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Ice resistance calculation method for a ship sailing via brash ice channel

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ABSTRACT

The main purpose of this study is to develop a method for calculating ice resistance of a ship sailing via channel clogged with brash ice. Theoretical treatment is attempted to address the processes involved in ship navigation via brash ice channel. A theoretical model is elaborated on the basis of full-scale and model test data. The results of this effort are applied to analyze how the key parameters of the developed mathematical model influence the ice resistance magnitude. Calculations by the suggested model are compared with experimental data obtained in ice model basin.

KEY WORDS: Brash ice; Ice channel; Ice resistance; Friction force; Ice basin; Experiments.

NOMENCLATURE

L – ship length on design waterline;

B – ship beam on design waterline;

T – ship draught;

R_{BC} –resistance of brash ice in channel;

R_1 – resistance due to brash ice displacement by ship hull over a distance equal to ship draught T;

R_2 – momentum resistance due to some instant velocity of brash ice particles imparted by ship hull;

R_{f1} –resistance due to friction of brash ice particles against ship bow, stern and bottom;

R_{f2} – resistance due to friction of brash ice particles against ship's side to be determined including ice piles by ship's side;

h_{BC} – thickness of brash ice ahead of ship;

l_{wl} – length of ship forebody in contact with ice;

f_I – coefficient of ice friction against hull plating;

f_{I-I} – coefficient of ice friction against ice;

φ_0 – angle of stem;

α_0 – angle of waterline slope at the bow with respect to the centerplane;

ρ_I – ice density;

ρ_w – water density;

$\Delta\rho$ – ice/water density difference;

n – brash ice porosity;

V_{bott} – volume of brash ice thrown from ship's bottom to one side;

S_{PM} – cross-section area of ice pile by ship's side;

ψ – natural angle of slope;

h' – height of ice pile by ship's side.

INTRODUCTION

One of the topical subjects of marine ice engineering today is investigation of ship behavior in channels clogged with brash ice (ice fragments less than 2 m across) (Sazonov, 2018). Brash ice is formed in navigable ice channels and harbours after repeated ship passages cutting ice into smaller pieces.

Operation of ships in brash ice has been studied for quite some time already. Two main vectors of research can be identified in this field today. Most of the studies look into the genesis of ice channels, processes of brash ice accumulation, and formation of consolidated ice layer. The first contributions by Arikainen A.I., Chubakov K.N.(1987), Kannari (1983) and Nortala-Hoikkanen (1999) were concerned with morphological features of ice channels. These studies have revealed that mature channels filled up with brash ice are featuring rather complex geometries. Brash ice in such channels is not uniformly distributed across their breadth. The thickness of brash ice layer varies from minimum around the channel axis to maximum by the channel edges (Fig.1). Moving away from the channel axis, we find "barriers" (i.e., local swells of brash ice) whose dimensions and quantity depend on the number of ship passages, hull breadth of passing ships as well as water depth. When the channel is navigated by vessels of similar breadth, one barrier is formed on each side to the channel. The studies have also revealed that a layer of brash ice may freeze up to form a consolidated ice layer. It is found that the consolidated layer of brash ice is growing faster than the intact ice cover under the same metocean conditions.

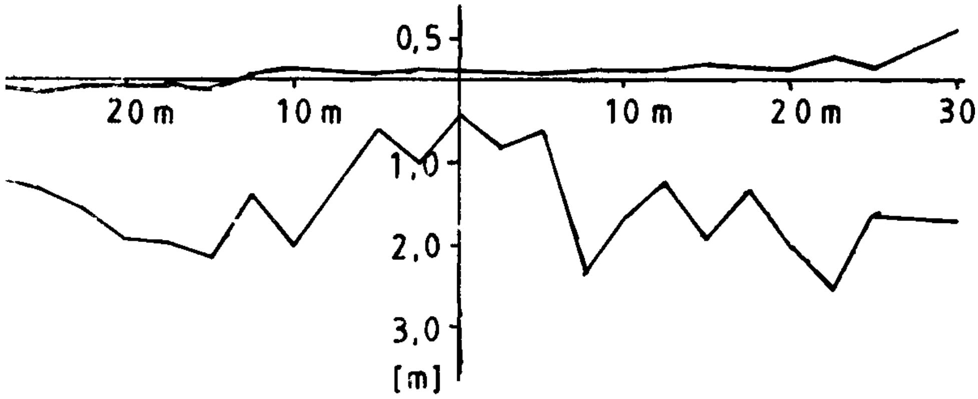

Figure 1. Ice channel profile [3]

