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The study of the Popov method for estimation of ice loads on ship's hull using full-scale data from the Antarctic sea

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ABSTRACT

When ice floe acts against a structure, there are three limit mechanisms in ice loads: limit ice strength, limit driving force and limit momentum. In this paper, Popov-type energy approach based on limit momentum mechanism is adopted and coupled with full-scale measurements conducted on board the S.A. Agulhas II in the Antarctic sea. This research aims to compare the ship-ice interaction parameters determined from energy considerations with processed full-scale measurements. This study focuses on the mechanism of ship-ice interaction, hull form, physical and mechanical properties of ice. Two cases of ship-ice collision processes including five types of ship-ice contact geometry are covered. The solutions for the collision processes are presented and compared for all geometry cases. The results of this study show that the estimated ship-ice interaction parameters have a relatively good correlation with full-scale measurements. Additionally, the limitations and future directions in the energy approach are also discussed.

KEY WORDS: Popov method; Full-scale data; Energy approach; Ice load; Ice strength

INTRODUCTION

Due to the increasing navigation through the Northern Sea Route, new regulations and standards related to the safety of operation in ice conditions are under development (Kujala et al., 2019). One of the key elements is ice class requirements for ships in ice. IACS Unified Requirements (UR) for Polar Ships (PS) are harmonized common rules for ships navigating in Polar Waters. In IACS UR for PS (IACS, 2016), the design ice loads are directly linked to the design scenarios, that are based on the energy collision model developed by Popov (1967).

The present research studies and compares Popov method for the estimation of ice loads on a ship's hull with full-scale measurements conducted on the polar supply and research vessel (PSRV) S.A. Agulhas II (Bekker, 2017). Particularly, the influence of mechanical properties and several geometry types of ice (Daley & Kim, 2010; Daley, 1999) are considered. This research aims to answer the following questions: is it possible to compare the ice load parameters obtained by energy method with full-scale results, and which assumptions or models can be used?

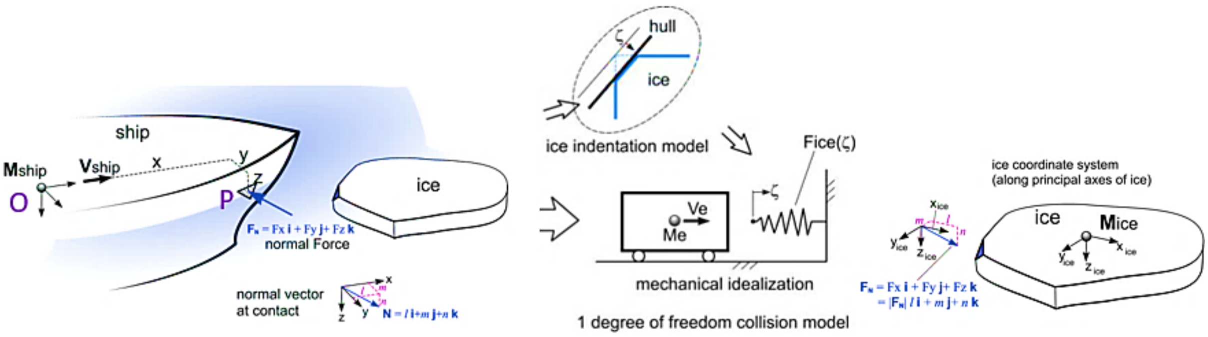
Firstly, the physical background of Popov method is studied, and the initial parameters included in the algorithm for the estimation of total ice load are identified. In order to obtain contact pressure assumed in the model, the experimental data of the ice properties and operational conditions measured on board S.A. Agulhas II are used (Kujala, 2017; Suominen et. al, 2013).

After obtaining contact pressure, the energy balance equations for two impact cases between the ship and ice are compiled. The obtained ice load parameters (total ice load, indentation, contact area, time of impact) for two contact points in the bow area are then analyzed.

Finally, for each contact geometry, total line load and total load length are determined. For the sake of comparison of the theoretical results with full-scale data, two models of load intensity functions (Popov, 1967; Kheysin, 1961) are used. Then the derived local line loads for the bow region are compared with the line loads deduced from full-scale measurements. The paper concludes with possible ways of application of Popov method in structural analysis and speed assessment.

