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Impact of Testing Method on Measured Flexural Strength of Model-Scale Ice

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ABSTRACT

Ice-going vessels and offshore structures designed to break ice in bending rely on the reported flexural strength of ice. Depending on the ice conditions, the flexural strength can be affected by the choice of testing method. Nonetheless, the International Towing Tank Committee (ITTC) recommends three different methods to determine the flexural strength of model ice. To address the uncertainty related to the choice of testing method, this study performs downward and upward cantilever beam tests and in-situ and ex-situ three-point bending tests in ethanol-doped and saline fine-grained model ice, respectively. Experiments were performed at the Aalto University Ice and Wave Tank and in the ice basin of Aker Arctic Technology using the same test setup. The results show that the flexural strength of both types of model ice is influenced by bending direction of the ice samples. The testing method has only a minor effect on the determined flexural strength. Consequently, the testing guidelines proposed by the ITTC sufficiently address the uncertainty in the flexural strength of ethanol-doped and of saline fine-grained model ice.

KEY WORDS: Mechanical properties; Fine-grained model ice; Cantilever Beam; Three-point bending, ITTC

INTRODUCTION

The design of ice-breaking vessels and inclined offshore structures often relies on the flexural strength of ice, as they are often intended to break ice in bending. Flexural strength is not an inherent material property, but it is commonly used in industry to quantify the strength of ice. Model-scale testing in ice is a state-of-the-art method to determine the performance of vessels and structures in ice during the design phase. As the testing aims to mimic the full-scale, the ice conditions must be modelled and measured correctly.

The International Towing Tank Conference (ITTC, 2021) proposes three different methods to determine the flexural strength of model-ice: (upward or downward) cantilever beam tests and three-point bending tests, and four-point bending tests. The latter two can be performed in-situ and ex-situ. Cantilever beam tests are the preferred test method, whereas three- and four-point bending tests are mainly intended for research on model ice or in situations where cantilever beam tests become impractical (ITTC, 2021).

Some studies have focused on the effect of the flexural strength testing method on different types of model ice, freshwater ice, and sea ice. Several studies have reported upward and downward cantilever test results. The difference in strength is often presented using a homogeneity factor, H, which describes the ratio of the upward flexural strength to the downward flexural strength. Only a handful of studies have compared results from in-situ to ex-situ three- or four-point bending tests or from cantilever beam tests to three- or four-point bending tests.

The homogeneity factor of sea ice is generally accepted to be 1. This ratio was found in field tests carried out in North Star Bugt, Thule, Greenland and in Hopedale, Labrador, Canada (Weeks & Anderson, 1958), in Mombetsu harbour, Hokkaido, Japan (Tabata, 1967) and in the port Notsuke-Odaito, Hokkaido, Japan (Kayo et al.,1983).

Dykins (1969) and Vaudrey (1977) compared cantilever beam tests to in-situ three-point bending tests at McMurdo, Antarctica and Barrow, Alaska. The tests were performed on large in-situ beams, and they resulted in similar flexural strength. Downward cantilever tests and three-point bending tests performed in Kalkkiranta and in Tvärminne, Finland, also resulted in similar flexural strength results (Enkvist, 1972).

Various studies have shown significant variability in the homogeneity factors for freshwater ice. Gow (1977) determined the homogeneity from simply supported beam tests. The reported average homogeneity factor varied from 1.2 to 1.6, while occasional values exceeded 2. Gow (1977) considered these results to be in line with Frankenstein (1961) where the values varied between 1.1-1.49. Suominen et al. (2022) reported contradicting results, as the average homogeneity factor was 0.43-0.76 in four cantilever test series and 1.26 in a three-point bending test series. Fransson (2002) conducted an extensive test series in three winter seasons using four-point bending tests. The homogeneity factors determined from the average values of the tests varied from 0.44 to 2.9. Fransson (2002) noted that the ratio is highly dependent on the general seasonal conditions, as the strength of the top layer varies depending on weather conditions. The strength of the bottom layer is more stable throughout the season.

Furthermore, the testing method is reported to have a significant impact on the determined flexural strength of freshwater ice. Gow (1977) noted that the strength determined through three-point testing is commonly 1.2-1.7 and occasionally two times higher (Gow, 1999) than the values obtained with cantilever beam testing. Similarly, Frankenstein (1961) has reported ratios of 1.3 to 3.4, and Suominen et al. (2021) values of 1.1-1.7. Aly et al. (2018) collected flexural strength measurements from several publications and estimated the factor to be commonly two.

