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An optimal route of a vessel with presence of drifting ice feature

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ABSTRACT

Marine transport systems have a special significance for operations in Arctic regions, including the Northern Sea Route: the ongoing or planned economic activity requires transportation management. Digital models of marine transport systems are mainly based on routing algorithms, which are designed for static map.

The navigation in high latitudes sometimes takes place in presence of drifting ice floes and icebergs. Hence, dynamics of ice features governed by weather conditions is important for time and risk management in navigation.

The paper considers the problem of finding the optimal route on the static ice map in presence of a single dynamic ice feature. To solve this problem, the composite model is suggested. The static part of the map with ice conditions is given by the graph on which route optimization is performed by means of A* algorithm. On the dynamic part of the map the ice feature thought as a velocity object performs both translational and rotation motion. To avoid possible collision of a vessel with this ice feature, an avoidance maneuver or vessel motion delay can be used. In the paper it is shown that simple avoidance maneuver calculated with respect to ice feature motion law can significantly decrease the travel time.

KEY WORDS: Routing; Ice; Drift; Map; Graph.

INTRODUCTION

During last decades the atmospheric processes triggered by warming led to changes of ice conditions in the Arctic regions. From an economic point of view, the melting of ice in the Arctic provides new maritime opportunities: the sailing time along the Siberian Coast of the Northern Sea Route has been reduced almost twice since 1990-s (Aksenov et al., 2017). Facilitating of ice conditions leads to better Arctic ports accessibility, reducing navigation limitations and transportation costs. Sailing in high latitudes has specific operational challenges: presence of drifting ice floes, ice features and icebergs. Their behavior is governed by wind and currents (Leppäranta, 2011; Turnbull et al., 2015; Yulmetov et al., 2016). Forecasting of trajectory in presence of such features is not an easy task.

Special navigation software, which would estimate ice risks on the way and contribute to finding a safe and economically effective route, may be a helpful instrument for the safe and effective operational employment of sea routes and ports in the Arctic regions.

The history of transport transit models in ice-covered waters development goes back to 2001, when Riska and coauthors studied influence of ice conditions on ship transit time in ice (Riska et al., 2001). Later in 2003 (Frederking, 2003) it was reported that Canadian Hydraulics Centre developed the software, which could calculate travel costs of given transit route in ice.

In 2009 Kotovirta and coauthors described the IRIS − a model for route optimization in ice conditions developed in VTT, Finland (Kotovirta et al., 2009). The system was tested for conditions of the Gulf of Bothnia, the Baltic Sea. IRIS model predicted appearance of ice ridges occurrence on the way and their geometrical parameters as well as took into account ship ice resistance.

The approaches to transportation system development for the Arctic are applied nowadays in Norway and Finland (Valkonen et al., 2013; Valkonen and Riska, 2014; Bergstrom et al., 2014). In 2016 the authors of this paper presented the multicriteria route optimization model on the graph based Dijksta algorithm (Zvyagin and Voitkunskaia, 2016). The study considered two criteria including additive criteria for travel route costs minimization and minimax criteria for less risky route finding. It was demonstrated that the solution is Pareto-optimal.

Later in 2017 Krylov State Research Centre (KSRC, St. Petersburg) presented ice-management system for Arctic routes, including training complex for training the crews of the supply vessels in operations in Arctic conditions (Kazantsev et al., 2017; Tarovik et al., 2017).

The concept of sea route planning system applicable for ice environment based on A* algorithm was presented in 2017 by Zhang et al. (Zhang et al., 2014). The A* method with modified components g and h of the objective function was implemented by means of MATLAB. In the concept, the travel risk was interpreted as additional travel cost along the graph edge. The modified way of g and h calculation allowed reducing ship-steering changes along the route.

The structure of the paper is as follows. The first section is introductive. In the second part the principles of composite routing model are proposed. In the third part the methods of route optimization used in composite model for static part of the map are discussed. In the fourth part a model of collision avoidance maneuver for dynamic part of the map is proposed. In the fifth part the computational example is provided. The last part is conclusive.