POPOV ENERGY-BASED COLLISION MODEL

The ship-ice interaction in Popov method is modeled as an equivalent one-dimensional collision with all motions taking place along the normal to the shell at the point of impact, where both ship and ice have unique mass reduction terms and the maximum load at the end of interaction. The location of the collision point is determined relative to the center of gravity of the ship and includes information about hull form. The scheme of the contact model is presented in Figure 1, which is also included in the IACS rules for PS as a standard ship-ice collision scenario with glancing impact on the bow region (Daley et al., 2017).



a *b* *c*

Figure 1. Sketch for standard ship-ice collision model proposed by Daley and Kim (2010)

As sketched in Figure 1*a*, if a ship moving uniformly at the speed of v_0 along the “OX” axis and at the time $t = 0$ comes into contact with a finite ice floe, it is assumed that collision takes place at a point P and results in a normal force F_n. The total reduced mass M_{red}, $[t]$ of the system ship-ice can be derived through coupling of 6-degree-of-freedom (6-DOF) equations of motion for ship with 3-DOF equations of motion for ice floe (ice is considered as ellipsoid) (for details see Popov (1967)), which lead to the following expression:

$$M_{red} = \frac{{M_{red}}^{ship} \cdot {M_{red}}^{ice}}{{M_{red}}^{ship} + {M_{red}}^{ice}}, [t] \tag{1}$$

where $M_{red}{}^{ship}, M_{red}{}^{ice}$ are ship and ice reduced masses, [t].

Thus, the ship-ice system is formulated as having only one DOF. The total ice load $F, [MN]$ is described as integral of contact pressure over the nominal contact area A, $[m^2]$, (hereinafter referred to as area), as shown in Eq. 2.

$$M_{red}\ddot{\xi} = F = \int p(\xi, \dot{\xi}) dA\,, [MN]. \tag{2}$$

As we can see from Eq. 2, the total ice load is a function of so called "effective contact pressure" $p(\xi, \dot{\xi}), [MPa]$ (Popov, 1967), which depends on the indentation $\xi, [m]$ and velocity of penetration $\dot{\xi}$, $[m/sec]$ – indentation rate. Due to complexity of this model, for real velocities of ship in ice it was supposed, that the "effective contact pressure" is equal to ice crushing strength $\sigma_c, [MPa]$ (Popov, 1967). According to this hypothesis, the maximum ice load corresponding to the maximum indentation depth $\xi = \xi_{max}, [m]$ is

$$F = \sigma_c \int_0^{\xi} dA, [MN]. \tag{3}$$

The area depends on the configuration of the edge of the ice and the indentation of the ship into the ice, as shown in Eq. 4.

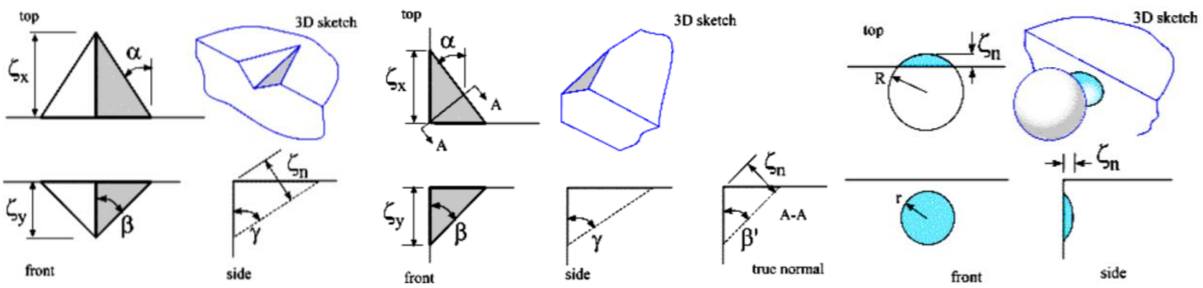
$$A = G\xi^a, [m^2]. \tag{4}$$

In Eq. 4 two unknown contact area parameters G and a were initially derived by Popov (1967) for impacts against an ice with round and angular edges, that are included in the current study. The "a" exponent depends only on the configuration of the ice edge at the point of impact, and "G" coefficient depends on the geometric parameters of the ship and ice (Popov, 1967).

The relationship between the normal indentation and nominal contact area can be found for any contact geometry. As a result, the contact force is a function of one unknown parameter, which is the indentation depth.