Homogeneity factors for several types of model ice have been reported in the literature: The homogeneity factor is 0.4-0.7 for NaCl ice and 0.6-0.9 for carbamide ice (Timco, 1980). Later reports state a homogeneity factor of 0.39 for carbamide (urea) ice (Timco, 1986). EG/AD/S ice resulted in a homogeneity factor of 0.69 (Timco,1986). For ethanol-doped fine-grained ice, the homogeneity factor was reported to be 0.5-1.2 (Jalonen and Ilves, 1990) and 0.61-1.54 (Li and Riska, 1996).

Borland (1990) performed in-situ cantilever beam tests, in-situ three-point bending tests, and ex-situ three point bending tests on urea ice grown in the CRREL test basin. Overall, the results show no significant influence of the testing method. All the testing methods led to similar strength results, however ex-situ three-point bending tests resulted in higher flexural strength than in-situ three-point bending tests if the bottom surface was in tension.

The above-cited literature shows that there is significant variation in the flexural strength results of ice due to the use of different testing methods. This leads to the question of whether the guidelines of the ITTC are suitable for consistently determining the flexural strength of ice. To answer this question, upward and downward cantilever tests, and in-situ and ex-situ three-point bending tests were performed in ethanol-doped and saline fine-grained model ice at the Aalto University Ice and Wave Tank and the Aker Arctic ice tank, respectively.

MEASUREMENTS AND METHODS

Testing Facilities and utilized model-scale ice

The measurements were conducted in Aalto University Ice and Wave Tank and Aker Arctic ice tank. The basin in Aalto University is a 40 m by 40 m square basin, while the basin in Aker Arctic is a 75 m by 8 m rectangular basin, see Figure 1. Both basins produced the model ice through a spraying process where a fine mist of an aqueous solution is sprayed in the air from a carriage moving over the basin. At Aalto University, the solution is doped with ethanol, while Aker Arctic applies saline solution. The sprayed solution lands on the surface of the basin, forming a thin layer of ice. This process is repeated until the desired ice thickness is reached, producing fine-grain model ice with grains of size equal to or smaller than one millimeter. Air temperature is kept below -10 to -15 degree Celsius during the spraying process and after the spraying to consolidate the formed ice. After the ice has consolidated, the air temperature in the basin is set to rise to obtain the desired strength level. A detailed description of the procedures at Aalto University and Aker Arctic can be found in Jalonen and Ilves (1990) and Nortala-Hoikkanen (1990). The resulting ice at Aalto University and at Aker Arctic has a harder top surface compared to the bottom surface due to a temperature difference between water and air in both basins. The crystal-structure of the ice is homogenic from top to bottom (Figure 2).

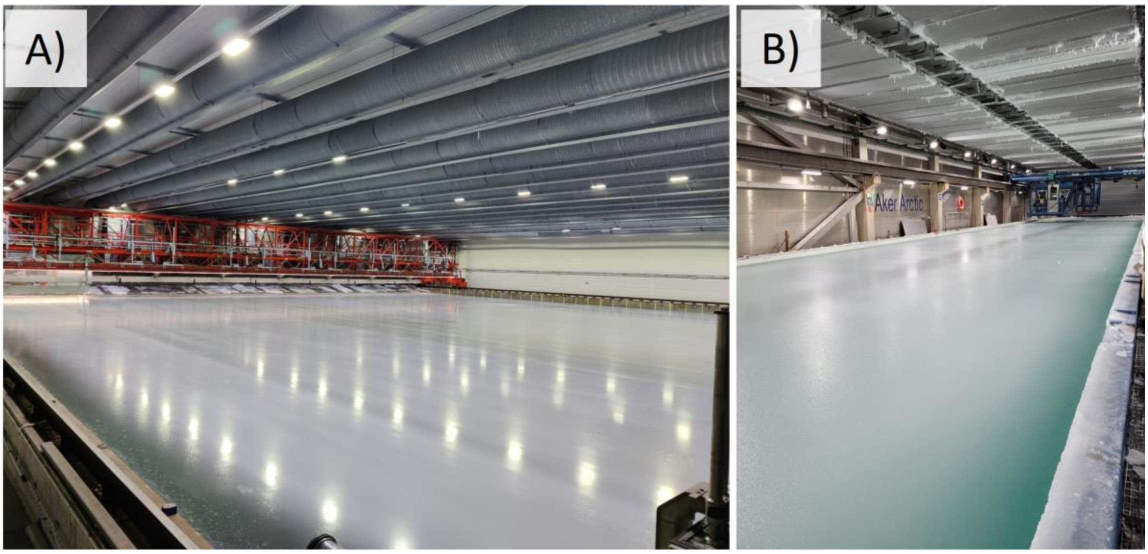

Figure 1. A) Aalto University Ice and Wave Tank, B) Aker Arctic ice tank.