COMPOSITE ROUTING MODEL

Besides the significant number of publications on navigation systems in ice-covered waters, which are listed in the introductive section of this paper, a few of them discuss an algorithmic part of such systems.

Land route planning usually employs graph model for environment description. Traditional algorithms are used for land route optimization and based on graphs are Breadth-first search, Bellman–Ford algorithm, Dijkstra (Dijkstra, 1959), and its modification A* (Cormen et al., 2009). For instance, the authors (Zvyagin and Voitkunskaia, 2016) had previously applied Dijkstra algorithm for finding Pareto-optimal route on static ice chart. A* algorithm is based on Dijkstra, but usually converges faster, so it was applied for route optimization by Tarovik et al. (2017) and Zhang et al. (2017). Again, in these papers the optimization was performed for static state of ice conditions, so the motion of ice floes and ice features on the graph were not considered.

In the following study we will split the map, on which routing should be performed, into two parts: first larger part, where ice obstacles are static, and the second smaller part with a moving ice floe/ice feature. In the second part of the map we will estimate the optimal route numerically from a set of starting points, taking into account ice feature motion and performing an avoidance maneuver if needed. We propose here a model which finds optimal route on the whole map by using A* algorithm with additional component in aim function, whose values are given by results of route optimization in the second ("dynamic") part of the map.

Figure 1 provides an image of model ice floes on the surface of the ice tank in Krylov State research Centre. The part bounded by blue rectangle is supposed to be dynamic with single moving broken ice feature, while other part is thought to be static. The image is marked with rectangular grid, on which graph will be built subsequently.

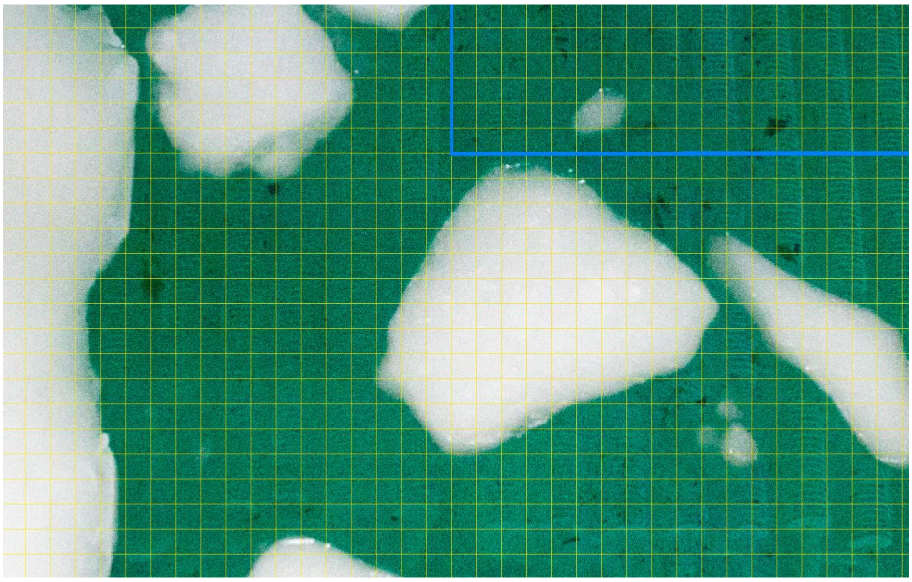


Figure 1. The image of the part of the ice tank surface with broken model ice

The method described in this paper is intended to find the route through leads and free-water areas around ice obstacles, avoiding moving ice feature floating in some part on the map. We consider motion with constant velocity. The aim will be to get from the start point to the finish point in minimum time. For constant velocity movement on the static map this is the same as getting the shortest route. In dynamic part of the map we consider the case of the shortest route blockade by floating ice feature (velocity object), so minimum passage time will be provided by avoidance maneuver with longer route, than the shortest available route with the period of stand-by.