Contact geometry cases

For comparing ice loads from several possible variants of contact geometry, additional cases from Daley (1999), that shown in Figure 2 are included in Popov method. For these geometry cases contact area parameters are derived from Daley (1999) energy equations, taking into account the changes in pressure-area relationship $P(A)$ (so-called "process" pressure-area curve) (Daley & Kim, 2010).



a b c

Figure 2. Additional cases of contact geometry from Daley (1999): a – Symmetric V Wedge, b – Right-Angle edge, c – Spherical Contact

Collision scenarios

In the present paper two collision scenarios for the bow area of the ship are presented:

1. Impact between a ship and a floating ice floe
2. Impact between a ship and the edge of an ice field

In the first scenario, the kinetic energy of the ship is partially converted into the kinetic energy of the moving ice floe and partially into the crushing of the ice edge. In the second scenario, the edge of the ice field crushes and the ice field bends due to the vertical component of contact force P_v, $[MN]$. In this case, the ship's kinetic energy is absorbed into crushing and bending of the ice field. The energy balance equations for the specified contact model are summarized in Table 1 (for details see Popov (1967)).

Table 1. Energy components in Popov type collision model

Popov case 1 Ice Floe	Popov case 2 Ice Field
$T_1 + T_2 = U$	$T_1 = U + V$
$T_1 = 1/2 \cdot M_{red}{}^{ship} \cdot v_1{}^2$	$T_1 = 1/2 \cdot M_{red}{}^{ship} \cdot v_1{}^2$
$T_2 = 1/2 \cdot M_{red}{}^{ice} \cdot v_2{}^2$	$V = 1/2 \cdot f \cdot P_v$
$U = \int_0^{\xi} F\, d\xi$	$U = \int_0^{\xi} F\, d\xi$

where T_1 is kinetic energy of the ship, reduced toward the line of impact, $[J]$; T_2 is kinetic energy of the ice, reduced toward the line of impact, $[J]$; U is work of contact forces which cause crushing of the ice floe, $[J]$; V is potential bending strain energy of a semi-infinite ice plate, $[J]$; v_1 is reduced speed of ship, $[m/sec]$; v_2 is reduced speed of ice, $[m/sec]$; $f = \frac{P_v}{2 \cdot \sqrt{\gamma \cdot D}}$, where γ is specific weight of ice, $[N/m^3]$; D is flexural stiffness of an ice plate, $[Pa \cdot m^3]$ which is determined as follows (Popov (1967)):
$D = \frac{E \cdot h^3}{12 \cdot (1-\mu)}$, where E is elastic modulus of ice, $[Pa]$; h is ice thickness, $[m]$; μ is Poisson ratio for ice.

Effective contact pressure and ice crushing strength

For the determination of ice load parameters, Popov hypothesis was applied: "effective contact pressure" is equal to ice crushing strength over the whole contact area. Ice crushing strength is an uncertain parameter which significantly depends on various factors (e.g., it can vary between $1.25 - 10\ MPa$ according to Popov (1967)). However, in order to calculate ice loads, a particular value of ice crushing strength should be input. Besides, the magnitudes of ice loads are quite sensitive to this value. To overcome this difficulty, the following scheme that uses measured data from Agulhas II was proposed:

1. Straightforward guidelines for estimations of ice crushing strength are not available in the literature. However, experiments of Michel and Blanchet (1983) have shown that ice crushing strength is proportional to ice compressive strength σ_{co}, $[MPa]$:

$$\sigma_{cr} = C \cdot \sigma_{co}, [MN] \tag{5}$$

where C is the indentation coefficient. For the brittle range for S2 sea ice, according to Michel (1978), C can be taken as 1.57, which is also supported by Kheysin (1961).

2. Ice compressive strength can be determined as function of ice salinity and brine volume, ice temperature, ice density and indentation rate as follows:

2.1. The average ice salinity can be estimated from the Cox and Weeks (1974) equations, using the ice thickness data.
2.2. The average ice temperature and ice density can be evaluated using air temperature data assuming that for sea ice the temperature at the bottom of the ice sheet is always at $-1.8\ °C$ (Timco & O'Brien, 1994).
2.3. The brine volume as a function of ice salinity and temperature can be determined from Cox and Weeks (1982) equations.
2.4. The indentation rate (Timco & O'Brien, 1994) can be evaluated using reduced speed of the ship and ice floe diameters.
2.5. The data obtained at steps 2.1 – 2.4 can be substituted into Timco and Frederking (1990; 1991) expressions for ice compressive strength.
2.6. The empirical equations of Timco and Frederking (1990; 1991) derived from small-scale tests on ice (sample size of $20 - 25\ cm$) have been reported by Kovacs (1997) to contain scaling uncertainties. Thus, to obtain the correct value of ice compressive strength for ship-ice interaction scale, correction formula from the semi-empirical method proposed by Exon (1997) can be applied.
3. The elastic modulus of ice included in flexural stiffness of an ice plate can be calculated from Timco and Weeks (2010) equation.