Sample Preparation and Testing Procedures

The samples were cut from the model ice sheet with a milling drill bit connected to an electric drill. At Aalto University, the electric drill was connected to the carriage system of the basin, and movements were controlled manually with the carriage positioning system. In Aker Arctic, the ice was cut manually using the drill and a metal stencil. In both cases, the target sample size followed ITTC (2021) guidelines, i.e., the length was five to seven times the thickness, and the width was two times the thickness. In the case of cantilever beam samples, the ice surrounding the free end of the beam was carefully removed to prevent possible interaction. In the case of three-point bending tests, each test sample was placed on the supports with the top surface facing down right before the test.

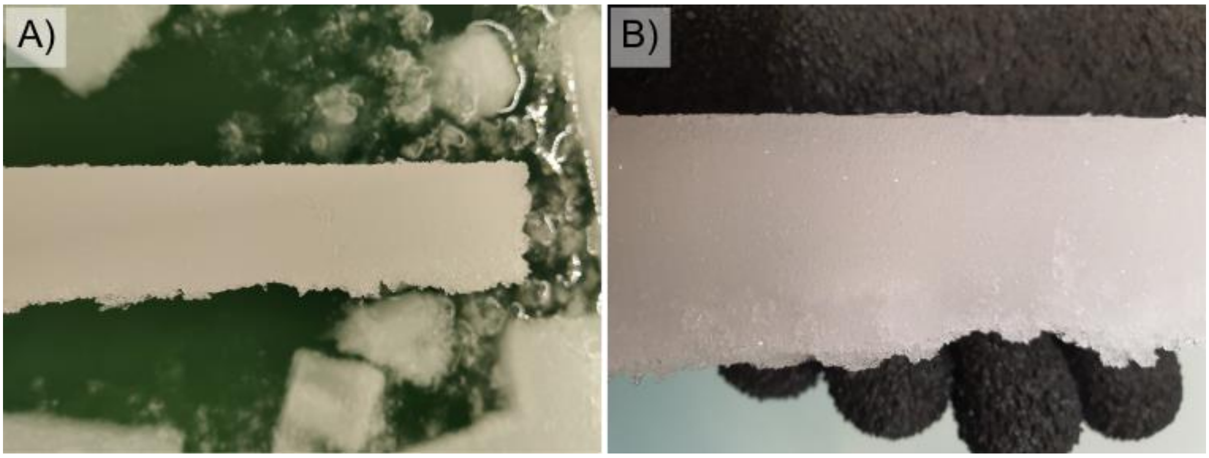

Figure 2. Ice samples from A) Aalto University ice basin and B) Aker Arctic ice basin.

Figure 3 presents the test set-up for different tests conducted within the study: a) downward cantilever beam, b) upward cantilever beam, c) in-situ three-point bending, and d) ex-situ three-point bending. In all tests, the samples were loaded with an indenter head connected to a load sensor. The load sensor was connected to a linear ball screw actuator, Hiwin KK60, to move the load sensor-indenter system. The ice was loaded under controlled displacement with an indentation speed of 5 mm/s for all the tests.

The indenter head used for loading was rounded at the contact point and had a width of a sample. It was connected to the load sensor through a joint that allowed the rotation in the length direction. This system was considered to prevent applying torsion to the sample in case of possible irregularities on the contact surfaces. In cantilever beam tests, the indenter contact line, i.e., loading point, was approximately 1 cm from the tip. In upward bending tests, the indenter was submerged and situated below the beam, whereas in the other tests, the indenter was in the air, above the loading location. In three-point bending tests, the loading was applied in the center, between the supports. The supports of the three-point bending test setup were connected to the aluminum profiles of the loading system. Both supports were rounded around the width direction of the system. One of the supports was fixed, while the other one was connected to the frame through a joint that allowed rotation around the length direction to prevent torsion caused by possible irregularities on the sample surface and thicknesses.

