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Groundwork for modelling propulsion shaft transverse vibrations due to propeller-ice interaction

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ABSTRACT

There has been an increase in maritime traffic in arctic regions in recent years. There is also data that shows that a large proportion of maritime incidents are related to failures of the propulsions system machinery. It thus becomes important to monitor a ship's propulsion system, especially under harsh conditions such as those expected when travelling through ice-covered waters. This work follows from research being conducted on the S.A. Agulhas II polar supply and research vessel. Previous research investigated the development of models for estimating the ice-induced propeller moment using the torsional response of the propulsion shaft. However, this propeller-ice contact would also cause bending loads on the propulsion shaft, which are necessary to quantify to understand their influence on the propulsion shaft bearings. The research presented here seeks to lay groundwork for modelling the transverse vibrations of the propulsion shaft to estimate the input bending loads. This forms part of a larger study to model and monitor the propulsion shaft of the S.A. Agulhas II, expanding the current capabilities to include the monitoring of transverse vibrations and estimations of transverse loads caused by propeller-ice interaction.

KEY WORDS: Propeller-ice interaction; Propulsion shaft transverse vibrations; Numerical modelling; Inverse problem.

INTRODUCTION

The need for safe and efficient shipping in Arctic regions is increasing, due to expected increases in maritime transport in ice-covered seas. It has been shown that there has been an increasing trend in the number of incidents occurring in Arctic waters, with damage to or failure of the propulsion system accounting for most of these incidents (Nejad, et al., 2021).

The propulsion systems of vessels travelling in icy waters are exposed to ice-related impact loading in addition to hydrodynamic loading. This additional loading affects the safety and efficiency of vessel operation.

Structural failure of propulsion system components could occur due to either a loading condition exceeding the ultimate strength of the component, or due to a cyclic loading resulting in fatigue failure. Both these loading conditions are exacerbated during propeller-ice interaction. This is due to increases in the maximum loading on the propeller blades during ice impacts.

Some of the major components of the propulsion system are the bearings that support the propulsion shaft. Part of the monitoring of these bearings would be to measure the loading conditions they are exposed to during operation. One way this can be done is by measuring the shaft response at one or more locations during operation, especially during propeller-ice contact. Then an inverse model can be used to estimate the forces that caused the response, as well as the response along the rest of the shaft. The estimated forces would include the transverse propeller loads and the total shaft response could be used to estimate the bearing loads.

There has been previous research in measuring the propulsion shaft response due to propeller-ice interaction. These works mainly focus on the torsional and thrust vibrations of the shaft, for example the work done by Zambon, et al. (2022). Though there is research on the transverse vibrations of propulsion shafts, such as Zou, et al. (2024), there is little with regards to the transverse response due to propeller-ice interaction.

Inverse models have previously been used to estimate ice-induced propeller loads (Browne, et al., 1998; Ikonen, et al., 2014; De Waal, et al., 2018; Polic, et al., 2019; Nickerson and Bekker, 2021; Nickerson and Bekker, 2022). Of these investigations, only Browne, et al. (1998) and Nickerson and Bekker (2021) considered axial loads and responses, while the rest considered only torsional loads and responses. Again, there is little on the use of inverse models to estimate transverse propeller loading from the shaft response due to propeller-ice interaction.

This paper presents the initial development of an inverse model of the propulsion shaft of the S.A. Agulhas II (SAA II) that can be used to estimate transverse loads on the propeller, and the associated bearing loads. The model is based on similar principles to one developed by Nickerson and Bekker (2022) for estimation of torsional propeller loads.

The aim is to be able to use this model with full-scale measured data from the SAA II propulsion shaft to estimate the propeller and bearing loads due to propeller-ice interaction. This work forms part of a larger study of ice going vessels, using the SAA II as a case study (Bekker, et al., 2019; Purcell, et al., 2024). Previously, the transverse propeller loads have been estimated based on the torque loads (Gilges, et al., 2024). This work seeks to provide another method to estimate these transverse loads.

MODEL DEVELOPMENT

A model of the SAA II port-side propulsion shaft has been developed. The port-side is chosen as this corresponds with the current measurement system on board the vessel.

For the model, a single propulsion shaft is considered between the propeller and motor. The shaft is modelled as a beam with free boundary conditions. The external transverse propeller loads are included and each of the bearings capable of supporting radial loads are modelled as elastic supports. The layout for this shaft model is provided in Figure 1. There are three bearings mounted in an external stern-tube (1 – 3), one intermediate shaft bearing (4), and two bearings supporting the rotor (5 and 6).