APPLICATION OF POPOV METHOD TO FULL-SCALE DATA

Measured data

The PSRV S.A. Agulhas II, which was instrumented with strain gauges for measuring local ice loads is taken as a case study. The determination of ice loads on the ship hull is based on strain measurements by implementing a classical beam theory and an influence coefficient matrix. More detailed information can be found in Suominen et. al. (2013). In addition to load measurements the following parameters were collected (Kotilainen et. al, 2018): ice thickness (from stereo camera measurements), floe diameters (from continuous visual observations), air temperature (from the ship's weather system), ship's speed (from the ship's GPS).

The strains were measured with a frequency of $200\ Hz$ and ship's speed was recorded with high precision at a frequency of $1\ Hz$.

In order to determine the ice induced loads from the measured time history, ice loads data for frame 134, frame 134.5 and for their combination was run through a Rayleigh separator. The separator value and the threshold value used for the data were ½ and $10\ kN$ respectively, below which the measurements are considered as noise and discarded.

Total ice load

The direct solution of Popov equations gives total ice loads exerted during ship-ice collision process. The focus is only put on two frames in the bow region – frames 134 and 134.5.

It is noteworthy that in Popov model time scale of impact process is much shorter compared with overall period of available observations. Therefore, it is reasonable to narrow the time interval of interest down to one day. The observations on 16.12.2013, chosen as time interval of study, indicate significant ice concentration, ice thickness and ice floe diameters, which is supposed to lead to high ice loads.

Before the actual comparison of ice loads it is reasonable to study interdependence of measured ice loads and other variables. For this purpose, all available measurements, calculated ice loads and ice crushing strength values were put in the comparison. An example of estimated ice crushing strength, measured ice thickness and diameters are presented in Figure 3.

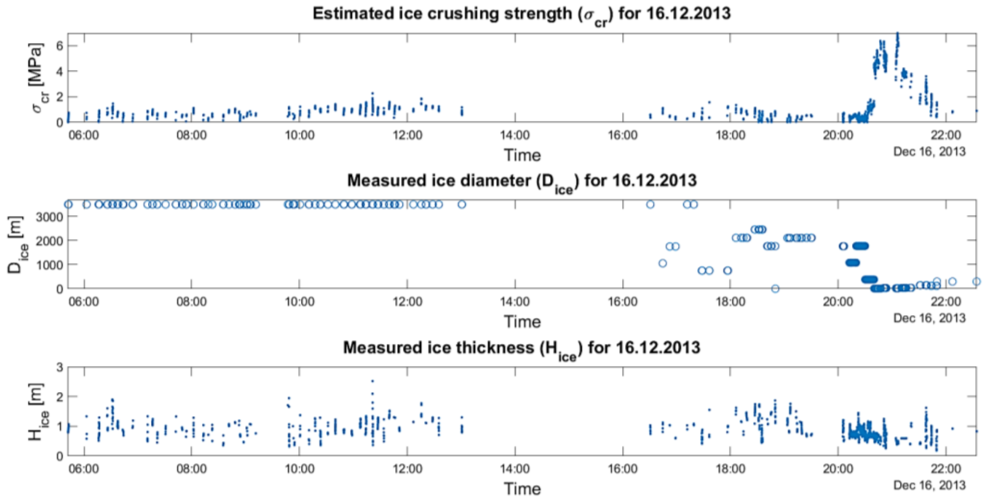


Figure 3. Example of estimated ice crushing strength and measured parameters for Agulhas II

After studying the interdependence of aforementioned parameters, total ice loads from Popov equations were found for all geometry cases. It should be noticed, that if output total loads are compared with measured local loads, unsurprisingly the strong disagreement which is more than of the order of magnitude takes places. To make it possible to correctly compare experimental data with theoretical results, different approaches should be applied.