Determination of Flexural Strength

The flexural strength was determined following the procedures described in ITTC (2021). Assuming the beam behaves in accordance with Euler-Bernoulli beam theory (Timco, 1981), the flexural strength from cantilever beam test, $\sigma_{Cantilever}$ [Pa], can be calculated as

$$\sigma_{Cantilever} = \frac{M}{W} = \frac{6Fl_b}{bh_i^2} \tag{1}$$

where, M [Nm] is the bending moment affecting the cross-section, W [m^3] is the section modulus of the cross-sectional area, F [N] is the measured force at the time of failure, l_b [m] is the length from the loading point to the location of crack, b [m] is the width of the beam, and h_i [m] is the ice thickness.

Following the same methodology, the following form can be determined for the flexural strength in three-point bending ($\sigma_{3\text{-}Point}$):

$$\sigma_{3-point} = \frac{M}{W} = \frac{3Fl}{2bh_i^2} \tag{2}$$

In upward bending, the bottom surface was in tension, while the top surface was in tension in the remaining tests.

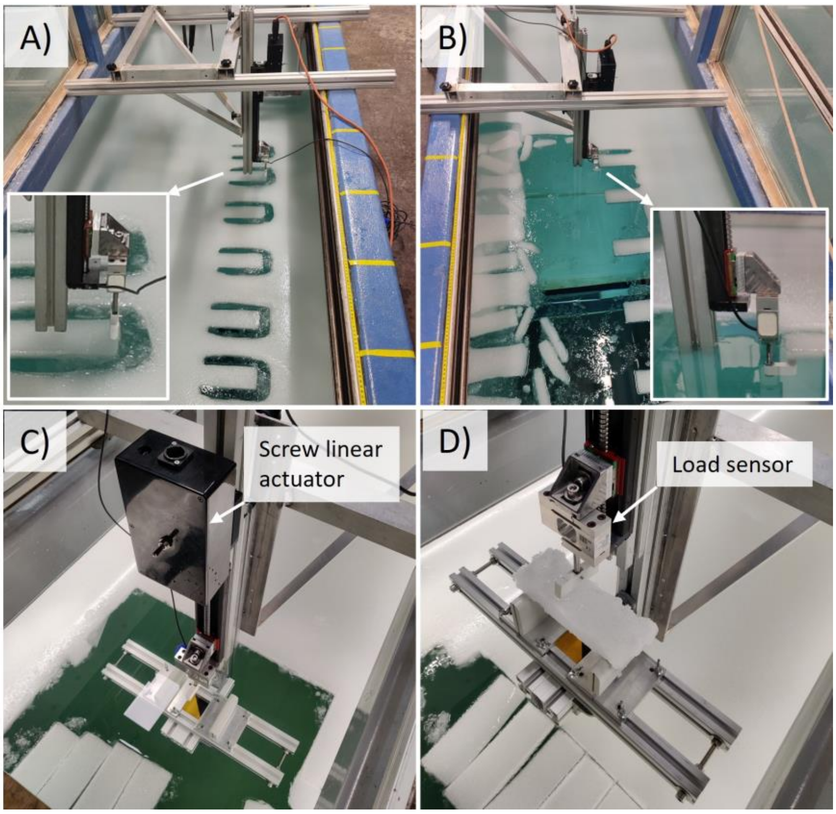

Figure 3. Test set-up in different tests: a) downward cantilever beam, b) upward cantilever beam, c) in-situ three-point bending, d) ex-situ three-point bending.

RESULTS

The measurements in Aalto University and Aker Arctic were conducted on December 9 and 13, 2022. In both instances, the basins produced the ice sheet following basin specific standard model ice sheet preparation procedures. The indicative target ice thickness for the sheets was 30 mm with a flexural strength of 50 kPa.

Nine and twelve sets of measurements were conducted in Aalto University and Aker Arctic, respectively. The number of tests and their order is described in Table 1. Five to ten measurements were performed for each test method. The measurements in Aalto University consisted of three sets of downward cantilever beam tests and two sets of upward cantilever beam tests, and in-situ and ex-situ three-point bending tests. The measurements in Aker Arctic consisted of three sets of all testing types.

Table 1. Location and time of the test sets in chronological order. In Aalto University, the origin of the basin is set to the southwest corner of the basin. In Aker Arctic, the frames run along the length of the basin starting from the door connecting the ice basin to trimming tank.