Lots of studies look into the processes of brash ice growth and consolidated layer formation. As a rule, these processes are examined in connection with the frequency of ship passages via channel. Practically, all investigations are based on the heat-balance equation solved on the assumption of constant temperature gradient in intact ice cover and consolidated ice layer. The temperature of brash ice layer is assumed constant and equal to the water-ice transition temperature. This approach is developed in Klyachkin et al (1999), Sazonov (2015) and Karulin et al. (2018) and allows us to find the rate of brash ice growth in channel.

Ice resistance of ships sailing in brash ice channels has been investigated in a limited number of publications. Practically all resistance estimates are based on the work of Riska (2000), which has been used as a basis for formulation of the current Finish-Swedish ice rules (Finnish-Swedish ice rules, 2010). An important drawback of the formulas used in the Finish-Swedish ice rules is that they do not take into account the influence of ship speed on resistance. The ice resistance values obtained from these formulas refer to the ship speed of 4 knots. In ref. Sazonov, (2015) it is attempted to supplement these formulas with the known scaling relationships applied in marine ice engineering (Sazonov, 2010), however, such analytical constructions are not always correct. In ref. Karulin et al. (2018) it is suggested to estimate the ice resistance using the relationships derived for the analysis of interaction between ice-ridge keel and fixed structure (Sazonov, 2010). In the authors' opinion, there is no good ground for application of these formulas because the physical processes of structure/ridge keel interaction are somewhat different, e.g. global shift of ridge keel.

Currently, most of the data on brash ice resistance to ship in channel are acquired from model experiments in ice basins (ITTC 7.5-02-04-01, 2017). However, more recent studies have shown (Franz von Bock und Polach and Molyneux, 2017) that model test data raise some doubt among experts. Therefore, research studies on ship operation in brash ice are going on.

Results of the efforts to develop a mathematical model of ship's behaviour in brash ice channels are described below. A case of ship sailing in channel without consolidated ice layer was considered. This scenario takes place when the ship sails via channel just after it has been brushed up by an icebreaker. The presence of consolidated ice layer can be included by one of the known methods available for calculating ship's ice resistance in continuous ice cover, e.g. see ref. Ionov and Gramuzov, 2013. If the channel is renewed by an icebreaker whose beam is not as wide as the breadth of ships going via this channel, then ice resistance estimates have to include an additional resistance component because the ship in question is to break a consolidated ice layer that is left unbroken. This component is conveniently taken into account with a method suggested by B.P. Ionov, as shown in two references of Dobrodeev et al., (2018). The model was elaborated by purely theoretical methods based on visual observations performed in the ice basin of Krylov State Research Centre.

MATHEMATICAL MODEL

According to Dobrodeev et al. (2018), in marine ice engineering practices it is usual to break down the ice resistance into various components for theoretical estimations. In this case a dedicated mathematical mode is elaborated for each component. A similar approach was applied to consider the brash ice resistance R_{BC}. Under the mathematical model this resistance was calculated as a sum of four components: resistance R_1 due to brash ice displacement by ship hull over a distance equal to ship draught *T*; momentum resistance R_2 due to some instant velocity of brash ice particles imparted by ship hull; resistance R_{f1} due to friction of brash ice particles against ship bow, stern and bottom; resistance R_{f2} due to friction of brash ice particles against ship's side to be determined including ice piling by ship's side:

$$R_{BC} = R_1 + R_2 + R_{f1} + R_{f2} \tag{1}$$

Let us employ the energy method to find the first two terms by considering variations in the potential and kinetic energy of brash ice particles contained within a certain volume dV. This volume is defined as:

$$dV = h_{BC} l_{wl} dx, \tag{2}$$

where the thickness of brash ice in channel ahead of ship h_{BC} can be given by as a function $h_{BC} = f(y)$ describing the channel section perpendicular to its axis;

$l_{wl} = 2\int_{L_{PM}/2}^{(L-L_{PM})/2} \sqrt{1+y'^2}\,dx$, where $y = y(x)$ - equation for the effective forebody waterline in the coordinate axes fixed to the CoG of the effective waterline, L - length of the effective waterline, L_{PM} - length of parallel middle body; dx - differential of ship advance in heading direction.

