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Modification of ship routing algorithms for the case of navigation in ice

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ABSTRACT

Navigation in ice-covered waters has a number of specific features that distinguish it from the open water operation; they are the non-stationary ice conditions, ice channels on the fairways, and the additional opportunities such as the possibility to involve an icebreaker and change the mode of movement (stern- or bow forward). All these features should be considered when solving the problem of ship route optimization in ice. In this paper, we propose the modifications of the well-known graph-based and cell-free (wave-based) mathematical methods of path finding to adapt them to the problem of route optimization in dynamic ice conditions considering all above-mentioned features. We formulated the versatile cost function that involves such factors as the total voyage time, fuel consumption, freight rates of ships and risks of ice operation. The optimization task is set in such a way to allow finding the route segments where the icebreaker assistance is economically proven and optimize the sailing mode of double-acting ships. We also proposed the original and computationally efficient algorithm to reduce the number of points in a wavefront based on the Concave Hull method (for the wave-based approach). The strengths and weakness of the proposed methods are demonstrated using the specially developed research software where we compare various approaches under the same input datasets.

KEY WORDS: Routing; Ice; Icebreaker; Double-acting Ships; Concave Hull.

INTRODUCTION

Offshore development and growing prospects of commercial shipping in the Arctic set the task of optimal ship routing in ice. Navigation in ice-covered waters has a number of specific features in contrast to open water operation. The important peculiarities are a) the substantial temporal and spatial instability of the environment where ship moves and b) the critical influence of ice conditions on ship performance. The essence of ship routing in ice is the effective use of the local areas with weak ice or ice lanes, even if the voyage distance increases significantly. It may be proven by the fact that the real tracks of ice-going ships and icebreakers are the complex labyrinthine curves that change from day to day reflecting the dynamics of ice conditions. Significant non-stationarity of ice cover leads to the fact that many well-known conservative methods of optimal path finding on a predefined spatial graph (Dijkstra, Dynamic programming) turn out to be inapplicable or require special modification to be applied to this task. Another important extension of the task of ship routing in ice is the new variables in the optimization problem. The subjects of route optimization in open water are the route itself and the speed of the vessel. For ice navigation, there are additional control variables: a) the ability to involve an icebreaker, b) the choice of a movement mode (stern- or bow forward) for the double-acting ship. Indeed, the use of an icebreaker and turnaround of the ship in ice are quite expensive operations both in time and cost meanings.

The active development of ice routing methods began in the 2000s and was governed by the growth of Arctic navigation and new possibilities of radar satellite imagery. Graph-based algorithm of step-by-step "greedy" optimization with the use of Powell's method was presented by Kotovirta et al. (2009). They set the task of ice routing in three-dimensional space (two space coordinates and time) taking into account a number of spatial restrictions, such as land area, pathways, and areas of shallow water. Choi et al. (2013) introduced the genetic optimization algorithm instead of the "greedy" one. They also analyzed the advantages of continuous routing in comparison with vessel movement through the fixed nodes and presented a new formulation of the objective function as the weighted sum of distance, smoothness, time and ship safety parameter. Nam et al. (2013) applied the Dijkstra's algorithm to choose the best way on a set of segments along the Northern sea route (NSR) and defined the objective function as the total cost, i.e. the sum of fuel price, operational and capital costs, port charges, and icebreaker fee. This is the only study we know, where the icebreaker assistance is considered in a frame of routing problem. At the same time, there is an assumption that the ship engages an icebreaker if the predefined condition is met, i.e. the optimization of the amount of icebreaker assistance is out of the investigation. Ice routing methods that are based on the A* heuristic algorithm have attracted much interest in recent time due to the favorable calculation time. There are several studies where the basic A* algorithm is modified for better applicability to the ice routing problem (Wang et al., 2018).

In this article, we address the issues related to the optimal ice routing of ships and icebreaker-escorted convoys in a non-stationary environment with the use of graph-based and cell-free (wave-based) mathematical programming methods. We modified the formulations of these routing approaches to consider independent (bow forward), icebreaker-assisted and stern forward operation of ship simultaneously, i.e. in the frame of a single mathematical formulation of the ship and convoy optimal routing problem in non-stationary ice conditions.

MATERIALS AND METHODS

Any optimization problem is described by the control variables, the optimization criterion, and the constraints. The control variables in case of ship routing in open water are the trajectory of movement and the speed of the vessel (Walther et al., 2016). Somewhere only the trajectory varies for a predetermined speed (Wang et al., 2017), somewhere only the speed for a given route (Sotnikova et al. 2018) or both characteristics together (Lin et al., 2013). At the same time, additional risks arise in case of ship navigation in ice; therefore, vessels always tend to pass the ice areas with the highest possible safe speed. The control variables in case of ice navigation remain the route and the mode of movement. Thus, the general mathematical formulation of ice routing problem could be done as follows:

$$\langle \mathbf{S},\mathbf{M}\rangle_{opt} = \arg\min_{\mathbf{S},\mathbf{M}} \int_0^T C\left(\mathbf{S}(t),\mathbf{M}(t)\right)\cdot dt \quad , \tag{1}$$

where $\mathbf{S}(t)=\langle X(t);Y(t)\rangle$ is the vector of the current ship (or convoy) position on the (X, Y) – plane of geographical coordinates; $\mathbf{M}(t)$ – vector of indices (Boolean variables or enumerators) that determines the regime of movement at time t; C – total cost per time unit.