Pressure-area relationship

The general solution from Popov (1967) equation is presented in a form of indentation depth and total load, however, supplementary load parameters can be also derived. For example, the relationships between the maximal contact area and effective contact pressure for angular and round edge in both collisions (with ice floe and ice field) for frame 134 (bow area) were constructed. It is important to point out, that there are currently no measured pressure data available from Agulhas II and thus there no $P(A)$ has been proposed before. Figure 4 shows data points for pressure-area relationship based on values calculated by Popov method.

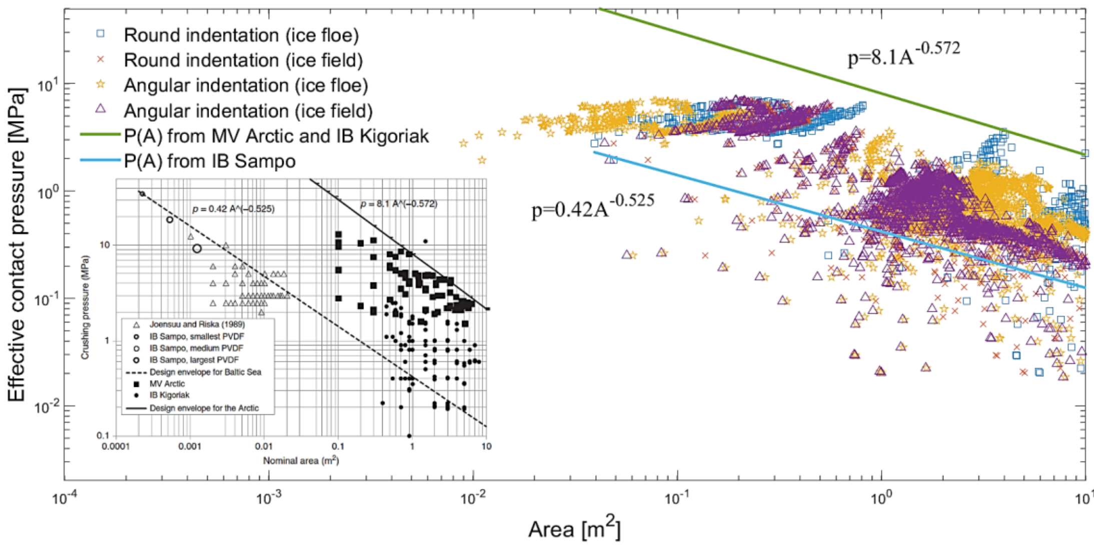


Figure 4. Example of $P(A)$ from Popov method and crushing $P(A)$ curves based on full-scale measurements both in the Canadian waters and in the Baltic Sea (Kujala, 2017)

Two $P(A)$ curves on Figure 4 represent crushing pressure-area relationship based on full-scale measurements both in the Canadian waters (from MV Arctic and IB Kigoriak) and in the Baltic Sea (from IB Sampo) (Kujala, 2017). When estimated and experimental data points from Figures 4 are considered jointly, the same decreasing trends can be observed in both plots.

The $P(A)$ provides necessary information relevant to the design of structures for local ice loads (Masterson and Frederking, 1993). Despite that, $P(A)$ possesses a great deal of scatter (Kujala, 2017), as can be seen in Figure 4, which can be explained by the complexity of ice crushing process. To mitigate this scatter, it is worthwhile to analyze ice loads on different areas.

LINE LOAD

As it was pointed out above, more adequate results require the analysis when measured ice loads are presented as a function of the length with applied load, so-called "load length" $l, [m]$. Then the term "line load" $q, \left[\frac{MN}{m}\right]$ can be introduced as ice load distributed over l. Thus, q and l can be used instead of $P(A)$ (Riska, 2018).

Earlier measurements have shown that the magnitude of the ice-induced q is decreasing as a function of l (Kõrgesaar & Kujala, 2017). This study also aims to find the local q from Popov (1967) method for the comparison with full-scale measurements for Agulhas II.

Using Popov (1967) method we could find a total q in a bow as function of a total ice load F_{max} on the frame and l, as shown in Eq. 5.

$$q = \frac{F_{max}}{l}, [kN/m] \quad (5)$$

Where total l can be derived for any contact geometry. For example, for round edge according to the Popov (1967) scheme a total load length $l_{round}, [m]$ is calculated using Eq. 6.

$$l_{round} = \frac{5}{3} \cdot \sqrt{\frac{2 \cdot R \cdot \xi_{max}}{cos\beta}}, [m] \quad (6)$$

where $R, [m]$ is a radius of curvature of ice and β is a frame angle.