Now equation (1) can be re-written in a differential form:

$$R_{BC}dx = dE_1 + dE_2 + R_{f1}dx + R_{f2}dx, \tag{3}$$

where dE_1 – change in the potential energy of volume dV at immersion depth *T*; dE_2 – change in the kinetic energy of ice particles in the volume.

For finding dE_1 let us consider the energy consumed to displace a prism of brash ice of volume dV to the depth equal to ship's draught *T*. In this case it should be noted that brash ice particles move not only along buttock lines. In reality ice class vessels may have wedge shaped bows characterized by waterline angles α_{0i} with respect to the stem. Wedge-shaped bows tend to push aside some part of the submerged ice prism away from the hull. Under these conditions, parts of the ice prism displaced to the sides of the ship hull are not submerged to the full-draught depth, but less so.

Thus, the volume dV is broken down into two components: dV_1 - part of the brash ice prism which is submerged to the depth equal to ship's full draught *T*, and dV_2 - part of the brash ice prism submerged to the depth less than ship's full draught *T* and causing ice piling by ship's sides.

Changes in the kinetic energy of ice particles within the volume dE_2 imply that some energy is consumed to instantly impart a certain velocity to brash-ice particles otherwise at rest. In this process a ship advance of dx entails motion of more and more brash ice particles. These are vertical and horizontal motions. In the first approximation it can be assumed that an ice prism of volume dV is engaged in these vertical and horizontal motions.

Due to its buoyancy properties the brash ice crawling over the underwater hull interacts with the same, generating friction forces.

First of all, it is necessary to find the limiting trajectory of ice particles to identify the parts of ship's forebody covered with ice and free from ice, and, therefore, to determine the force acting on ship's forebody:

$$R_{f_{bow}1} = f_I \Delta\rho(1-n) g h_{BC} \frac{\mathrm{T}\cos\theta}{\sin\alpha_0 \sin\varphi_0}(B-2T)\cos\varphi_0, \quad (4)$$

where $\theta = \arcsin\sqrt{1-\frac{\cos^2\alpha_0}{\cos^2\varphi_0}}$.

Brash ice enclosed by two limiting straight lines will come to ship's bottom. The width of this band on each side of the ship is $(B-2T)/\sin\alpha_0$. The length of this band is equal to the ship's bottom length, which in the first approximation can be considered as equal to the length of parallel middle body L_{PM}. As it is seen from Fig. 2 obtained from model tests at Krylov's ice basin, there are two patches of brash ice on the model bottom, while the middle part of the bottom is free from ice.