Temporal dynamics of $\mathbf{S}(t)$ determine the route of the ship, while the vector $\mathbf{M}(t)$ defines the fact of icebreaker assistance, type of navigation (stern- or bow forward) and other relevant parameters of the movement regime at the route point $\mathbf{S}$ and at time t. The set of constraints on the optimization problem (1) are the boundary conditions:

$$\mathbf{S}(0)=\mathbf{S}_0,\ \mathbf{S}(T)=\mathbf{S}_T\ , \tag{2}$$

where $\mathbf{S}_0$ and $\mathbf{S}_T$ are the fixed initial and destination points of the route. The time boundary T that corresponds to the time of reaching the point $\mathbf{S}_T$ can be either free or fixed value. In the latter case, there should be an additional constraint on the allowable date of voyage end. The operational limitations on the ship speed in specific ice conditions may be formulated as:

$$\left\|\dot{\mathbf{S}}(t)\right\|=V\left(\mathbf{K}(\mathbf{S},t),\mathbf{M}(t)\right)\ , \tag{3}$$

where V is the vessel or convoy operational speed at the point $\mathbf{S}$ at time t and in the current movement regime $\mathbf{M}$. V is calculated using the vector $\mathbf{K}$ of environmental parameters (ice, weather, depth, etc.) and the calculation model of ship movement (ship transit model). We suppose that environmental parameters $\mathbf{K}$ at any time and at any geographical point as well as the ship transit model (i.e. the internal logic of $V(\mathbf{K}, \mathbf{M})$ algorithm) are known.

The set of equations (1-3) defines a problem of the calculus of variations, which exact analytical or even numerical solution is impossible at practice. Typically, the simplification is necessary; it relates to the spatial and/or temporal discretization of the originally continuous problem. The resulting formulation has a finite dimension and can be resolved by different applied methods. Below we present the corresponding formulations for the graph-based and the wave-based approaches.

Graph-based approach. The essence of the classical graph-based or cell-based path finding method is the replacement of a continuous geographical space with a predefined set of discrete points that form a regular grid. The grid is converted into a graph by connecting vertices by the edges. One can connect each point with its nearest vertical and horizontal neighbors only (quasi-rectangular graph), but the diagonal edges between vertices through several layers are also possible. In such a way, one can increase the graph coherence.

Calculation of the total unit costs C for every edge can be done by means of direct simulation of vessel movement along this edge. Ship speed and all operational costs are determined by the values of ice- and weather parameters at the corresponding geographic point and at a particular time. The latter creates difficulties when using such classical methods of optimal path finding on weighted oriented graphs as dynamic programming (de Wit, 1990) or Dijkstra's algorithm (Zhu et al., 2016). This is because the cornerstone of these approaches is the successive marking of the vertices by tentative distance values. For the backward (dynamic programming) or forward (Dijkstra's) algorithms the weight of each edge must be assigned to a constant time-independent value, known in advance. The latter is impossible for the unsteady ice conditions, when the environmental parameters, ship speed, transit times and associated costs depend on when the ship reaches a particular point, i.e. from the entire prehistory of its voyage.

The possible and widely used approach to overcome this restriction is the use of a 3D graph instead of 2D one when the third dimension is a discrete time (Veremei, Sotnikova, 2016). Such an approach allows taking into account any type of environmental non-stationarity, but significantly increases the computational complexity of the problem. Another way is to use various heuristic methods of search by the first best match on the graph, for example, the A* algorithm. Despite that this algorithm does not guarantee to find the exact solution, it can be easily adapted to solve the problem of path optimization in deterministic time-dependent dynamic networks. Therefore, we chose the A* algorithm as the basis to formulate the ice routing problem taking into account the mentioned aspects of ice navigation.

We introduced the three-layer graph (see Fig. 1) in order to consider the alternative edge prices for all possible combinations of ship motion regimes (icebreaker assisted operation, stern- and bow forward navigation) with corresponding operational costs and speeds of movement. The middle layer of the graph defines the parameters of independent bow forward ship operation;

bottom layer describes the icebreaker assisted operation (we suppose that in theconvoy ships always move bow forward); upper level relates to the stern forward independent movement of a double-acting ship.

