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Is Linear Hydrodynamics Sufficient to Predict Breaking Lengths Correctly?

Chris Keijdener[1,2], Andrei Metrikine[1,2]
[1] Delft University of Technology, Delft, Netherlands
[2] SAMCoT, Trondheim, Norway

ABSTRACT

The bending failure of level ice in 2D is mainly characterized by its breaking length and interaction forces. In this paper a study is carried out to assert if a 2D ice-structure interaction model based on an incompressible, inviscid and irrotational fluid together with the *linearized* Bernoulli equation for ice can reproduce the dependence of the breaking length on the interaction velocity as observed in experiments. To this end a 2D model is composed in which the ice, modelled as a semi-infinite Euler-Bernoulli beam, is pushed into a rigid, immovable, downward-sloping structure with a fixed horizontal velocity. The beam rests on the finite depth, infinitely wide fluid layer.

To assert whether this linear description of the fluid suffices, the velocity dependence of the breaking length predicted by this model is compared with the experimentally validated 2D model of (Valanto 1992). The comparison shows that linear hydrodynamics gives significant error in certain velocity ranges. Recommendations are given as to a fluid model that would result in an improved prediction of the velocity dependence of the breaking length.

KEY WORDS: Bending failure, hydrodynamics, modeling, level ice.

INTRODUCTION

The breaking length of level ice is a widely studied topic in ice-structure interaction. A correct prediction is important for several reasons. Firstly, because the breaking length is directly linked to the time of failure, and therefore to the duration of the interaction, it governs the amount of work done by the ice on the structure and therefore governs the magnitude of the structure's response. Secondly, the breaking length, together with the interaction velocity, determines the period of ice failure and so is important for the dynamic response of the structure. A correct predictions of the breaking length is therefore imperative for a dynamic ice-structure interaction model.

To the authors' knowledge, only very few hydro-elastic ice-structure interaction model have been implemented to date, namely (Valanto 1992; Valanto 2001; Valanto 2006; Lubbad et al. 2008; Lu et al. 2012). Of these models only one is in 2D, namely (Valanto 1992). In this paper the dependence of the breaking length of level ice on the interaction velocity, i.e. the *dynamic breaking length*, is presented and shows good agreement with experimental results.

It remains unclear, however, which components of Valanto's model are essential in its correct prediction of the dynamic breaking length. As each component can incur a significant implementation cost, knowing which ones may be neglected is essential.

The goal of this paper is to determine whether a linear description of the fluid can replicate the dynamic breaking lengths observed by Valanto. This linear model is introduced in the next section. After this, the model's breaking lengths are compared with the experimental data by Valanto. This comparison will show that the proposed linear model has significant errors in certain velocities ranges. A discussion follows to estimate which assumption has led to this discrepancy.

In this paper 2D models are used for the sake of simplicity but the qualitative findings are postulated to also apply to 3D.

MODEL DESCRIPTION

In this section a brief description of the linear model is given. The purpose of this section is to highlight the assumptions on which the model is based so that any discrepancies in the breaking behavior can be traced back to a difference in the assumptions made. Because of this, details regarding the implementation are kept to a minimum for brevity. The model is depicted in the e below.

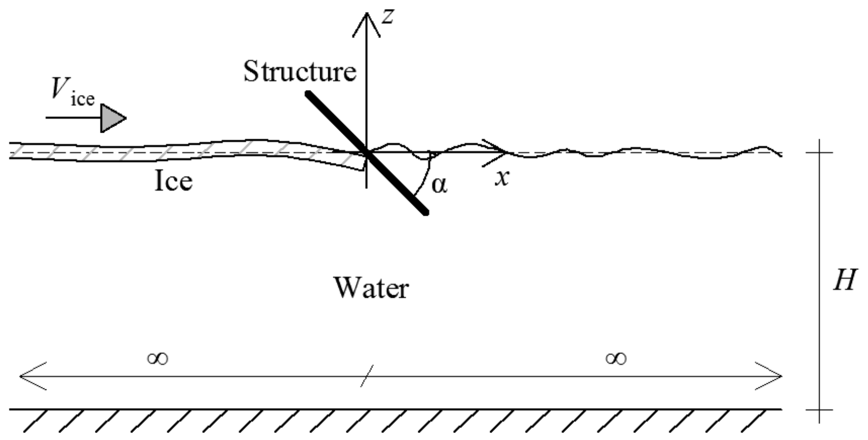

Figure 1. Problem overview of the linear 2D model

For simplicity the structure itself is considered in a simplified manner. Its geometry is not taking into account as a boundary condition for the fluid. However, for determining the ice-structure interaction loads the structure is modeled as a rigid, immovable, downward sloping wall with an angle α. The ice itself, located at $x \leq 0$, moves with a constant velocity V_{ice} towards the structure. The ice then fails dynamically because of the initial impact or quasi-statically as it is gradually pushed further down the wall. In both cases the ice fails in pure bending.

The ice rests a the fluid layer that has a depth H. The fluid layer is infinite in horizontal direction. As the ice is deflected, surface waves are generated that propagate away from the interaction point. These radiated waves act as a form of energy dissipation. Additionally, the fluid underneath the ice has to be mobilized when the ice deforms, resulting in additional resistance. Both effects thus resist the vertical motions of the ice. Since the ice is kinematically forced to move downward because of the structure, the extra resistance results in higher stresses in the ice and therefore in earlier times of failure and shorter breaking lengths. Note that these forms of resistance are not present in models that only capture hydrostatics. The mathematical description of the model is given next.