ROUTING METHODS FOR STATIC PART OF THE MAP

In routing problems formulated for water-covered areas the whole area with appropriate depths is covered by the grid. Usually this grid has rectangular cells. Nodes of this grid are considered as vertices of the graph, and sides of rectangular cells – as edges of the graph. Weights are assigned to edges according to travel costs or risks associated with them. Such graph model allows application of corresponding algorithms for solving a routing problem. Usually the weight function for route is to be minimized. Dijkstra algorithm finds the shortest routes from start vertex to finish vertex in the graph, whose edges have non-negative weights (Cormen et al., 2009). It belongs to class of greedy algorithms, which make locally-optimal choice on every step, and have polynomial complexity. Dijkstra algorithm builds spanning tree of the graph. This spanning tree is a basis of the graph and contains shortest routes from start vertex. Due to the Rado-Edmonds Theorem (Edmonds, 1971) this greedy algorithm will find optimal route on every graph matroid consisting of connected acyclic edge sets.

Moreover, it is proven that the found solution can't be enhanced by selecting greater number of edges with smaller edge travelling costs.

The A* algorithm implies basic ideas of Dijkstra algorithm on the modified weight function: there besides the travel cost from the start vortex, the estimate of distance to finish is added. This algorithm is greedy, has polynomial complexity and operates mainly with those vertices, which provide targeted propulsion to the finish. So the A* method considers only the subgraph of spanning tree instead of complete spanning tree as Dijkstra method does. This advantage allows avoiding paths which obviously lead away from the finish vortex.

Another advantage of A* algorithm is the possibility of modifying the weight function according to our purposes: we can assemble it as a sum of separate functions, each of them describing the special practical need. Two summands in the aim function are classic – the estimate of route length from the start point to current position, and the estimate of remaining distance from current position to finish.

For fine tuning of the aim function we use constant coefficients of each summand in that weight function. Such coefficients will regulate the contribution of each summand in the weight function, for example, increase the velocity of convergence in free waters and decrease velocity of convergence with considering alternative routes on maps with ice conditions.

The solution obtained by modified A* algorithm with composite weight function as mentioned above can be suboptimal, but practically it can be the best available option. The A* algorithm with modified weight function remains in class of greedy algorithms and has polynomial complexity.

On the static part of the map we implement an ordinary A* algorithm to find sub-optimal route. In our study we define cost function in time units.

Let us consider route T_{x_i} from the start node x_S to the node x_i, which contains l edges:

$$T_{x_i} = (x_S, x_1, \dots, x_i),\ \ |T_{x_i}| = l \qquad (1)$$

Let us denote the destination node on the static part of the map with x_{F1}, while destination point on the whole map will be defined with x_F. The usual weight function of A* algorithm at graph node x_i for route T_{x_i} has the following form:

$$f(T_{x_i}) = g(T_{x_i}) + h(x_i) \qquad (2)$$

where $g(T_{x_i})$ is the cost of the route T_{x_i}, and $h(x_i)$ is the component, responsible for the closeness of the node x_i to the finish node x_{F1}. Let us denote desired speed of the vessel with V_0. Then

$$g(T_{x_i}) = \frac{L(T_{x_i})}{V_0} \qquad (3)$$

where $L(T_{x_i})$ is the length of the route T_{x_i}. Here heuristic function $h(x_i)$ can be defined in the same way, using Euclidian or Manhattan distance from x_i to x_{F1}. The function (2) is optimized using the logic scheme of Dijkstra algorithm.

Dijkstra algorithm considers all available vertices of the graph, which have some connection with starting point. In standard A* algorithm the component $h(x_i)$ from function (2) pulls the algorithm to the destination point. This increases speed of convergence on areas free of obstacles, but does not let algorithm investigating surroundings of the route in the case of complex map. Taking into account that the route found by the A* can be suboptimal, an examination of surroundings can let algorithm to find other proper route.