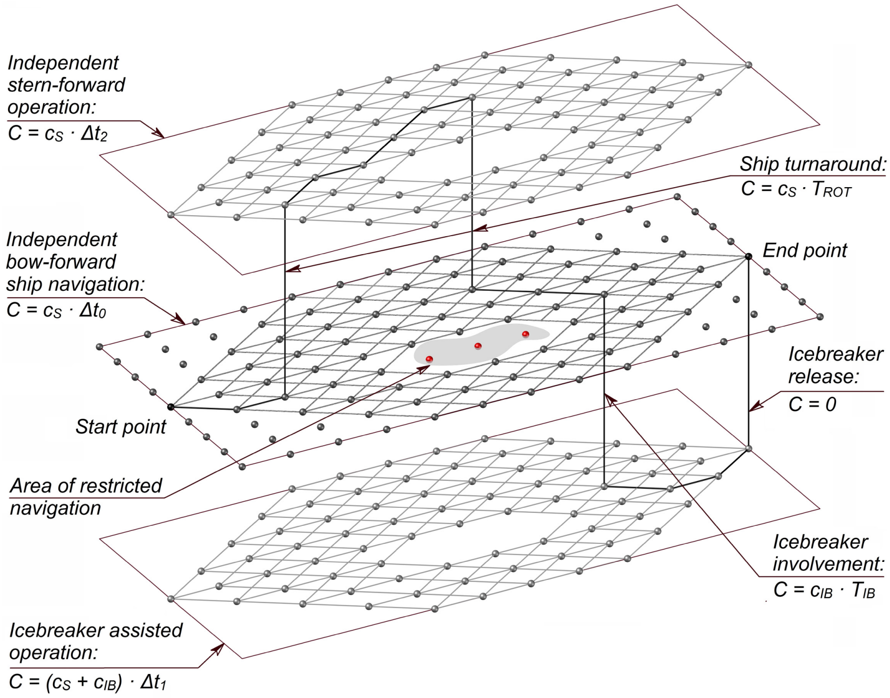

Figure 1. Graph-based formulation of ice routing problem considering icebreaker assistance and the ability of the ship to change the mode of movement.

Below we consider the simplest case of calculation of operational costs, which are based on the ship and icebreaker freight rates only (c_S and c_{IB} respectively), with no fuel consumption and no additional costs. In this case, the cost of the lower layer edge could be expressed as:

$$C_E = (c_S + c_{IB}) \cdot \Delta t_1 = (c_S + c_{IB}) \cdot \frac{|E|}{V_{SIB}(E,t)} \ , \tag{4}$$

where Δt_1 – time of convoy movement through edge E, $V_{SIB}(E, t)$ – the attainable speed of the convoy (ship + icebreaker) on the edge E at time t.

Similarly, the edge price in the middle layer is calculated by the same principle, where the movement time Δt_0 is determined by the attainable speed of vessel independent movement and unit cost equals to ship freight rate only. The transition from the middle layer to the lower one is possible at any point; the transition cost C_E is resulting from two factors:

$$C_E = c_{IB} \cdot T_{IB} \ , \tag{5}$$

where T_{IB} – the conditional time that icebreaker needs to reach the point where it meets the ship.

This conditional cost of involving an icebreaker approximately consider the fact that icebreaking resources are limited and ship cannot get an icebreaker free of charge at the required point, as well as the ship is not able to involve an icebreaker as many times as necessary. The parameter T_{IB} has only cost-dependent meaning and does not correspond to any time delay of the ship in the voyage. The cost of the reverse transition to the middle layer from the lower one is assumed to be zero, i.e. the release of an icebreaker is free of charge. Transitions from the middle layer to the upper one and backwards describe the turnaround of a double-acting ship to move in the stern or bow forward modes. Formally, such operation is allowed at any node of the graph, but its cost and required time might be excessively high. The cost of the corresponding vertical edge can be expressed as:

$$C_E = c_S \cdot T_{ROT}\left(t, \mathbf{S}\right) \text{ ,} \tag{6}$$

where $T_{ROT}\,(t,\boldsymbol{S})$ is the estimated time of ship turnaround that depends on ice conditions at a particular location at a given time t. High-tonnage ships need much more time for the turn than the small ones, also sometimes ship cannot turn due to severe ice conditions (tick ice, very close drift ice, ice compressions, etc.). In the latter case, the corresponding value of $T_{ROT}\,(t,\mathbf{S})$ is equal to infinity. It should be noted that, in contrast to the case of involving an icebreaker, the edges that describe the turnaround have not only cost but also duration.

It is not necessary to set the edges that connect the top and bottom levels because the corresponding transitions can be done by passing through the middle level. The reason is that, according to our assumptions, both involvement and release of an icebreaker are the operations with zero duration, therefore the costs of simultaneous ship turnaround and icebreaker involvement must be really summarized.