Tests at Aalto University on Dec 9			
Test set	Time	x [m]	y [m]
Cant. Down1	8:30	20	16
3-point Exsitu1	9:00	20	16
3-point Insitu1	9:30	20	16
Cant. Up1	10:00	20	16
Cant. Down2	13:00	23	10
3-point Exsitu2	13:45	23	10
3-point Insitu2	14:18	24	10
Cant. Down3	15:00	24	9
Cant. Up2	15:00	24	9

Tests at Aker Arctic on Dec 13		
Test set	Time	Frame [m]
Cant. Down1	8:45	10
Cant. Up1	8:45	10
3-point Insitu1	9:40	11
3-point Exsitu1	10:00	12
3-point Exsitu2	10:30	29
3-point Insitu2	10:50	29
Cant. Down2	11:05	29
Cant. Up2	11:05	29
Cant. Down3	12:30	42
Cant. Up3	12:30	42
3-point Insitu3	13:00	42
3-point Exsitu3	13:15	42

Figure 4 presents typical force time histories from each testing types. Table 2, Table 3, Figure 5, and Figure 6 present the variation and mean value of each test set. In some of the three-point bending tests, the samples failed at support or started sliding during the measurements. Upon analyzing the force time histories more closely, it was found that some of the force histories exhibited multiple instances intermediate force decreases, and in some cases even significant force decreases, before the ultimate failure of the ice sample. These force drops were identified as preliminary failures that occurred before the final failure, and as a result, these ice samples were not included in the subsequent analysis. The number of samples presented in Table 2 and Table 3 indicated the number of successful measurements.

Table 2. Summary of measurements at Aalto University on December 9. Each column summarizes the results from a different measurement set.

	Cantilever Beam					**3-point bending**			
	Down1	**Down2**	**Down3**	**Up1**	**Up2**	**Insitu1**	**Insitu2**	**Exsitu1**	**Exsitu2**
max [kPa]	42.5	59.3	59.1	53.4	68.0	47.5	46.9	67.5	50.4
min [kPa]	34.6	42.8	42.7	42.9	55.7	35.9	41.8	52.6	43.9
mean [kPa]	39.0	49.3	50.3	48.9	62.7	42.1	43.6	60.8	47.2
samples [-]	5	6	5	5	6	5	7	5	7

Nonetheless, all three-point bending measurements conducted at Aalto University contained some medium to significant intermediate failures, as illustrated in the typical force time histories in Figure 4. These were considered to result from the crushing failure of the bottom layer of the model ice, which consisted of ice flakes formed from dendric growth of the ethanol-doped aqueous solution. These measurements were included in the subsequent analysis. The remaining flexural strength measurements are consistent throughout the day for each test type and test location (Figure 5). Especially, the values obtained at Aker Arctic show small variation.

Table 3. Summary of measurements at Aker Arctic on December 13. Each column summarizes the results from a different measurement set.

	Cantilever Beam						3-point bending					
	Down1	Down2	Down3	Up1	Up2	Up3	Insitu1	Insitu2	Insitu3	Exsitu1	Exsitu2	Exsitu3
max [kPa]	54.2	55.4	66.1	24.1	26.8	26.8	58.3	64.7	76.4	58.1	70.3	70.9
min [kPa]	41.0	43.0	50.8	20.7	20.9	20.7	48.9	45.5	56.3	48.2	52.7	56.9
mean [kPa]	46.0	48.6	57.3	22.7	24.5	23.9	55.0	54.9	64.0	52.3	60.8	65.9
samples [-]	6	4	4	5	4	3	4	6	5	5	5	3

Comparison of the testing methods where the top surface of the ice was in tension, i.e., comparison of downward cantilever beam tests to three-point bending tests, indicates that the testing method does not have a significant impact on the flexural strength. The ratio of flexural strength measured using downward cantilever beam tests to three-point bending tests is 1.12-1.20 for in-situ tests and 1.15-1.25 for ex-situ tests. The corresponding values for test sets at Aalto University are 0.89-1.08 and 0.96-1.56. However, the ratio of 1.56 is the result of a local variation as it is shown to be an outlier from the general trend (Table 2, Figure 4). Thus, the results indicate that the testing method has no significant effect on the determined value for ethanol-doped fine-grained ice, and only a minor to insignificant impact to saline fine-grained ice. On contrary, the model ice types show deviation in the flexural strength of different surfaces. For ethanol based fine-grained model ice, the homogeneity factor is 1.25, whereas for saline fine-grained ice the factor is 0.42-0.50.