To obtain flexibility of algorithm convergence the modifications and improvements of this function (Hao et al., 2005) can be used. In this paper we consider the following modification of weight function for the edge between nodes x_i and x_j:

$$f\left(T_{x_i}, x_j\right) = g\left(T_{x_i}, x_j\right) + k_1 h\left(x_j\right) + k_2 b\left(x_j\right) \quad (4)$$

Here $b\left(x_j\right)$ is a barrier function. Tuning of algorithm convergence speed can be performed by variation of constant factors k_1, k_2.

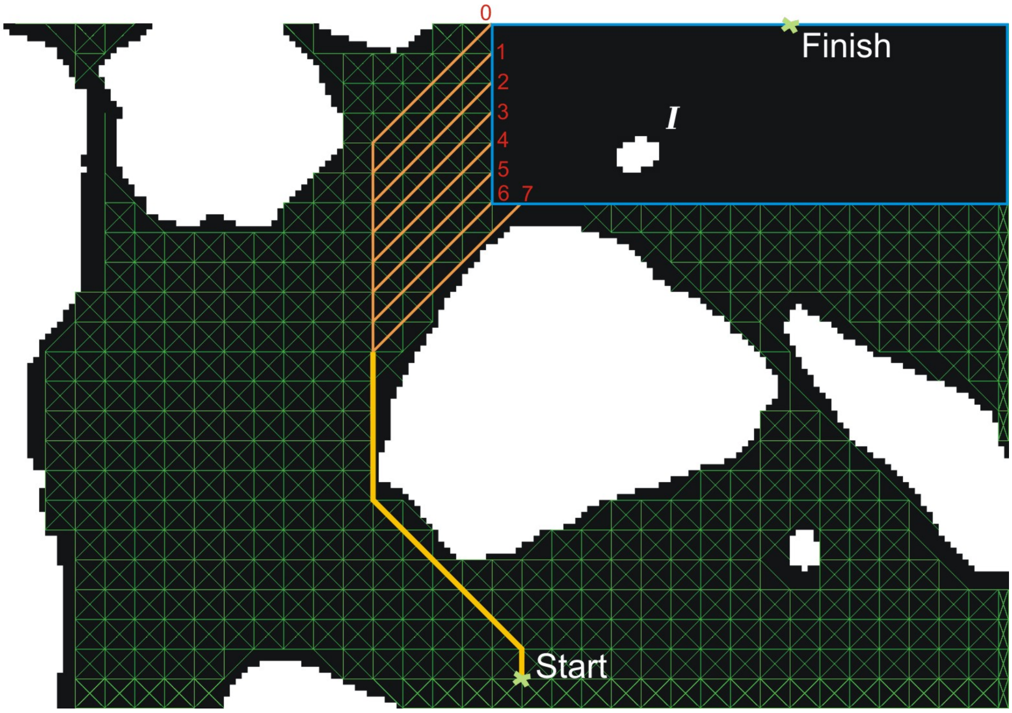


Figure 2. Routing on static map by A* algorithm in composite model

COLLISION AVOIDANCE MANEUVER

Drift of icebergs and sea ice floes are governed by atmospheric and ocean forces (Lepparanta, 2010). Direction and velocity of an iceberg may significantly change within 24 hours. Turnbull et al. (2015) provided observations of ocean current speed and direction affecting the keel of an iceberg. Basing on plots provided there we put forward a supposition for our model, that ice feature performs both translational and rotation motion. Forces, which influence iceberg drift, are summarized by Kubat et al. (2005). However, the trajectory of a drifting iceberg is usually complicated due to wind and current drag forces variation.

Developing the model of vessel's optimal route in ice conditions, we consider the situation of possible collision with moving ice formation in the small-size free-water area. Here we can consider an ice formation, which can be an iceberg or an ice floe, as a velocity obstacle (Fiorini and Shiller, 1998). Collision preventing at sea is mostly studied for a "vessel-vessel" case (Pedersen et al., 2003; Huang et al., 2018). In general, the implemented models can be taken for a "vessel-ice formation" case with the difference that ice formation velocity cannot be regulated to avoid collision. In our case we consider deterministic models both for a vessel and an ice formation.