Wave-based approach. An alternative group of cell-free numerical algorithms for path optimization includes different methods that do not require prior explicit spatial discretization. Below we consider the wave-based method that solves the routing problem by means of the consistent construction of lines of equal criterion value (isolines) in the geographical space. The transparent physical analogy of this approach is the propagation of a surface wave in an inhomogeneous medium. This physical analogy implies that the reasonable way of mathematical modeling of wave propagation is the use of the Huygens-Fresnel principle, which states that every point on a wavefront is itself the source of the spherical wave. The sum of these secondary waves determines the form of the wave at any subsequent time (Topaj et al., 2019).

The earliest interpretation of wave-based approach assumes that the objective function is the total voyage time. In this case, isolines are the lines that ship can reach during the same time, i.e. the isochrones (James, 1957). Extension of this principle allows using other optimization criteria. In case of minimizing the fuel consumption, isochrones are converted into isopones (Klompstra, 1992), in case of a purely economic criterion – into the isocosts, i.e. the lines that ship can reach with equal expenses. When discretizing, the wavefront lines are replaced by the sets of points that form the discretized equal-level lines of optimization criterion C (see Fig. 2).

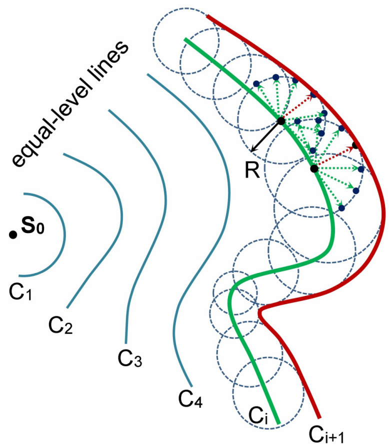

Figure 2. The layout of wave propagation in the path optimization problem (Topaj et al., 2019).

The general principles of the wave-based numerical algorithm in case of independent vessel navigation are the following. Each step of wave propagation consists of two stages: a) generation of points in a new wavefront and b) reduction of their number by excluding the "unpromising" ones. The first stage means that each point of the previous level generates the "fan" of points of the next level set, where the angle between neighboring rays is determined by the selected spatial resolution. The distance R_i, that is the radius of propagation of the secondary wave from each point i, is calculated as:

$$R_i = V_i\left(\mathbf{K}\left(\mathbf{S},t\right)\right)\cdot\Delta t_i\left(\mathbf{S},t\right) = V_i\left(\mathbf{K}\left(\mathbf{S},t\right)\right)\cdot\frac{\Delta C}{C_i\left(\mathbf{S}\left(t\right)\right)}\ , \tag{7}$$

where ΔC is the selected constant resolution of contour lines in terms of the optimization criterion. In case of searching the fastest route, equation (7) degenerates to $V_i\left(\mathbf{K}\left(\mathbf{S},t_i\right)\right)\cdot\Delta t$, i.e. the classical problem of isochrone method when Δt is a constant predetermined value.

Reduction stage allows decreasing the number of points on the next isoline by constructing the envelope curve. This stage is essential to overcome the "curse of dimension" because if we leave all the generated points on the next isocost, their total number will grow exponentially and the corresponding computational algorithm will fail. The main idea of this stage is to leave only the most promising points on the current wavefront and exclude all others. Promising points form something like a Pareto set for the current wavefront.

There are several techniques to solve this task. The most popular one is a sub-channel-based method introduced in (Hagiwara, 1989) and improved by several researchers (Szłapczyńska, Śmierzchalski, 2008; Wang et al., 2017). The main idea of the method is to divide the region of the possible route into 2N stripes (sub-channels) by means of the parallel lines arranged in the equal distance on both sides of the vector of global displacement. At each discrete time step only one best point, which is the closest in distance to the end point, remains on each stripe and forms the next isocost. This heuristic approach is simple and has high computational efficiency, but leads to the so-called "greediness" of the routing algorithm. In (Topaj et al., 2019) we constructed special counterexample (a dead-end channel of open water area), where the strip-based method gives the result that is very far from the optimal one. Below we propose the alternative original method to reduce the number of points based on Concave Hull technique.

The final stage of the routing algorithm is to check whether the propagating wave has reached the end point. If the distance from progenitor point to some heritor point, generated on the current step, is less than the distance to the end point, then all heritor points are substituted by the destination point. The resulting optimal route may be built step by step using the reverse motion principle, starting from the destination and passing from each point to its progenitor.

Below we introduce several improvements of the described general algorithm in order to take into account the icebreaker assistance and double-acting concept of ship operation.

Each point on the isocost wavefront is described by an extended set of parameters, including: a) geo-coordinates, b) progenitor point, c) time T_a to reach this point from the initial one, d) the status *ST*. The latter parameter is a two-dimensional vector $\mathbf{ST} = \left[ST_1\ ST_2\right]^{\mathrm{T}}$ that characterizes the type and genesis of the corresponding mode of movement. The components of the status vector are: ST_1 – status of icebreaker involvement; ST_2 – status of heading or turnaround stage. The semantics of possible ST_1 values is as follows:

$ST_1 = -1$	means that the vessel moves from the point without icebreaker assistance;
$ST_1 = 0$	means that the vessel moves from the point assisted by an icebreaker, i.e. there is no need to pay for icebreaker involvement;
$ST_1 = 1...N_{IB}$	means that the current point is fictitious and represents the rest of the payment to involve an icebreaker. Such values of status determine the number of elementary trenches in ΔC portions left to pay at the current point to get an icebreaker.