It is assumed that the vertical load due to ice impacting the propeller blade is significantly higher than the axial load. Hence the moment, M_{prop}, applied to the propeller is considered negligible. For simplicity, it is also assumed that the cross-section and material properties remain constant throughout the length of the shaft.

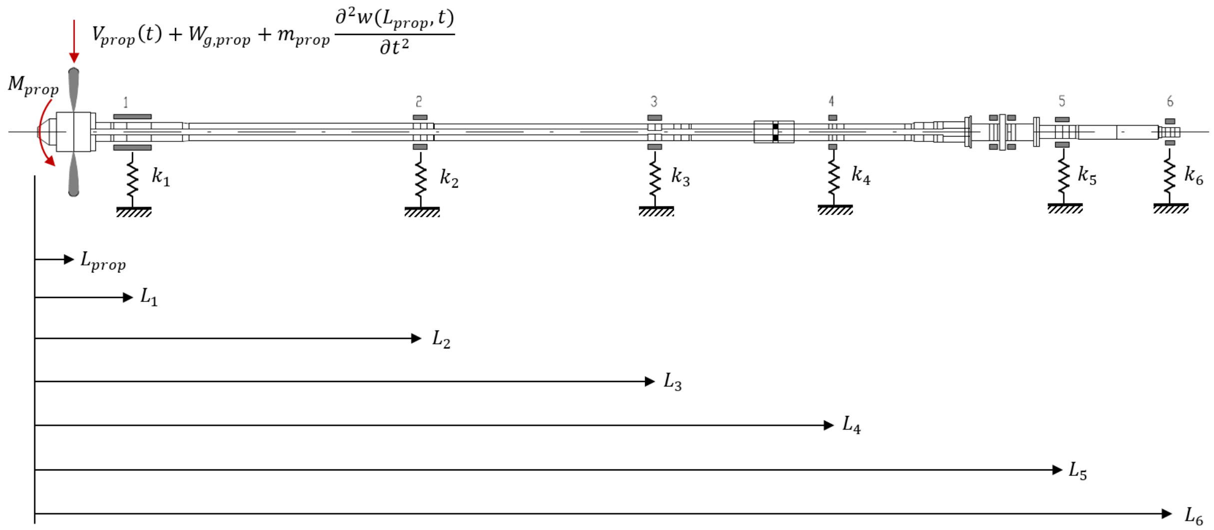

Figure 1. Shaft model layout

Forward model

When used to solve the forward problem the input propeller loads would be known, and the shaft response would be unknown. The wave equation for the transverse vibrations of a beam is (Rao, 2007),

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = f(x,t) \tag{1}$$

where E is the elastic modulus, I is the cross-section area moment of inertia, ρ is the material density, A is the cross-section area and $w(x,t)$ is the transverse displacement. The applied forces, $f(x,t)$, are given by

$$\begin{aligned} f(x,t) = &-V_{prop}(t)\delta(x - L_{prop}) - W_{g,prop}(t)\delta(x - L_{prop}) \\ &- m_{prop}\ddot{w}(L_{prop},t)\delta(x - L_{prop}) - \sum_{j=1}^{6} k_j w(L_j,t)\delta(x - L_j) \end{aligned} \tag{2}$$

where V_{prop} is the transverse propeller load, $W_{g,prop}$ and m_{prop} are respectively the weight and mass of the propeller, and the k terms represent the bearing radial stiffnesses. The delta function, δ, is given by

$$\delta(x - a) = \begin{cases} 1, & x = a \\ 0, & x \neq a \end{cases} \tag{3}$$

The transverse displacement of the shaft can be represented using modal superposition,

$$w(x,t) = \sum_{n=1}^{N} \varphi_n(x) q_n(t) \tag{4}$$

where N is the number of mode shapes used to describe the displacement of the shaft, $\varphi_n(x)$ are the mode shapes, and $q_n(t)$ are the corresponding modal coordinates as functions of time. The mode shapes and natural frequencies are described respectively by,

$$\varphi_n(x) = \cosh(\beta_n x) + \cos(\beta_n x) - \sigma_n(\sinh(\beta_n x) + \sin(\beta_n x)) \tag{5}$$

$$\omega_n(x) = {\beta_n}^2 \sqrt{\frac{EI}{\rho A}} \tag{6}$$

The values for β_n and σ_n depend on the boundary conditions and are provided by Inman (2014). Note that the rigid modes were not considered in the development of this model. The orthogonality condition for the modes can be expressed as (Rao, 2007),

$$\int_0^L \varphi_n(x)\varphi_m(x)dx = \begin{cases} 0, & n \neq m \\ \dfrac{1}{\rho A}, & n = m \end{cases} \tag{7}$$