Thus, for any geometry case a total l can be derived as a function of the indentation depth.

In order to compare theoretical results with empirical, the line loads from measured data are determined by the method described in Suominen and Kujala (2015). The q affecting over one frame spacing $s, [m]$ are calculated by dividing the measured loads on one frame with s, which is 0.4 m. The q affecting over two frame spacing are calculated by dividing the combination of measured ice loads on two frames with the sum of two frame spacing, which is 0.8 m. The results of the q obtained from full-scale measurements for 16.12.2013 are shown in Figure 5.

Since in Popov method estimated line load characterizes the distribution of total ice load over a whole loading length, for studying a line load intensity in a range of local structures two schemes of line load intensity proposed by Popov (1987) and Kheysin (1979) were applied.

Firstly, the solutions of total line loads on each frame for all considered collision scenarios and geometry cases are analyzed. As an example, for ship-ice field impact in Table 2 the estimated peaks of line load parameters for 16.12.2013 are summarized.

Table 2. Line load parameters from Popov method for Agulhas II (ship-ice field impact)

Contact geometry case (Popov, 1967 & Daley, 1999)	Total line load, [kN/m]	Total load length, [m]	Total line load, [kN/m]	Total load length, [m]
	Frame 134.5		Frame 134	
Round	1758.7	1.37	1793.6	1.4
Angular	1405.4	2.36	1443.6	2.39
Right-apex Oblique edge	2887.8	1.15	2966.6	1.16
Symmetric V Wedge	2552.5	1.21	2603.6	1.23
Spherical	9082.8	0.46	9257.3	0.47

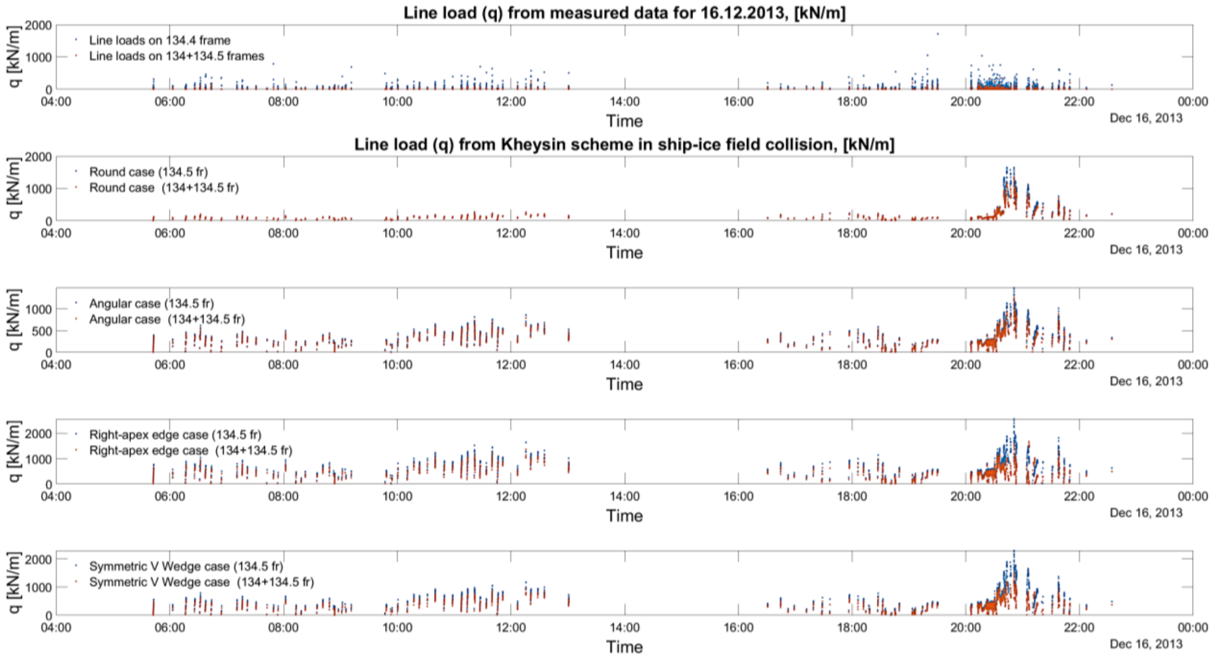


Figure 5. The example of comparing of line loads from Kheysin scheme (1979) in ship-ice field collision with obtained measurements from Agulhas II