Heading-related status ST_2 can have the following values (here we use conditional codes):

$ST_2 = 99$	means that the vessel moves bow forward;
$ST_2 = -99$	means that the vessel moves stern forward;
$-99 < ST_2 < 99$	means that the vessel is turning around and is in an intermediate position now. The modulus of the value is the remaining number of single steps of

the algorithm until the turnaround is completed, and the sign is the direction of the turn (positive – to bow forward position, negative – to astern).

The proposed modification of the algorithm of wave-based isocost propagation consists of several steps:

1. Generation of the first isocost.

At the start location of the route, the algorithm generates the single point forming the wavefront of initial isocost. Status of the first point is set to [-1; 99] (for the simplicity, we accept that the voyage always starts with independent bow forward movement).

2. Run of the recursive algorithm of wave propagation.

2a. Generation of next isocost. The algorithm consequently generates the next isocost from progenitor points, until fulfilling the condition of reaching the end point or facing the situation when there are no points on the next isocost (complete stop due to heavy ice). The technique of point generation at each step is closely connected with the status of progenitor point. As a rule, each such point generates a "fan" of isocost propagation paths in a geographic space according to the common principles of the wave-based algorithm. If propagation distance is equal to zero, the set of destination locations degenerates to a single point. At the same time, in each geographical point of the wave propagation, several points of the next isocost of different status can be generated. The radius of propagation vector R_i as well as the time stamp T_a and the statuses of child points depend on the corresponding characteristics of the progenitor. Table 1 presents all possible transitions between status values (state machine algorithm).

Table 1. Transitions and parameters in wave-based multi-mode propagation algorithm

Parent point status	ΔT_a	R_i	Status of child points		
[–1; 99]	$\Delta C / c_S$	$V_{BF}(\mathbf{S},T_a)\cdot\Delta T_a$	[–1; 99]	[–1; $-K_S$]	[N_{IB}; 99]
[–1; $-K$]	$\Delta C / c_S$	0	$[-1; -K+1]$		
[–1; –1]	$\Delta C / c_S$	0	$[-1; -99]$		
[–1; –99]	$\Delta C / c_S$	$V_{SF}(\mathbf{S},T_a)\cdot\Delta T_a$	[–1; –99]	[–1; K_S]	
[–1; K]	$\Delta C / c_S$	0	$[-1; K-1]$		
[–1; 1]	$\Delta C / c_S$	0	$[-1; 99]$		
[N; 99]	0	0	[N–1; 99]		
[0; 99]	$\Delta C / (c_S + c_{IB})$	$V_{SIB}(\mathbf{S},T_a)\cdot\Delta T_a$	[0; 99]	[–1; 99]	

In Table 1, we used the following notation: N_{IB} (8) is the initial delay of an icebreaker – the integer value, which determines the number of isocost steps ΔC to compensate the "instant" cost of icebreaker involvement; K_S (9) is the number of isocost steps to compensate the delay of ship movement for turnaround (or 99 in marginal case where the turnaround duration is small); ΔT_a is the increase in T_a value of the child point compared with its progenitor.

$$N_{IB} = \left[\frac{c_{IB} \cdot T_{IB}}{\Delta C} + 0.5 \right], \tag{8}$$

$$K_S = \begin{cases} \left[0.5 + c_S \cdot T_{ROT}(\mathbf{S},T_a)/\Delta C \right], & c_S \cdot T_{ROT}(\mathbf{S},T_a)/\Delta C > 0.5 \\ 99, & c_S \cdot T_{ROT}(\mathbf{S},T_a)/\Delta C \le 0.5 \end{cases}, \tag{9}$$

where V_{BF}, $V_{SF,}$ and V_{SIB} are the velocities of independent movement bow forward and stern forward and icebreaker assisted convoy respectively, $K = 2..K_S$, $N = 1..N_{IB}$.

In contrast to the constant value of N_{IB}, the isocost-intended compensation of the turnaround K_S is a variable that depends on both the coordinate of the generating point and its timestamp. Relations in Table 1, as well as the formulas (8) and (9), assume the following:

- the explicit scheme of numerical integration of the ship motion is applied, i.e. the ice conditions at the start point determine the speed of ship movement along the whole propagation vector;
- the dependence of ship speed on the course is not considered, i.e. we assume the medium to be isotropic and neglect the direction of winds, currents, and cracks in ice field;
- in the graph-based approach, we do not consider a case of a ship sailing stern forward under icebreaker escort since it is a rare situation;
- we do not take into account the case of releasing an icebreaker and ordering it again immediately (i.e. generation the point with status N_{IB} from the point with status 0), because such strategy looks evidently unprofitable;
- the cost and duration of ship turnaround depend on ice conditions at the time of the start of this operation only, i.e we suppose the temporal dynamics of ice conditions to be low;
- we replaced the real variables N_{IB} and K_S in (8-9) with the discrete integer values (the corresponding numbers of "cost quants") using the rounding operations.