From the definition of the delta function,

$$\int_0^L \delta(x-a)\varphi_m(x)dx = \varphi_m(a) \tag{8}$$

Equations 2 and 4 can be substituted into Equation 1 and the resulting equation multiplied by $\varphi_m(x)$ and integrated over the length of the shaft,

$$
\begin{aligned}
EI \sum_{n=0}^{N} \beta_n{}^4 q_n(t) \int_0^L \varphi_n(x)\varphi_m(x)dx + \rho A \sum_{n=0}^{N} \ddot{q}_n(t) \int_0^L \varphi_n(x)\varphi_m(x)dx \\
= -\left(V_{prop}(t) + W_{g,prop}\right) \int_0^L \delta\left(x - L_{prop}\right)\varphi_m(x)dx \\
- m_{prop}\ddot{w}\left(L_{prop}, t\right) \int_0^L \delta\left(x - L_{prop}\right)\varphi_m(x)dx \\
- \sum_{j=1}^{6} k_j w\left(L_j, t\right) \int_0^L \delta\left(x - L_j\right)\varphi_m(x)dx
\end{aligned}
\tag{9}
$$

Using Equations 4 and 6 – 8, Equation 9 can be simplified to one equation representing the equation of motion for each mode, Equation 10.

$$
\begin{aligned}
\ddot{q}_n(t) + m_{prop}\varphi_n\left(L_{prop}\right) \sum_{i=0}^{N} \varphi_i(L_{prop})\ddot{q}_i(t) + \omega_n{}^2 q_n(t) \\
+ \sum_{j=1}^{6} k_j \varphi_n\left(L_j\right) \sum_{i=0}^{N} \varphi_i(L_j) q_i(t) = -\left(V_{prop}(t) + W_{g,prop}\right)\varphi_n\left(L_{prop}\right)
\end{aligned}
\tag{10}
$$

Equation 10 can then be written into matrix form, which facilitates solution using a numerical time integration scheme (Kadapa et al., 2017). In this case, the vector containing the q_n variables would be the solution to the model. These variables can then be used with Equation 4 to provide the response at any location along the length of the shaft.

Inverse model

While the forward model is useful when the input loads are known, this is often not the case for operational measurements. Often, the input loads are unknown, and some output response is known. In this case, for the inverse model $V_{prop}(t)$ would be unknown. Thus, an additional equation would be necessary. This would come from a physical measurement of the system response for use as input in the inverse problem. For this model, the measured transverse displacement of the shaft at a given point is chosen,

$$
w(x_a, t) = \sum_{n=1}^{N} \varphi_n(x_a) q_n(t) \tag{11}
$$

where x_a is the location at which the displacement is measured. This additional equation would be included in the matrix formulation, and the solution to the inverse problem would be the vector containing the q_n variables as well $V_{prop}(t)$.

Measurement of the transverse displacement could be achieved using laser displacement sensors, or machine vision methods (Zou, et al., 2024).

Estimating bearing loads

Both the forward and inverse models could be used to infer the bearing loads. Once the q_n variables have been solved for, the bearing forces can be estimated using modal superposition to determine the displacement at the bearing location and multiplying with the bearing stiffness.

$$F_j(t) = k_j w(L_j, t) = k_j \sum_{n=1}^{N} \varphi_n(L_j) q_n(t) \tag{12}$$

RESULTS AND DISCUSSION

The model is simulated using the parameters provided in Table 1.

Table 1. Model parameters (Rolls-Royce AB, 2010)

Parameter	Symbol	Value
Elastic modulus for steel	E	207 GPa
Density for steel	ρ	7850 kg/m^3
Propeller mass	M_{prop}	13 427 kg
Propeller location	L_{prop}	1.102 m
Motor rotor mass	M_{rot}	11 300 kg
Propeller location	L_{rot}	33.140 m
Shaft outer diameter	d_o	0.500 m
Shaft inner diameter	d_i	0.175 m
Shaft length	L	35.095 m
Bearing stiffness	k_1	0.691 GN/m
	k_2	0.576 GN/m
	k_3	0.563 GN/m
	k_4	0.590 GN/m
	k_5	1.000 GN/m
	k_6	1.000 GN/m
Bearing locations	L_1	3.137 m
	L_2	11.959 m
	L_3	19.174 m
	L_4	24.552 m
	L_5	31.687 m
	L_6	34.940 m

The first three natural frequencies of the modelled shaft are compared to those provided in the design documentation of the SAA II (Rolls-Royce AB, 2010) and are provided in Table 2.