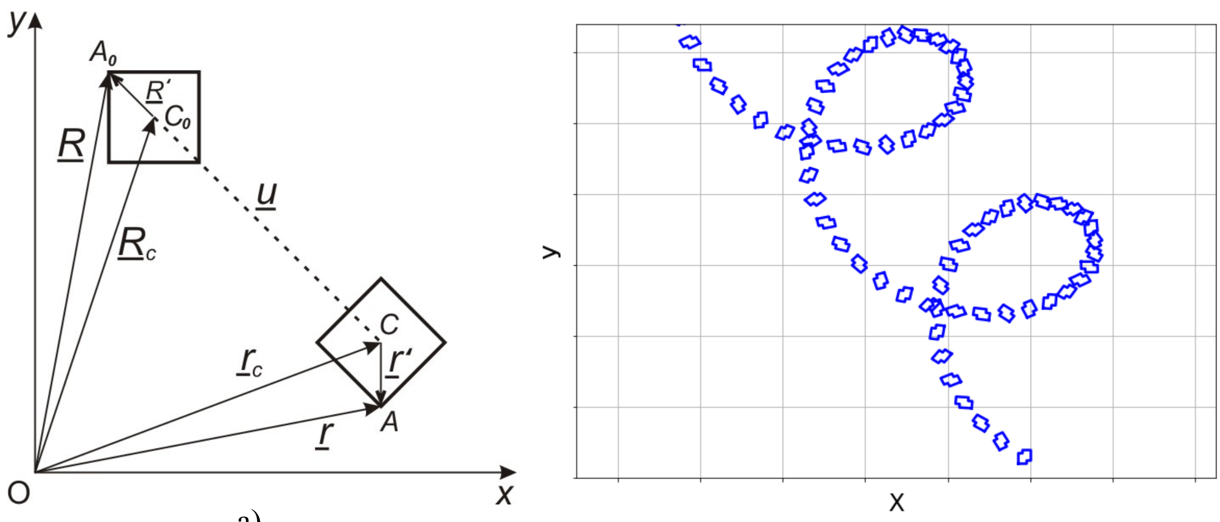
Let us describe the kinematics of ice formation motion. According to Maddock et al. (2011) in the southeast Beaufort Sea ice drift inertial oscillations were observed, which had a period of slightly more than 12 hours and resulted in cusps or loops in the ice drift path. In our model an ice formation given by its contour performs two types of movement: rotational and translational. The motion law of the ice formation contour is illustrated in Figure 3 a. Let the point A of the moving contour have its position A_0 in the reference configuration and given by the radius vector $\underline{R}$, while the center of the mass C_0 of an ice formation is given by the radius vector $\underline{R_c}$. In the current configuration new position A_1 of the contour point A and new position C_0 of the center of mass are given by radius vectors $\underline{r}$ and $\underline{r_c}$, respectively. Radius vectors R_c and r_c, which define centers of the ice formation mass in reference and current configurations, are related with each other by the displacement vector $\underline{u}$. We associate the moving reference system with the center of the ice formation mass. Thereafter, the radius vector $\underline{r'}$ of the new position A_1 of the point A is related to the radius vector of its initial position by the $\underline{\underline{Q_1}}$ rotation tensor. In general, we describe the law of motion of each point of the ice formation contour by the following relation:

$$\underline{r} = \underline{\underline{Q_1}} \cdot \left(\underline{R} - \underline{R_c}\right) + \underline{R_0} + \underline{\underline{Q_2}} \cdot \left(\underline{R_c} - \underline{R_0}\right) + \underline{v} \cdot t \tag{5}$$