2b. Reduction in the number of points in a new-generated isocost. When point generation is done, the algorithm eliminates the internal points and generates the front line of the next isocost. The algorithm for the reduction of a number of points must be applied separately for the sets of points that have the equal status values. So, the resulting set of points that form the next isocost is the assembly of reduced sets of points for any possible combinations of icebreaker-related and heading-related indices.

We developed the original method to reduce the number of points in any finite set, which is based on a Concave Hull computation technology (Moreira, Santos, 2007). The idea of the algorithm is to preserve only the external points that form the Pareto subset on the current wavefront after propagation stage. It could be done by constructing a convex or non-convex curve, which envelopes a set of points and covers the area occupied by them. For the case of non-convex shape, the problem of constructing an envelope curve does not have a single solution and the one can obtain several polygon boundaries by varying the internal parameter of the method. We have implemented our algorithm based on α-shape approach. A transparent analogy for understanding the technique was proposed in (Edelsbrunner, Mücke, 1990), where the set of points that needs to be enveloped is imagined as "hard" chocolate pieces in a huge mass of ice cream. If one tries to carve out all parts of the ice cream mass by means of the sphere-formed spoon (or just a flat circle in 2D case) without shifting chocolate pieces, the set of boundary points in remaining shape will form the Concave Hull. It is clear that the larger the radius of the spoon, the closer the resulting contour will be to the convex shape; the smaller the spoon, the higher number of internal chocolate pieces you can achieve. Following this analogy, we replaced the spoon with a variable-radius wheel that rolls along the outer boundary of a given set of points (see Fig. 3). In more details, the algorithm of point reduction is the following:

a) We find and mark an arbitrary outer point belonging to a given set. For simplicity, this is the rightmost one, i.e. the point with the greatest longitude. To the right of this point, we locate a fictive wheel (circle), which radius is equal to the current radius of isocost propagation, i.e. the distance from the contact point to its progenitor.

b) We successively roll the wheel counterclockwise until the moment when it contacts any other point. Mathematically, the next contact point is the point for which the increment of the angle $\Delta\varphi$ of wheel rotation from the current value is the smallest. Fig. 4 shows the mathematics of the next contact point calculation. When the found point is marked, the procedure continues until finding the next contact point, and so on. Wheel radius D_i can be either a constant value in each i-th contact point or a variable. In the latter case, D_i may be a function of the current speed of wavefront propagation (i.e. the function of propagation radius R_i); this results in a more accurate rendering of the wavefront in the areas where vessel movement slows down.

c) All unmarked points are eliminated from the initial set.

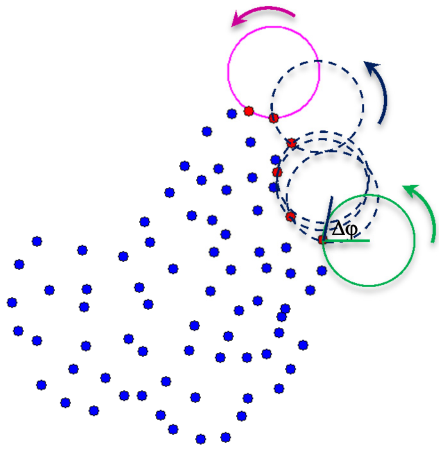

Figure 3. On the method of Concave Hull construction by the rolling wheel approach

$$\mathbf{P}^{\mathbf{next}} = \underset{\mathbf{P}^{\mathbf{i}} \in \Omega}{\arg\min}\left(\Delta\varphi^{i}\right)$$

$$\mathbf{P}^{\mathbf{i}} \in \Omega \quad if \quad \left\|\mathbf{P}^{\mathbf{i}} - \mathbf{P}^{\mathbf{prev}}\right\| < D_i \ ; \ \Delta\varphi^{i} = \varphi^{i} - \varphi^{prev}$$

$$\varphi^{i} = \begin{cases} \varphi^{i}_{+1}\,, & \left(\mathbf{P}^{\mathbf{i}} - \mathbf{P}^{\mathbf{prev}}\right)\times \mathbf{G}^{\mathbf{i}}_{\mathbf{+1}} < 0 \\ \varphi^{i}_{-1}\,, & \left(\mathbf{P}^{\mathbf{i}} - \mathbf{P}^{\mathbf{prev}}\right)\times \mathbf{G}^{\mathbf{i}}_{\mathbf{-1}} < 0 \end{cases} \quad (*)$$