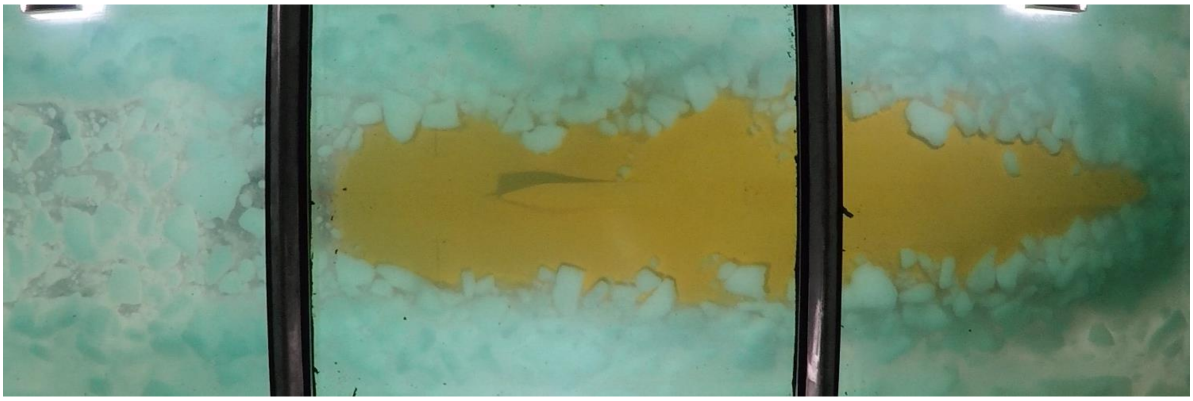

Figure 2. Brash ice distribution over bottom of ship model

The thickness of brash ice layer on ship's bottom is equal to h_{BC}. Ref. Karulin et al. (2018) describes how the brash ice on ship's bottom is shifting towards ship's sides. This process can be regarded as natural slipping of loose materials characterised by the angle of natural slope (repose) ψ. If brash ice is described in the first approximation as a loose material without cohesion, then the natural angle of slope is equal to the internal friction angle (Tsytovich, 1973). The volume of brash ice slipping from ship's bottom V_{bott} towards one side depends on the width and height of the ice layer in question:

$$V_{bott} = L_{PM}\left(h_{BC} - \frac{B-2T}{8\sin\alpha_0} tg\,\psi\right), \text{ subject to } \frac{B-2T}{8\sin\alpha_0} tg\,\psi < h_{BC} \quad (5)$$

$$V_{bott} = \frac{L_{PM} h_{BC}^2}{2tg\,\psi}, \text{ subject to } \frac{B-2T}{8\sin\alpha_0} tg\,\psi \geq h_{BC}.$$

In this case the friction force on bottom can be estimated from:

$$R_{f_{bott}1} = f_I \Delta\rho(1-n) g h_{BC} \left[\frac{L_{PM}(B-2T)}{\sin\alpha_0} - V_{bott} \right]. \tag{6}$$

Summation of eq. (4) and eq. (5) yields the final expression for the friction force acting on ship's forebody and bottom:

$$R_{f1} = f_I \Delta\rho(1-n) g h_{BC} \frac{(B-2T)}{\sin\alpha_0} \left[L_{PM} - \frac{V_{bott}\sin\alpha_0}{B-2T} + \frac{\mathrm{T}\cos\theta}{tg\varphi_0} \right]. \tag{7}$$

Brash ice comes to the parallel middle body due to flow around bow and natural ice sloping on ship's bottom. According to ref. Sazonov (2008) an ice pile with natural slope angle is formed, which rests on ship's side and unbroken ice cover or intact edge of ice channel. Pressure of this ice pile on the vertical side of hull gives rise to a friction force. This pressure can be estimated by the methods applied in granular material mechanics, e.g. ref. Emelyanov (1987).

Some amount of brash ice comes to one side of the ship in way of her parallel middle body and piles up by the side with natural slope angle. One can find the cross-sectional area of this pile, assuming that the ice pile porosity is equal to that of the brash ice.

The ice pile has a triangular cross section with its base angle equal to the natural angle of slope (repose) ψ. The pile's height h' is found from:

$$h' = \sqrt{2S_{PM} tg\psi}\ . \tag{8}$$

The formulas given in this paper are valid if:

$$h' + h_{BC} \le T\ . \tag{9}$$

If condition (9) is not satisfied, i.e. the pile's height is greater than the ship draught, the ice pile would collapse back on ship's bottom. With this process in mind, equations (6) and (7) have to be corrected.

The friction force induced by ice pile on the sides of parallel middle body can be estimated using the methods of granular material mechanics:

$$R_{f2} = 2 f_I \Delta\rho(1-n) g S_{PM} L_{PM} \frac{\sin\psi\cos\gamma}{\cos(\psi+\gamma)}, \tag{10}$$

where $\gamma = arctgf_{I-I}$.