It is found, that frame 134 experiences greater total ice loads in comparison with frame 134.5 and it is used as basic for the estimation of the local q. Secondly, using the results from Table 2 in ship-ice field impact we can find local q for any contact geometry within the total l by analyzing one frame. For example, for round indentation (frame 134) total q is distributed over 1.4 m (see Table 2) or as the span of intermediate frames is 0.4 m, it is distributed over 3.5 frames approximately. Thirdly, in Popov (1967) scheme the area of crushing of the ice edge has a parabolic segment's form, which can be applied as a line load intensity function for two frames (134 and 134.5). In the Kheysin scheme (1979) the ice load is presented in the form of trapezoidal prism shape, which takes into account the peak character of contact pressure over $0.2l$. In both schemes in each cross-section line load is kept constant.

In the estimation of local q spherical contact geometry is not included, as it presents a case with iceberg impact and results in extreme response from ship's hull. The examples of line loads in ship-ice floe collision according to Kheysin scheme (1979) are shown in Figure 5.

In Figure 6 the examples of the estimated mean, standard deviation and maxima values of q from measured data and from energy method for Kheysin (1967) scheme are also presented. In Figures 6 the left square combines the results from measured and estimated q for different indentation cases affect over $l = 0.4\,m$, which corresponds to one frame spacing. The right square in Figures 6 includes the q affect over $l = 0.8\,m$, which corresponds to two frame spacing.

Additionally, the same solutions for the line loads from Kheysin scheme (1979) in ship-ice field impact and from Popov scheme (1967) in ship-ice floe and field impact are also obtained.

At this stage we studied how to link the energy method with empirical data.

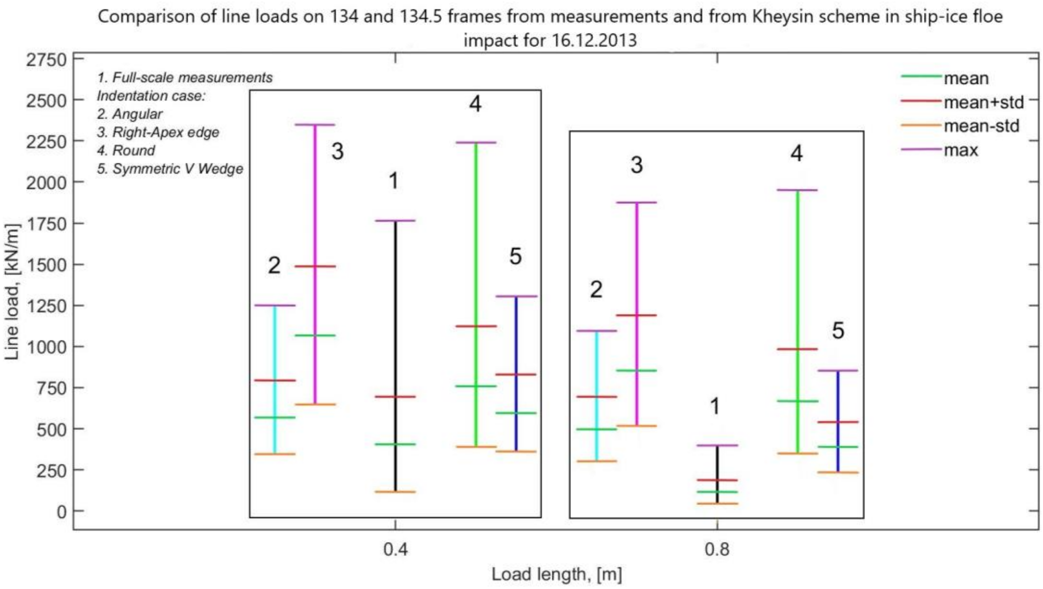


Figure 6. The example of comparing of line loads from Kheysin scheme (1979) in ship-ice floe impact with obtained measurements from Agulhas II

DISCUSSION

Contact pressure is one of the challenging questions in the estimation of ice loads on ship's hull. A lot of research was done into the effect of $P(A)$ on different impacts and a variety of formulations were derived (Riska, 2018). In the current research, the ice crushing strength is assumed as "effective contact pressure". Obtained solution for $P(A)$ in ship-ice floe collision scenario from Popov method for different contact geometry cases shows good correlation with crushing pressure-area curves based on full-scale measurements (see Figure 4). The values of estimated ice crushing strength are used in the determination of ice load parameters, where the line loads for different contact geometries are also analyzed.