$$\tan\left(\varphi^{i}_{K}\right) = \frac{Y^{i}_{K}}{X^{i}_{K}}\,;\ \mathbf{G}^{\mathbf{i}}_{\mathbf{K}} = \left(X^{i}_{K}, Y^{i}_{K}\right);\ K = -1; +1$$

$$X^{i}_{K} = 0.5\cdot\left\{\left(\mathbf{P}^{\mathbf{i}}\right)_{X} - \left(\mathbf{P}^{\mathbf{prev}}\right)_{X} + K\cdot d^{i}\cdot\left(\left(\mathbf{P}^{\mathbf{i}}\right)_{Y} - \left(\mathbf{P}^{\mathbf{prev}}\right)_{Y}\right)\right\}$$

$$Y^{i}_{K} = 0.5\cdot\left\{\left(\mathbf{P}^{\mathbf{i}}\right)_{Y} - \left(\mathbf{P}^{\mathbf{prev}}\right)_{Y} - K\cdot d^{i}\cdot\left(\left(\mathbf{P}^{\mathbf{i}}\right)_{X} - \left(\mathbf{P}^{\mathbf{prev}}\right)_{X}\right)\right\}$$

$$d^{i} = \sqrt{\frac{D_i^2}{\left\|\mathbf{P}^{\mathbf{i}} - \mathbf{P}^{\mathbf{prev}}\right\|} - 1}$$

Figure 4. Mathematics of the next contact point calculation: $\mathbf{P}^{\mathbf{prev}}$, φ^{prev} – previous contact point and wheel rotation angle; (*) – demand for the contact with the external wheel rim

3. Checking the end point of the voyage and building the optimal route. Checking whether the propagating wave has reached the destination point is done in a way described above. The resulting optimal route is constructed using the reverse motion method. The only difference from the general approach is that the statuses of intermediate points of the received route determine the recommended mode of movement (bow- or stern forward) and the segments where icebreaker escort is profitable.

RESULTS

To investigate and compare different numerical methods of vessel and convoy routing in ice, we developed the specific subject-oriented research software. It allows us to preprocess, plan, tune, perform, visualize and analyze the results of various pathfinding algorithms. The main representation scene is an electronic map of the built-in GIS system associated with a spatially referenced database of weather and ice conditions. The software has a modular structure and contains several components from the earlier developed integrated solution for simulation modeling and pre-design analysis of maritime transportation systems (Tarovik et al, 2017). The important component is the database of ice conditions patterns, each of which represents a benchmark picture of the temporal dynamics of ice severity in terms of the set of ice characteristics (concentration, age composition, shape, ridging, snow cover, decay rate, compression, etc.). Spatial distributions of ice parameters can be presented in a grid form (spatial resolution is 0.25°) or as a vector geo-referenced graphic (shapefiles) depending on data genesis. This data was provided by the Russian Arctic Antarctic Research Institute (AARI) for the period 1960-2014.

We made several calculations to demonstrate the results of various routing approaches. Table 2 represents the parameters of optimal ship routes for the selected pair of source and destination points at the Northern Sea Route for the January. Illustrations with route visualization are presented in (Topaj et al., 2019). Results are obtained for the ice-going tanker "Kirill Lavrov" and diesel-electric icebreaker LK-25 under the assumption that the ship and convoy move with the maximum (attainable) speed. In addition, we assume that the freight rates of tanker and icebreaker are equal ($c_S = c_{IB}$) and the conditional overpayment time of the icebreaker (T_{IB}) is 2 days. The optimal routes for independent (when icebreaker is unavailable) and icebreaker-assisted voyages by A* and isocost methods for the light type of ice conditions have been obtained. These paths demonstrate the efficiency of using the local areas of weak ice (large polynya to the west of Novaya Zemlya), despite the increase in sailing distance. In case if the icebreaker is available, the shape of optimal routes varies insignificantly, but all of them pass through the Cape Zhelaniya.

Table 2. The results of route optimization for test cases

No	Icebreaker type	Calc. method	Calc. time (sec)	Total cost (10^6 RUR)	Total time (hours)	Time in convoy (hours)	Overall distance (nm)	Distance in convoy (nm)
Light ice conditions								
1	–	A*	1.42	32.7	157	0	1027	0
2	LK-25	A*	1.71	30.9	126	19.8	1004	140
3	–	Isocost	4.11	31.9	153	0	935	0
4	LK-25	Isocost	13.49	29.2	100	23.3	937	173
Severe ice conditions								
5	–	Isocost	4.32	37.1	178	0	828	0
6	LK-25	Isocost	15.00	32.9	137	17.3	912	102

Calculations for the case of heavy ice conditions lead to thefundamentally different geography of optimal routes. In this case, the optimal way passes through the Kara gate. Moreover, icebreaker escorting turns out to be cost-effective in two sections of the route (close to the end point and when going around Yamal), despite the one-time costs of icebreaker involvement.