General expression for calculation of brash ice resistance. In consideration of the above, formula (3) can be written as the final equation for estimation of the brash ice resistance of ship:

$$R_{BC} = \Delta\rho(1-n) g T h_{BC} \frac{B}{2\sin\alpha_0}\left[1 + \frac{1}{\sin\alpha_0}\left(1 - \frac{2T}{B}\right)\right] + \rho_I (1-n) \frac{B}{2\sin\alpha_0} h_{BC} V_s^2 \left(\frac{1}{\sin\alpha_0} + \frac{1}{\sin\varphi_0}\right)^2 + \\ + f_I \Delta\rho(1-n) g h_{BC} \frac{(B-2T)}{\sin\alpha_0}\left[L_{PM} - \frac{V_{bott}\sin\alpha_0}{B-2T} + \frac{\mathrm{T}\cos\theta}{tg\varphi_0}\right] + 2 f_I \Delta\rho(1-n) g S_{PM} L_{PM} \frac{\sin\psi\cos\gamma}{\cos(\psi+\gamma)} \tag{11}$$

This equation makes it possible to calculate the brash ice resistance in function of ship's main dimensions, bow shape as well as brash ice properties.

TESTING OF THE DERIVED MATHEMATICAL MODELS

Below are given details of estimations done to analyze how the key parameters of the derived mathematical model influence the final results of formula (11) in application to a large-size vessel in brash-ice channel.

Calculations were done for two hypothetical large-size vessels of ice class. Table 1 provides the main ship data. In all cases the ship speed was varied from 0.5 to 5 m/s.

Table 1 Hypothetical large-size vessels of ice class: basic data

Characteristics	Ship 1	Ship 2
Beam, m	50	32
Draught, m	12	12
Length of parallel middle body, m	220	150
Waterline angle at 0 station, deg.	45	45
Stem angle, deg.	30	30
Ice/hull friction coefficient	0.1	0.1
Ice/ice friction coefficient	0.5	0.5
Water density, kg/m^3	1000	1000
Ice density, kg/m^3	870	870
Brash ice porosity	0.2	0.2
Thickness of brash ice layer, m	3	3

The parameters varied in mathematical model tests are specifically indicated in each case on the figures, while default parameters are as per Table 1. Figures illustrating the calculation results are grouped into three bins each of which illustrates the impact of respective factor groups: ship's main dimensions, hull shape, medium properties.

Fig. 3 presents the impact of ship's main dimensions on brash ice resistance in channel.

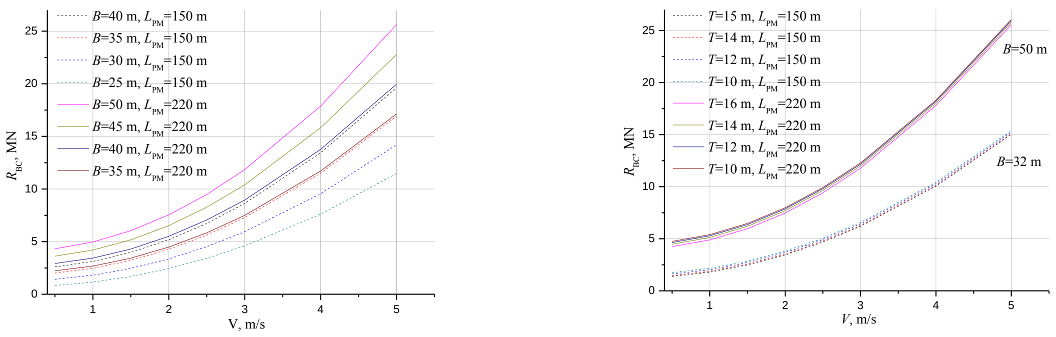

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Figure 3. Impact of ship's main dimensions on brash ice resistance in channel

Fig.3 shows that the most important factors for ice resistance are ship's beam and length of parallel middle body, while draught variations have little effect. In accordance with the suggested mathematical model the ship beam governs the volume of brash ice in contact with hull. The larger amount of ice come to interact with hull, the greater is the resistance of medium. The length of parallel middle body affects the energy losses in friction; a longer middle body would increase the losses. The ship draught only affects the energy losses related to an increase in potential energy of ice layer, which grows with ice immersion. As seen from Fig.2, only part of the ice volume is submerged to the full ship draught. Also, it should be noted that the potential energy changes are caused by buoyancy rather than gravitational force. It appears that these factors explain why variations in ship's draught have little effect on total resistance.

Fig. 4 shows the influence of effective waterline angles at 0 station and stem angle on the brash ice resistance of ship in channel. These angles provide a certain characteristic of the bow shape.