where, R_0 is radius-vector to the center of circular motion of ice feature, which also moves translationally, and v – its circular velocity. Rotation tensor $\underline{\underline{Q}}$ can be given by the mechanical formula:

$$\underline{\underline{Q}} = \underline{mm} + \cos(\omega t) \cdot \left(\underline{E} - \underline{mm}\right) + \sin(\omega t) \cdot \left(\underline{m} \times \underline{E}\right) \tag{6}$$

where $\underline{m}$ is rotation axis, which is orthogonal to the plain of moving ice formation, and ω is angular velocity of ice formation. Figure 3 b shows an example of long-term trajectory of ice formation moving according to (5) and (6). In this study we will consider only a part of such complicated trajectory, because velocity of ice is usually much less than the velocity of vessel.



a) b)

Figure 3. a) Ice formation motion configurations scheme, b) long-term trajectory of ice formation contour

Let the vessel get from the start point to the finish point positioned on the water area without shores and maintain variable speed from 0 to V_0, being maximum possible speed. Let an ice formation have complicated geometric shape defined by its contour, and move according to (5) and (6). Let the trajectory of an ice formation cross the shortest route of the vessel from its start point to the finish point, as it is shown in Figure 4. Let current positions of vessel and ice formation, maximum vessel's speed and ice formation's speed components be defined in such a way that collision should happen. So the vessel needs to perform an avoidance maneuver to avoid the collision.

The simplest avoidance maneuver suggests a vessel speed change (Fiorini and Shiller, 1998) with maintaining its direction. This will let the ice formation pass, and the vessel go its way straight to the finish with no obstacles. This is an optimal decision when route length is needed to be minimized, but it is not optimal when we need to minimize the time of the journey. In opposite, the direction maneuver provides longer way, but can significantly reduce the travel time.

In Figure 4 a vessel is presented as the material point. It is moving with its maximum speed V_0. Corresponding positions of a vessel and ice formation are designated with the same numbers. If a vessel will maintain its direction and speed, the collision will be unavoidable. Let us change the direction of the vessel's route maintaining its velocity, as it is shown in Figure 3. This way, a vessel passes astern the ice formation and then goes to the final point course. Numerically we can adjust trajectory of avoidance maneuver to decrease the route length as much as possible. In this example, performing this maneuver increases the route length by 0.79% in comparison with the straight direction, but decreases the passing time by 11.13% compared with the case, when a vessel does not change the direction, but slows down to let ice formation pass by.

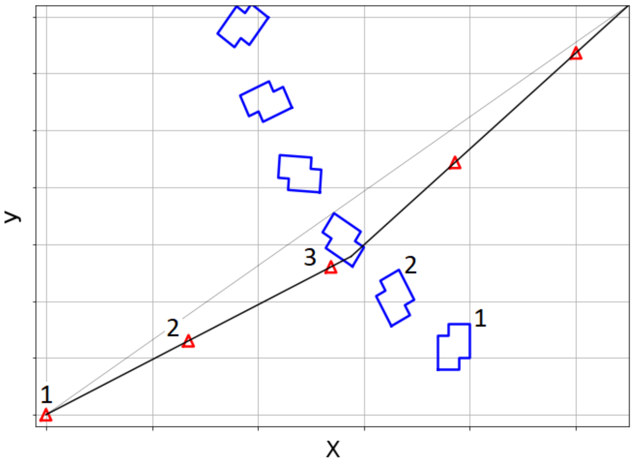


Figure 4. Principle of the model of velocity ice feature avoidance maneuver

AN EXAMPLE OF APPLICATION

To test the proposed composite model we consider routing on a map built on the basis of broken ice and leads image and provided in Figure 1. By means of segmentation tool developed in St. Petersburg Polytechnic University we get binary unnoised segmented map, provided in Figure 2. The graph is built on the basis of rectangular grid with given cell size and with diagonal edges in each cell. Applying the approach used earlier by the authors (Zvyagin and Voitkunskaia, 2016), we leave in graph only those edges and nodes which lay on free water. We assign the following scale to the grid in Figure 2: every cell has size 200 x 200 m.