As an example of route optimization for double-acting ships, we consider the case shown in Fig. 5. It presents a retrospective analysis of the real voyage of the vessel "Monchegorsk" (project Aker ACS-650) from Dudinka to Murmansk on April 3-10, 2018. We investigated only the part of this route when ship navigated in the Kara Sea.

As the input data, the electronic maps with attributive information in the SIGRID format and the forecasts of ice cohesion, thickness, and compaction based on the results of calculations of the ice field drift model, were used. Data set corresponds to the operational information support for navigation of ice going vessels, supplied by the staff of the AARI under the agreement with the "Norilsk Nickel" company.

Curve I in Fig. 5 is the optimal route obtained by a wave-based approach using the forecast of ice conditions change during the voyage. The segments marked by a dotted line are the route parts where the stern forward movement was found to be profitable. The results were obtained under the assumption that the rotation for the considered ship is rather an easy operation, i.e. the turnaround is free of charge. Curve II is the actual route according to AIS data and curve III is a route that was recommended to the captain when leaving Dudinka. As itis seen, the latter route runs through the Cape Zhelaniya, but the captain ignored these recommendations based on his experience. At the same time, the route obtained by the mathematical methods of routing is close to the actual trajectory of the vessel movement. However, the estimated duration of movement was 166 hours compared to 123 hours of a real voyage time.

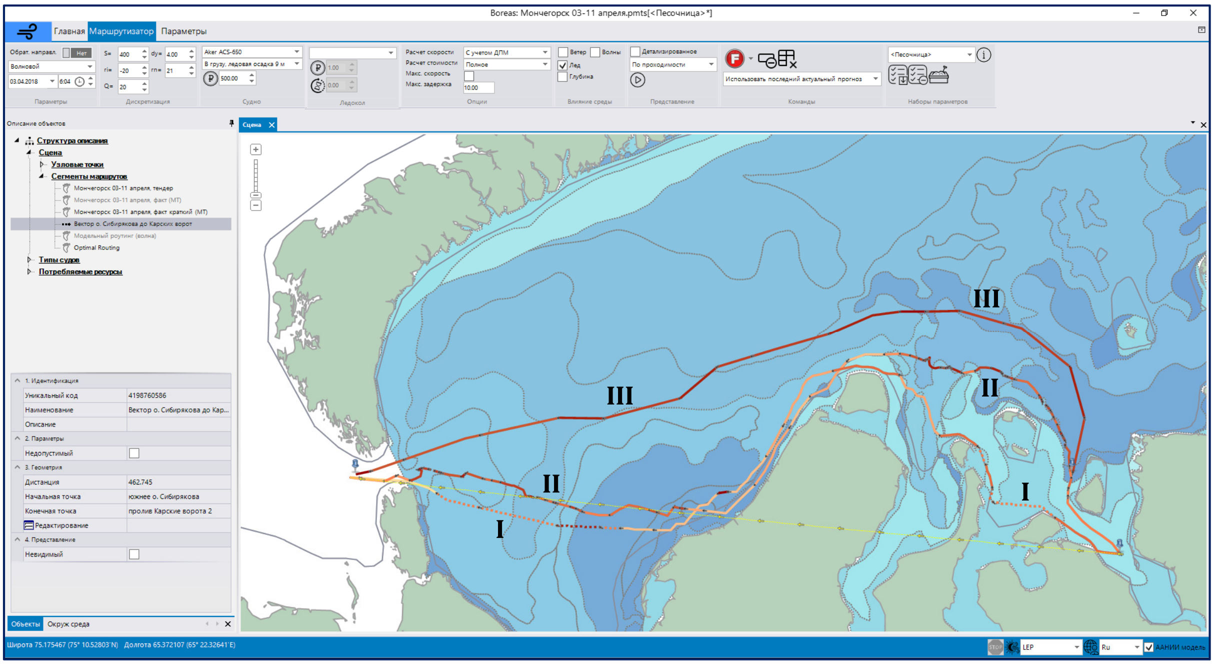

Figure 5. Example of route optimization for the double-acting ship in the user interface of subject-oriented research software. The route I – optimal route by wave-based approach, II - actual route according to AIS, III – route recommended by the expert

CONCLUSIONS

In this paper, we proposed the modifications of well-known methods of optimal path finding in ice-covered waters taking into account the possible involvement of an icebreaker and ability of double-acting ships to change its movement mode in order to facilitate passage through the heavy ice. In contrast to previous works, we considered the icebreaker assistance and stern forward movement as an integral part of the overall formulation of the route optimization problem. At the same time, the proposed approach contains a large number of simplifications and limitations, such as an explicit integration scheme, description of the limited amount of icebreaking resources by the single variable, etc. However, it seems that the weakest link in the practical implementation of automatic ship routing techniques in Arctic waters is not the mathematics of routing itself. The main issue is the informational support of routing services by the high quality actual and forecasted data on weather and ice. Anyway, the presented algorithm can serve as the initial component of the complex computer-aided system for planning and managing the exploitation of icebreaker and transport fleets in the Arctic.

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