At the stage of the determination of local line loads one important limitation in Popov (1967) method is identified. When trying to find theoretical line loads on two-frame spacing by the method that was applied to full-scale data (Suominen & Kujala, 2015), the results for two frame spacing are found almost the same as for one frame spacing. This phenomenon can be described by the physical model of the energy method when all ship and ice parameters are reduced to a collision point and each of collision points is isolated from one another. Consequently, there is one significant difference from Popov method and full-scale measurements is observed: using full-scale data it is possible to examine line loads taking into account the combination of several frames at the same time, when in Popov method each frame is studied independently. In this wise, in order to find theoretical line loads on two frame spacing additionally line load intensity schemes are used.

The closest results in comparing of the line loads estimated from the full-scale measurements for Agulhas II with the line loads from Popov method (1967) gives Kheysin scheme (1979). Where in ship-ice field collision scenario "Angular" and "Round" indentation cases, and in ship-ice floe collision scenario (see Figure 6) "Angular" and "Symmetrical V Wedge" indentation cases show the best correlation. Furthermore, it is found, that for comparison of the estimated and measured line loads ice conditions have to be taken into consideration. For example, from measured ice floe diameters in Figure 3 it can be observed that between $06:00$ and $13:00$ there is evident ice fields concentration and for this period of time the best correlation with full-scale measurements show the results from ship-ice field collision scenario (see for comparison Figure 3 and Figure 5). The same trend was found between $18:00$ and $20:00$ in ship-ice field impact.

Between $20:00$ and $21:30$ approximately ice diameter decreases significantly which indicates the occurrence of ice floes. Thus, in this period in the case with ship-ice floe impact theoretical line loads shift toward to the measured. Consequently, in order to avoid uncertainties and mistakes theoretically estimated ice load parameters have to be put in comparison with both measurements of ice loads and ice conditions.

Since line loads from Popov method were estimated based on the results for one frame, line loads over two frame spacing in all collision scenarios exceed the value obtained from full-scale data. Thus, it is still questionable how to estimate line loads from Popov method on several frame spacing taking into account the combination of ice loads affect on the corresponding frames.

Besides of that, in Popov model a lot of assumptions have been done that require additional research. For example, in the case with angular indentation an opening angle is a crucial factor which affects on ice load parameters. Popov (1967) from experimental studies suggested the value of the opening angle in a wide range of 45-145 degrees. According to the geometrical consideration, this angle depends on the size of cusps that form when ship is moving in ice field.

Consequently, Popov model provides a way to investigate a wide range of ship-ice impact scenarios and to estimate ice load parameters for various ship and ice properties (ship and ice mass, ice geometry, hull shape and speed). However, the energy-based method is not straightforward in analyzing of ice loads, and for today, no studies on the validation of Popov model with full- or model-scale results have been presented.

CONCLUSIONS

This research aims to analyze applicability of Popov method to full-scale data in different collision cases. In order to compare ice load parameters estimated from Popov method with experimental results, theoretical and empirical line loads are used. When comparing the line loads obtained from full-scale measurements (maxima on one frame spacing is $1762.4\ kN/m$ and on two frame spacing is $398\ kN/m$) with the line loads from Kheysin scheme (1979) in ship-ice floe collision scenario, the best correlation show:

a) "Angular" indentation case (maxima on one frame spacing is $1247.6\ kN/m$ and on two frame spacing is $1093.9\ kN/m$),

b) and “Symmetrical V Wedge” indentation case (maxima on one frame spacing is 1305.1 kN/m and on two frame spacing is $851.7\ kN/m$).

In ship-ice field impact a good correlation with empirical line load appears for one frame spacing from Popov scheme (1967) in:

a) “Angular” indentation case (maxima on one frame spacing is $1403.0\ kN/m$),
b) and “Round” indentation case (maxima on one frame spacing is $1747.3\ kN/m$).

Since design ice loads in the IACS UR for PS are based on the energy approach, it is essential to validate theoretical results with empirical. This research indicates that it is possible to put in comparison ice load parameters obtained from Popov method with full-scale measurements. In order to overcome this question different measured parameters (ice loads, ice conditions, ship’s speed) have to be included in Popov model. For further research other contact geometry cases, operational conditions and ice properties should be studied.

For practical application as an engineering tool, the energy method could be assigned in derivation of plastic structural response and improvement of safe speed assessment for ships navigating in ice.

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