the W–M model and the extracted signal tend to converge. The overestimation of low-frequency energy in the W–M-based simulation likely contributes to the observed differences in time-domain statistics. . For example, the maximum values of the simulated signal are higher, and its variance is greater, than those of the extracted component.

This behavior suggests that, while the W–M function is effective in reproducing the scaling characteristics of the fluctuation, it may amplify low-frequency components more than observed in the measured data. The following section will further examine the underlying cause of this spectral bias and its impact on time-domain characteristics.

4 DISCUSSION

4.1 On the Selection of Frequency Range

An essential consideration in generating the fluctuation component of CBR ice loads is the definition of the frequency interval. This interval which is close to the power law decay range reflects a scale-invariant energy transfer process, analogous to the inertial subrange in turbulence theory. In turbulent flows, the inertial subrange refers to the frequency band where energy cascades from larger to smaller eddies without significant energy loss due to viscosity. Similarly, in the case of sea ice, this frequency range characterizes the regime in which load fluctuations follow power-law decay. This indicates energy transfer and dissipation as the ice transforms from larger to smaller sizes.

However, unlike fluids such as wind or waves, sea ice is a heterogeneous material with complex failure mechanisms that cannot be accurately represented by a linear superposition of harmonic functions. As a result, the boundaries of the power-law decay region cannot be derived purely from theoretical assumptions. Instead, a combined approach grounded in both physical reasoning and empirical observation is required.

In this study, an initial frequency range of 0.16 to 8.00 Hz was proposed based on spectral observation across multiple events. While this interval captures the dominant fluctuation band in most cases, fitting results revealed significant variability across different events and even across different load panels within the same event. This variability introduces uncertainty in the subsequent spectral fitting and raises the need for more rigorous criteria, either theoretical or engineering-based to define this frequency range more precisely in future applications.

4.2 On the Discreteness of the W–M Function Spectrum

Another critical issue arises from the inherent structure of the Weierstrass–Mandelbrot (W–M) function. Although its theoretical formulation suggests a continuous power spectral density (PSD) under ideal conditions, this continuity is valid only when the frequency scaling parameter γ approaches 1 and the number of terms n tends toward infinity. In practical implementations, however, the W–M function must be truncated to a finite n, particularly within the limited frequency range relevant to engineering applications. This results in a PSD that is intrinsically discrete.

While the use of the autoregressive (AR) method provides a smoothed approximation of the PSD, it cannot fully eliminate the spectral discontinuities caused by finite truncation. In contrast, the ISO 19906 recommended spectral model is based on Welch method, which estimates the PSD from ensemble averages of measured CBR ice load data. The discrepancy between the discrete nature of the W–M function's PSD and the smoothed PSD derived using Welch's method leads to mismatches during curve fitting in the log–log domain, especially in the low-frequency range where spectral energy concentration is most sensitive.

This mismatch has direct consequences for parameter estimation—particularly for determining the amplitude scaling coefficient A and the frequency scaling factor γ. Therefore, more robust and physically consistent methods are needed to reconcile the PSD differences between measured and simulated signals, which would improve the accuracy of W–M-based models in reproducing realistic ice load fluctuations.

5 CONCLUSIONS

This study presents a numerical simulation approach for modeling CBR ice loads based on fractal theory, with the Weierstrass–Mandelbrot (W–M) function employed to replicate the time-domain fluctuation characteristics observed in field measurements. The key parameters of the W–M function, including the amplitude coefficient A, were determined by minimizing the difference between the theoretical PSD of the W–M function and the double-logarithmic PSD curve fitted from the extracted fluctuation component. The applicable frequency range for the simulation was selected based on the observed power-law decay behavior of ice load spectra within a defined band, analogous to the inertial subrange in turbulent processes.

This modeling framework captures both the self-affine nature and the spectral decay law of CBR ice loads. By superimposing the simulated fluctuation component onto the time-varying mean signal extracted from real measurements, the reconstructed load series retains the essential temporal and spectral characteristics of the original signal. The results demonstrate that the proposed method is capable of bridging fractal modeling concepts with empirical spectral features, offering a more comprehensive representation of ice loading processes.

It is evident that the accuracy of the simulation is highly sensitive to the selection of the frequency range and the theoretical spectrum used for fitting. Therefore, refining the criteria for determining these parameters remains a critical area for improvement.

The method proposed in this paper integrates the energy decay properties and scale-invariant behavior of ice loads, and provides an alternative path for generating CBR ice load .Based on the dataset collected at the Norströmsgrund lighthouse, future work should focus on two key aspects: (1) analyzing the physical mechanisms driving energy transfer within the identified power-law decay range, and (2) investigating the dynamic evolution of load fluctuations in the time domain. These efforts will help to further enhance the theoretical foundation for CBR ice load simulation and support the refinement of design methodologies aligned with ISO 19906, particularly in terms of spectral-based load generation.

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