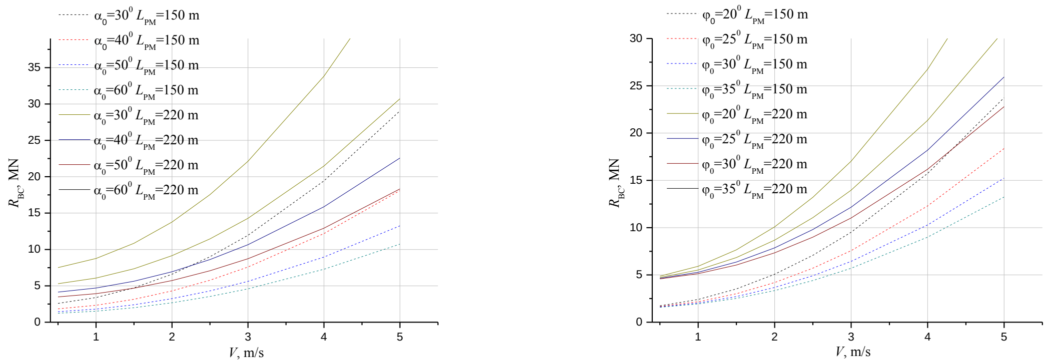

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Figure 4. Brash ice resistance versus hull shape

Fig.4 demonstrates that the bow shape has a rather significant effect on ship's resistance. The proposed model suggests that when the said angles are increased, the ice resistance is reduced. The hull form influence is mainly seen in the speed-dependent component of ice resistance.

Fig. 5 illustrates the influence of environment factors, such as ice/hull friction coefficient and thickness of brash ice in channel.

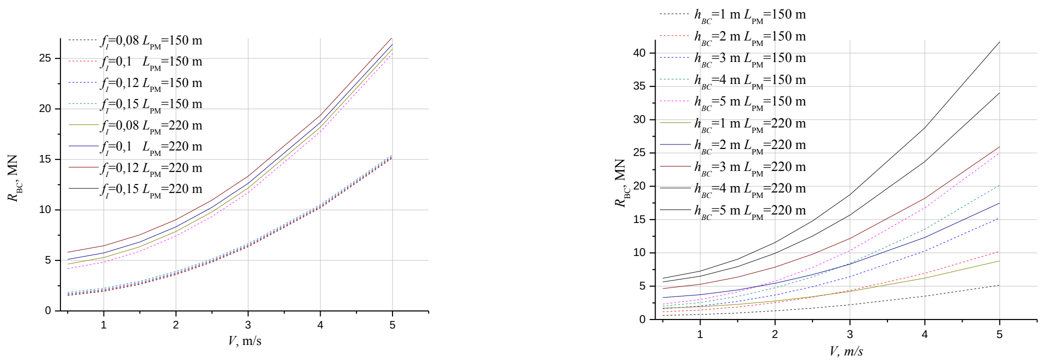

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Figure 5. Ice resistance versus ice/hull friction coefficient and brash ice thickness in channel

The friction coefficient within the examined range has little effect on ice resistance. It should be noted that this influence is increased with the ship dimensions, which is apparently explained by larger volumes of brash ice in contact with the hull and, therefore, greater friction losses. Fig. 5b indicates that the thickness of brash ice in channel is essential for ship's resistance.

EXPERIMENTAL INVESTIGATIONS

Physical model experiments were carried out at the Krylov's ice basin to verify the calculation method and confirm conclusions from the mathematical model tests. The main purpose was to compare the calculation results with experimentally obtained ice resistance data for a chosen ship in brash ice channel. It was also examined how the ice/hull friction coefficient influences the ice resistance.

Table 2. Ship data for model case study

Characteristics	Ship
Length, m	100
Beam, m	22
Draught, m	8.0
Waterline angle at 0 station, deg.	52
Stem angle, deg.	20
Ice/hull friction coefficient	0.05

Towing tests of an icebreaking ship model were performed in accordance with the ITTC guidelines and recommendations ITTC 7.5-02-04-02.1 (2017). The scale model manufactured for this purpose represented the ship whose main characteristics are given in Table 2.