Table 2. Comparison of natural frequencies

	Model	Rolls-Royce AB (2010)	Error
f_1	10.3 Hz	11.1 Hz	7.2 %
f_2	17.1 Hz	20.9 Hz	18.2 %
f_3	24.0 Hz	27.0 Hz	11.1 %

Though the frequencies are close, there are still significant errors with the results provided by the model. It may be that assumptions made during the development of the model need further investigation.

For the shaft layout as presented in Figure 1, there would be an additional two rigid body modes. These were not considered in the development of the model; however, this should still be examined in further detail.

The presented model development does not include any damping. This is included by modelling damping elements similar to how the spring elements are modelled. This results in an additional term in Equation 10,

$$\sum_{j=1}^{6} c_j \varphi_n(L_j) \sum_{i=1}^{N} \varphi_i(L_j) \dot{q}_i(t) \tag{13}$$

The damping coefficients for these elements need to be determined through experimental measurements. Furthermore, propeller damping due to interaction with water can be modelled using Schwanecke's method (Bertram, 2012).

The presented models only consider two-dimensional transverse vibrations, or displacement in a single plane. It is, however, expected that the shaft would bend in multiple planes as ice impacts could occur at any point during the propeller's rotation. Thus, to properly model this, the three-dimensional transverse vibrations need to be considered. This can be achieved with the current model by simulating the shaft in two planes, the *xy* plane and the *xz* plane, and combining the results to get the transverse vibration response in three dimensions.

For operational measurements, this would necessitate that the transverse vibrations are measured in two perpendicular directions. The displacement of the shaft can then be considered relative to these measurement directions.

It should also be noted that one could easily consider the effect of additional masses on the transverse vibrations of the shaft. The weight of additional components, such as bearings, couplings, or the motor rotor, could be added to Equation 10 in a manner similar to the propeller weight.

To show the functionality of the model, the forward problem is solved when the propeller is subjected to an impulsive load, $V_{prop}(t)$, with a magnitude of 100 kN and duration of 0.04

seconds. For this simulation, the masses of the propeller and motor rotor are included, and damping is modelled as described in Equation 13. The damping coefficients are chosen such that the response dies out in a reasonable time and are not necessarily representative of the actual damping in the system. Other parameters, such as the mass and bearing stiffnesses, are taken from design documentation (Rolls-Royce AB, 2010).

The results are given in Figure 2. The propeller deflects when the load is applied and then oscillates around its equilibrium position, as is expected for this type of loading. The bearings can be seen deflecting away from their positions initially and then oscillating back to equilibrium. The bearing force is then the product of the bearing displacement and stiffness.

Since the bearings deflect away from their initial position throughout the shaft, it will likely be necessary to account for the rigid body modes for the model to be accurate.

PLANS FOR FUTURE WORK

The model needs further work before it can be useful for the estimation and monitoring of transverse propeller and bearing loads. Work on a small-scale laboratory test set up is underway which will be used to verify the development of the model. For this set up, a beam will be supported by springs to simulate a model similar to that shown in Figure 1. This set up will be instrumented such that the input transverse loads, beam displacement, and spring loads can be measured simultaneously. The model will then be used to estimate the loads from a measured response or input and verify whether the correct measured loads can be obtained.

Further development of the model itself, including the interaction of the rigid body modes, is ongoing. Full-scale measurement of the transverse response of the SAA II shaft line is planned in order to determine the necessary damping coefficients for the model.

CONCLUSIONS

A model for the transverse vibrations of a ship propulsion shaft has been presented. The model allows for simulating the shaft response and bearing loads both when the applied loading conditions are known and when they are unknown. This allows the model to be used to estimate the loading conditions during operation, where it is typically easier to measure the responses.

The model does not currently match the natural frequencies from design documentation well. Further investigation into the assumptions made during the development of the model is necessary.