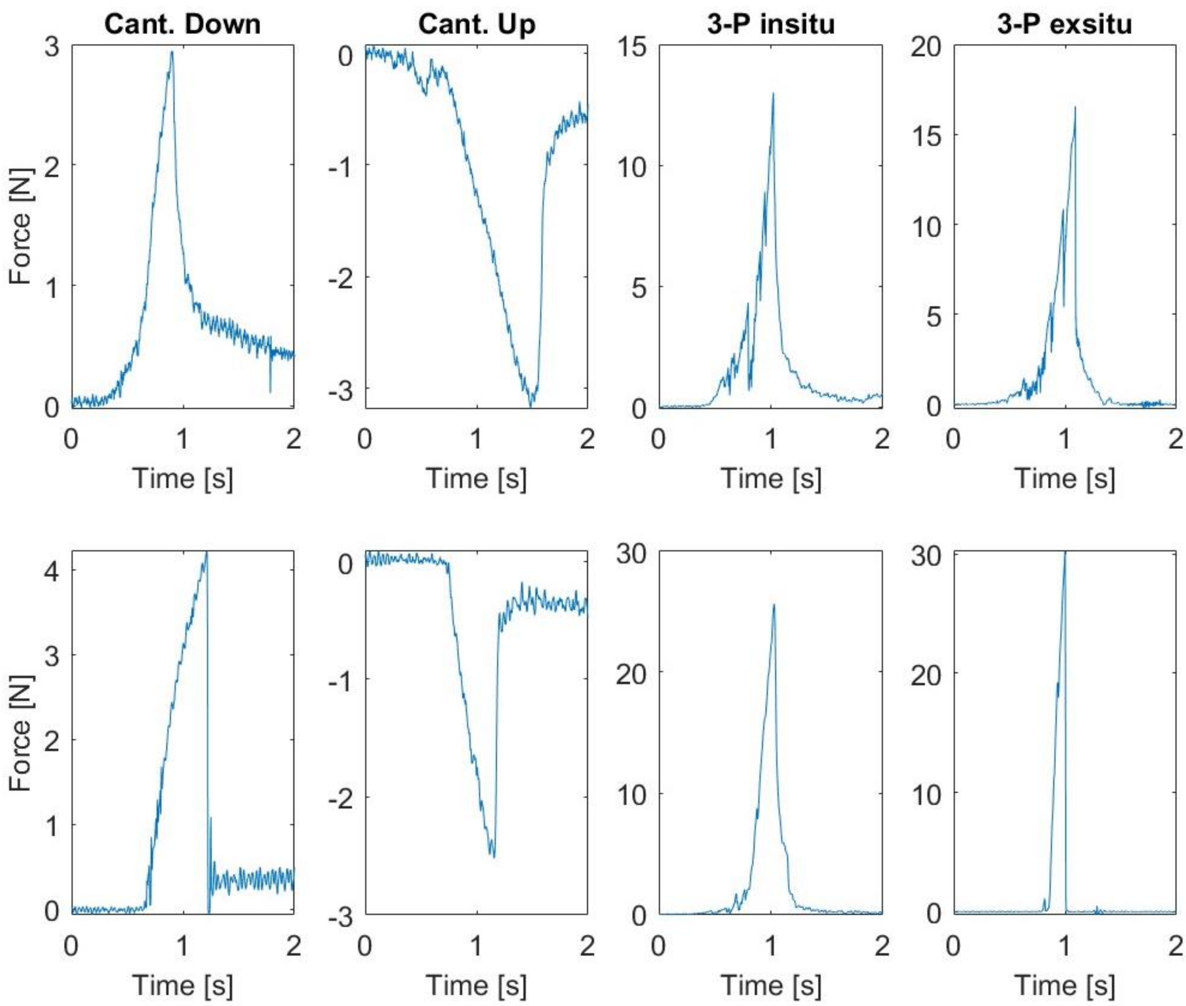

Figure 4. Examples of measured force time histories from cantilever beam tests downward

and upward, and in-situ and ex-situ three-point-bending tests in Aalto University (on top) and Aker Arctic (on bottom).