Rectangular area in the top right corner of the map is the area of dynamic motion not covered with graph. There the ice feature I performs translational and rotation motion according to the model (5) and (6) with the following parameters: horizontal and vertical components of translational motion velocity are -0.162 and 0.6156 knots, respectively, rotation velocity component ω around mass center is 0.5 (rad/sec) and circular velocity v is -0.2 (rad/sec).

A vessel needs to get from the start point to the finish point in the shortest time. Moving by the given graph edges, it can reach the dynamic area at 8 points, which are denoted in Figure 2 by numbers 0-7. Hence, applying the A* algorithm for route optimization we can find an optimal route on given graph from the start point to each of that 8 points. Such routs are shown in Figure 2 with orange lines; they have a common part, which is shown with yellow color. Supposing that a vessel moves with constant cruising speed 15 knots, we find the time, which is required for getting from the start point to the border of dynamic area (Table 1).

Inside the dynamic area, according to the parameters of ice feature motion, the direct route of a vessel to the finish point with the same velocity as on the static part of the map can lead (from nodes 5, 6, 7) or cannot lead (from nodes 0, 1, 2, 3, 4) to collision. However, the parameters of this example are the ones making the direct routes free of collision are too long. From nodes 5, 6, 7 avoidance maneuver described in previous section is performed. In our model we do not consider afore passing of an ice feature for safety reasons, and route only ice feature astern passing.

The final calculations for possible routes are provided in Table 1. In this case avoidance maneuver allows decreasing the travel time compared to the shortest route with a vessel waiting for an ice formation to pass by.

Table 1. Characteristics of composite routing model for example provided in Figure 2

Node number	0	1	2	3	4	5	6	7
Time needed to get from the start to the node, min	11.1	10.69	10.26	9.83	9.39	8.96	8.53	8.7
Distance from the node to the finish, m	2000	2010	2039.6	2088	2154	2236	2332.4	2163.2
Time needed to get from the node to the finish by direct route with vessel's stand-by if needed, min	4.32	4.34	4.41	4.51	4.64	6.67	6.05	5.18
Route length from the node to the finish with avoidance maneuver, m	2000	2010	2039.6	2088	2154	2278	2345.2	2168
Time needed to get from the node to the finish with avoidance maneuver, min	n/a	n/a	n/a	n/a	n/a	4.92	5.08	4.67
Time savings in dynamic part, %	n/a	n/a	n/a	n/a	n/a	26.2	16.1	9.6
Route increasing with avoidance maneuver in dynamic part, %	n/a	n/a	n/a	n/a	n/a	1.88	0.55	0.22
Total time needed for the route from the start to the finish, min	15.44	15.03	14.66	14.34	14.04	13.89	13.61	13.39

CONCLUSIONS

The composite model proposed in the paper allows combining advantages of route optimization on a graph by means of A* algorithm and dynamic model for predicting single ice floe motion. A* algorithm successfully allows finding suboptimal routes from the given start point on the static map to nodes on the border of a dynamic part of the map. In the proposed composite model all time-dependent changes are located only in a dynamic part of the map. In this part the moving ice floe is situated, collision of the vessel with it should be avoided. Dynamic tensor-based model describes spiral ice feature motion, which imitates tidal influence.

In a short-term period, which corresponds to small-size dynamic part of the map, the track of the ice feature is accepted as non-linear, but smooth. With known trajectory of the moving ice feature, this velocity obstacle can be avoided by vessel stand-by and waiting when obstacle will pass by or by an avoidance maneuver. In this study, the route of such maneuver is adjusted numerically.

The avoidance maneuver considered in this paper allowed saving up to 9.6% of the travel time inside the dynamic map area increasing the route length only by 0.22%. Resulting optimal (suboptimal) route on the map consists of the route on static part found by A* and the route on the dynamic part with respect to velocity obstacle avoidance maneuver.

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