The model will be validated at small scale in laboratory testing where both inputs and responses can be measured to verify whether it can be used to accurately estimate the applied loads. Once the model is validated, its use in estimating bearing loads based on the loading conditions or response of the shaft can be investigated.

Finally, full-scale measurements of the transverse vibrations of the propulsion shaft of the SAA II will be conducted. These measurements will be integrated into the current measurement and monitoring efforts conducted on board. Once a working model is established, it would allow for the monitoring of bearings through operational measurements.

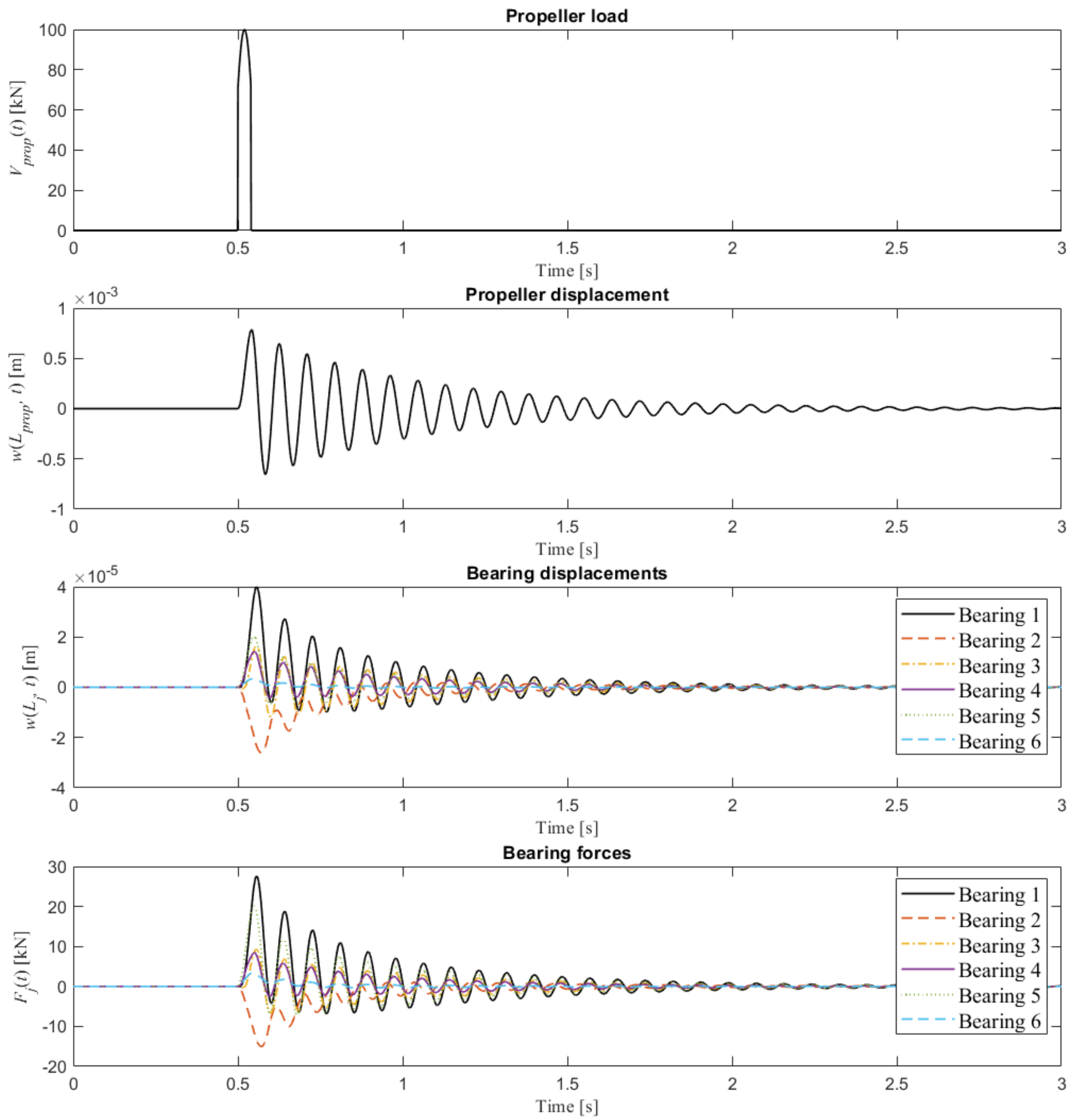

Figure 2. Results from solution of forward problem

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