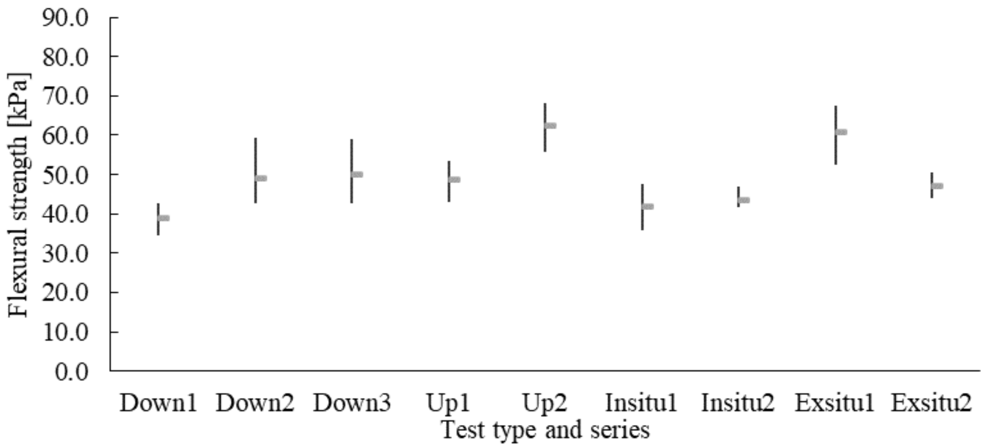

Figure 5. Summary of measurements at Aalto University on December 9. The line and marker indicate the variation and the mean value in each measurement set.

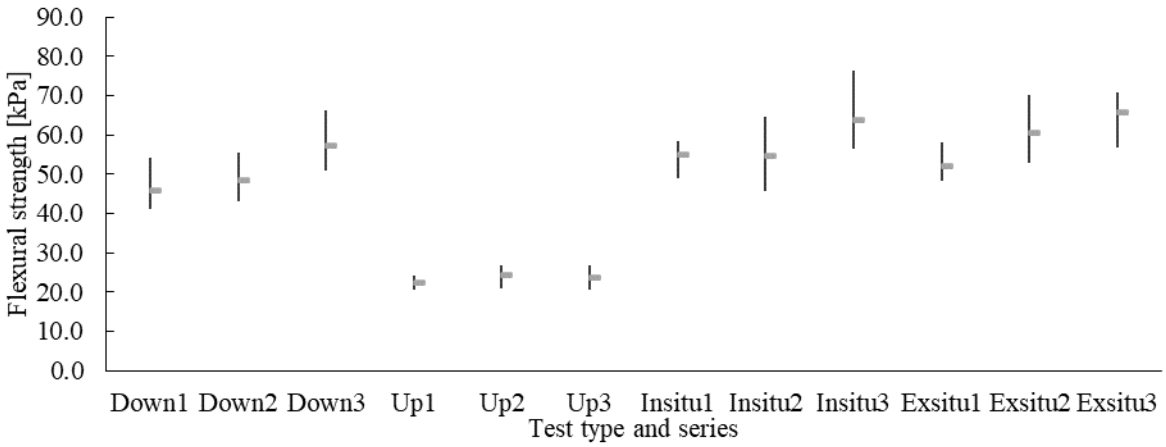

Figure 6. Summary of measurements at Aker Arctic on December 13. The line and marker indicate the variation and the mean value in each measurement set.

DISCUSSION

The buoyancy and gravitational effect were not accounted for when determining the flexural strength, as the applied testing setup did not allow an ambiguous analysis. In the interest of exploring the effect of these forces, we can consider the scenario where the surrounding ice field is not lifted or submerged, the sample beam remains straight, and the displacement of the beam tip increases linearly as a function of the loading tip movement. In that case, the determined flexural strength results from the upward cantilever tests decrease by approximately 6-10 % due to changes in the gravitational force. The results from the downward cantilever tests also decrease by approximately 3-4 % due to changes in the buoyancy force. Similarly, assuming the supports are rigid, and the ice is submerged linearly, the determined flexural strength in in-situ three-point tests would decrease by approximately 1 %. In contrast, the flexural strength of ex-situ three-point tests increases by approximately 7-10 % if the gravitational force is calculated based on the sample dimensions and density, gravitational acceleration, and this load is applied as a uniform line load over the length of the beam. To summarize, accounting for the buoyancy or gravitational force would not affect the comparison between downward cantilever beam and in-situ three-point test results but could affect the comparison to ex-situ three-point tests.