Captive model tests were carried out in a brash ice channel at specified speeds (fig. 6). The dynamometer was installed at the model bow. The channel width was 1.5 times the ship beam. This experiment is noted for a special technique employed to obtain brash ice by automatic fragmentation of fine-grain ice plates into smaller pieces using special-purpose cutters on a rotating drum. This jig was mounted on a service carriage and moved along to cut ice into smaller pieces sized from 0.8 to 1.2 m (in full scale), as well as to mix and compact the same in the direction of drum rotation. Prior to the experiment, the thickness of brash ice layer was measured to confirm the specified characteristics. According to the measurements brash ice was uniformly distributed along and across the channel. Experimental results were extrapolated to full scale using a standard method based on the Froude similarity criterion.

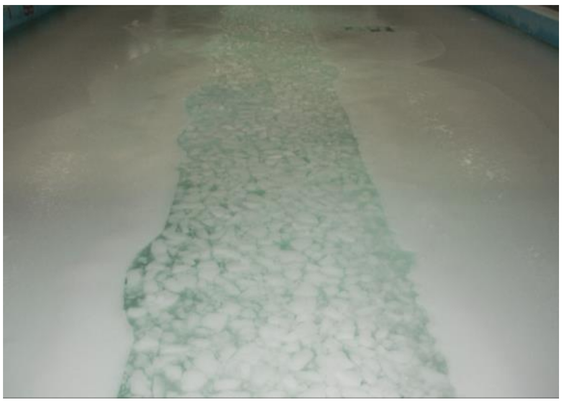

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Figure 6 – Ship hull interaction with brash ice in channel.

a – prepared brash ice channel; b – towing of ship model in brash ice channel

The pattern of brash ice/model hull interaction is in good agreement with the data obtained during full-scale observations of ice channel evolution (Arctic Passion News, 2018; Sandkvist, 1978). The ship bow submerges brash ice and in doing so push some ice away from the hull, producing underwater ice piles along the channel edges. These piles act on the ship side in way of the parallel middle body.

Table 3. Comparison of ice resistance in brash ice channel calculated by the derived formula and obtained experimentally

No	Ice thickness, m	Ship speed, knots	Friction coefficient 0.05		Friction coefficient 0.18	
			Formula (7), %	Experiment, %	Formula (7), %	Experiment, %
1.	2.0	5.4	100	106.5	108.2	115.4
2.	2.0	3.6	100	109.4	115.8	112.1
3.	2.0	1.8	100	93.4	135.9	126.7
4.	2.5	5.4	100	101.3	108.2	111.7
5.	2.5	3.6	100	96.8	116.0	123.5
6.	2.5	1.8	100	95.2	136.3	132.1

The model test data are in good agreement with the results obtained by calculations based on formula (10). Table 3 compares the obtained values of ice resistance. Special attention should be paid to the relationship of ice resistance and ship speed. Discrepancies between the results remain within permissible limits at speed variations. In addition the experiments looked at how the ice/hull friction coefficient influences the ice resistance; the results are also shown in Table 3. According to the calculations (Fig. 5) changes in the friction coefficient have little effect on the value of ice resistance. In order to verify this conclusion the ship model was painted twice with different types of paint systems. In one case the ice/hull friction coefficient f_l was 0.18, in the other case it was 0.05. The experimental data confirms the assumption that the friction coefficient has little effect on the ice resistance. The difference between the ice resistance values obtained at different ice/hull friction coefficients does not exceed 8%.

CONCLUSIONS

Based on the results of this study a mathematical model of ship motion in a brash-ice channel without consolidated ice layer has been developed. Resistance is represented as a sum of components: resistance due to displacement of brash ice accumulations by the ship hull over a distance equal to the ship draught, momentum resistance, resistance due to friction of brash ice particles against ship's bow, stern and bottom, and resistance due to friction of brash ice particles against ship's sides. Calculations performed by the mathematical model for a number of hypothetical cases of large-size ice-class vessels indicate that the width and length of parallel middle body, as well as the bow shape have the most significant effect on the resistance magnitude. Variations in the ship draught and ice/hull friction coefficient show little effect. The results of experimental investigations conducted in the ice basin are in good agreement with the ice resistance estimates obtained by the developed method.

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