Turning back to the results presented in this paper, downward cantilever beam tests and in-situ and ex-situ three-point bending tests led to similar flexural strength results for both ethanol-doped and saline model ice. These results support earlier findings for urea ice (Borland, 1990), and are in line with full-scale measurements with saline ice where the testing method has not shown significant impact on the obtained strength value. In contrast, the measurement method has been shown to have an impact on the obtained values in full-scale measurements with freshwater ice. Stress concentrations at the root of the cantilever beams can affect the flexural strength measurements, especially in brittle freshwater ice. As saline ice and model ice are more ductile, the root stress effect is decreased.

In contrast, upward cantilever beam tests resulted in significantly different flexural strength results compared to downward bending for both ice types. This difference can be explained by the homogeneity factor of the ice. The measured homogeneity factor of ethanol-doped ice is 1.2 and of saline ice is 0.42-0.50. Similar homogeneity factors have been reported for ethanol-doped ice (Jalonen and Ilves, 1990; Li and Riska, 1996) and for saline ice (Timco, 1980). The difference in tensile strength of the top and bottom surface of the ice are reflected in the flexural strength measurements. As such, the bending direction (top surface in tension or bottom surface in tension) of the ice beam has the largest influence on the flexural strength result.

For comparison, sea ice and freshwater ice have a reported homogeneity factor of 1 and approximately 1, respectively, implying that the flexural strength is not affected by the loading direction (upward or downward). Although the reported homogeneity factor provides a basis for comparing different ice types, it is a significant oversimplification, as it does not consider the impact of temperature gradients throughout the ice, nor the existence of porosities, both of which could have a significant through thickness variation and a considerable influence on the homogeneity factor. Nonetheless, the homogeneity of full-scale and model-scale ice should be taken into consideration when planning model-scale experiments, and flexural strength should be tested keeping the full-scale bending direction in mind.

For saline ice, the variation of flexural strength was higher in three-point bending tests than in cantilever beam tests. Handling the beams may introduce some stresses that can lead to more scatter in strength results (Borland, 1990; ITTC, 2021). It is not clear if handling the beams caused additional variation as this trend is not reflected in the flexural strength of ethanol-doped ice. The results of the downward cantilever beam tests showed similar levels of variability in both model ice types. However, the upward bending tests carried out on ethanol-doped ice exhibited greater variability compared to those conducted on saline ice. The larger variability in ethanol-doped ice may be a result of the dendritic undergrowth. This undergrowth led to noisy force-time histories with intermediate load decreases, which introduced some uncertainty in how to define the force peak associated with bending failure.

Finally, the flexural strength results obtained from both ice basins show differences in flexural strength across different testing locations and times. These differences can be attributed to the challenge of maintaining a stable environment in these large testing facilities. As such, the condition of the ice can differ depending on the testing location and time. Keeping this in mind, it is recommended that only flexural strength results obtained from the same location should be compared. To strengthen this point, an uncertainty analysis was performed on the test setup. Both ice basins had a measurement variability of 3 mm in thickness and width, which could have been introduced during cutting or measuring of the ice samples, as the sample size was measured using a ruler with a resolution of 1 mm in both ice basins. Additionally, the same test setup was used for both in-situ and ex-situ tests in both ice basins. Moreover, the average and maximum deviation from the mean of flexural strength measurements taken at the same location were 11% and 20% at Aalto University and 12% and 19% at Aker Arctic, respectively. These findings suggest that the variability observed in flexural strength was not due to the test setup but rather represents the actual variability in the flexural strength of the model ice.

CONCLUSIONS

This study analyzed the influence of different flexural strength measurement methods on the flexural strength of ethanol-doped and saline fine-grained model ice. The tests were performed at Aalto University Ice and Wave Tank and at the ice basin at Aker Arctic Technology using the same flexural strength testing setup. The results show that, when the top surface of the ice is in tension, cantilever beam tests and in-situ and ex-situ three-point bending tests result in similar flexural strength. Upward cantilever tests lead to significantly different results, as the homogeneity factor of ethanol-doped model ice is 1.2 and of saline model ice is 0.42-0.50.

Overall, this study shows that the current ITTC guidelines for cantilever beam tests and three-point bending tests are appropriate to determine the flexural strength of the tested ethanol-doped and saline model ice. The guidelines address limitations, such as natural scatter in results, choice of loading direction, and the risk of damaging three-point bending samples during handling. Additional experiments are needed to confirm whether these guidelines are sufficient to determine the flexural strength of model ice with higher flexural strength or of columnar model ice.

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