Mathematical Description of the Model

The fluid is assumed to be incompressible, inviscid and irrotational and so is governed by the Laplace equation:

$$\Delta\phi(x,z,t) = 0 \qquad \forall\, x \in (-\infty, 0] \cap z \in [-H, 0] \tag{1}$$

Its solution is sought in the form of the displacement potential $\phi(x,z,t)$. The boundary condition (BC) at the seabed prevents penetration of the fluid into the seabed:

$$\phi_z(x,-H,t) = 0 \tag{2}$$

where the subscript denotes derivatives. The fluid pressure is given by the *linearized* Bernoulli equation:

$$p(x,z,t) = -\rho_w\left(\ddot{\phi} + g(\phi_z + z)\right) \tag{3}$$

where the linearization removes the *dynamic pressure term* $1/2\left(\dot{\phi}_x^2 + \dot{\phi}_z^2\right)$, ρ_w is the fluid density, g the gravitational constant and the dot denotes time derivatives. At the surface the fluid pressure has to satisfy the following x-dependent BC:

$$p(x,0,t) = \begin{cases} \rho_i A\ddot{w} + EIw'''' & \forall\, x \in (-\infty, 0] \\ 0 & \forall\, x \in (0, \infty) \end{cases} \tag{4}$$

where at $x \leq 0$ the ice is present and so the fluid pressure must balance with the stresses in the ice (modeled by a semi-infinite Euler-Bernoulli beam) and at $x > 0$ the fluid must satisfy the pressure release condition. Additionally, $w(x,t)$ is the displacement of the ice, ρ_i its density, A the area of its cross-section and I the second moment of its area. Continuity between ice and fluid dictates that their vertical displacement must be the same along their interface and so:

$$w(x,t) = \phi_z(x,0,t) \qquad \forall\, x \in (-\infty, 0] \tag{5}$$

Lastly, at the contact interface with the structure, the ice is assumed to crush and so the contact force is given by:

$$F_{contact}(t) = \begin{cases} \sigma_c A(t) & \text{when crushing (stage 1)} \\ \sigma_c^*(t) A^* & \text{when not crushing (stage 2)} \end{cases} \tag{6}$$

where σ_c is the crushing strength of the ice and $A(t)$ is the contact area. The contact model distinguishes between two stages. In the first stage the ice crushes and consequently the contact area is growing. Vibrations of the ice can cause a temporary drop in the required contact force and so crushing stops, entering stage 2. The time moment the contact force starts to drop is defined as t^*. During stage 2 the area remains constant at $A^* = A(t^*)$ and the pressure becomes variable within $\sigma_c^*(t) \in (0, \sigma_c)$. If at some point $\sigma_c^*(t) \leq 0$ contact is lost and if $\sigma_c^*(t) \geq \sigma_c$ crushing recommences (back to stage 1). $\sigma_c^*(t)$ is computed using

Lagrangian Multipliers as the contact is assumed to be stiff during the second stage. The force has an arm with respect to the neutral axis of the beam, generating a tip moment. Based on these loads the BCs for the beam are:

$$\begin{cases} EIw'''(0,t) = F_{contact}(t) \\ EIw''(0,t) = M_{contact}(t) \end{cases} \tag{7}$$

Further details of the contact model are given in (Keijdener & Metrikine 2014) but are omitted here as they are not relevant for this paper.

Solution Method

The defined problem is solved semi-analytically using an approach very similar to the Boundary Element Method. First, a Green's Matrix $\boldsymbol{G}(t)$ is computed that relates the excitation by contact loads $\boldsymbol{F}(t)$ to the vertical displacement and slope of the beam tip $\boldsymbol{u}(t)$. $\boldsymbol{G}(t)$ is thus a 2x2 matrix. Time-integration is then done based on the convolution of this Green's Matrix with the contact loads, so $\boldsymbol{u}(t) = \boldsymbol{G}(t) * \boldsymbol{F}(t)$, where $*$ denotes convolution.

The Green's Matrix is given by the numerically computed Inverse Fourier Transform of the frequency domain response $\widetilde{\boldsymbol{G}}(\omega) = \widetilde{\boldsymbol{T}}^{\boldsymbol{-1}}(\omega)\widetilde{\boldsymbol{E}}(\omega)$ of the system to the elementary loading functions $\widetilde{\boldsymbol{E}}$**.** These loading functions act on both of its inputs (the tip force and moment) and are introduced to be able to perform the convolution numerically. They can be seen as a consequence of approximating the impulse (delta function) with a load that acts over a finite duration. The non-linear nature of the contact loads is handled using iterations at each time-step.

The transfer functions of the system $\widetilde{\boldsymbol{T}}$ are sought using two potentials: $\phi_{ice}(x,z,t)$ and $\phi_{ow}(x,z,t)$. ϕ_{ice} satisfies the ice-covered surface BC and the open water potential ϕ_{ow} satisfies the free-surface condition, see Eq. (4). Both potentials satisfy their respect BC along their entire surface, so $\forall\, x \in (-\infty, \infty)$. One half of each potential is then taken and these two halves are effectively 'glued' together using Eigenfunction Matching so that together they again span the full domain $(-\infty, \infty)$. This 'gluing' process assures that the fluid acts as a smooth continum but, since each potential satisfies a different surface BC, one half is now covered with ice and the other half is not. An overview of the solution method is shown below in figure 2.

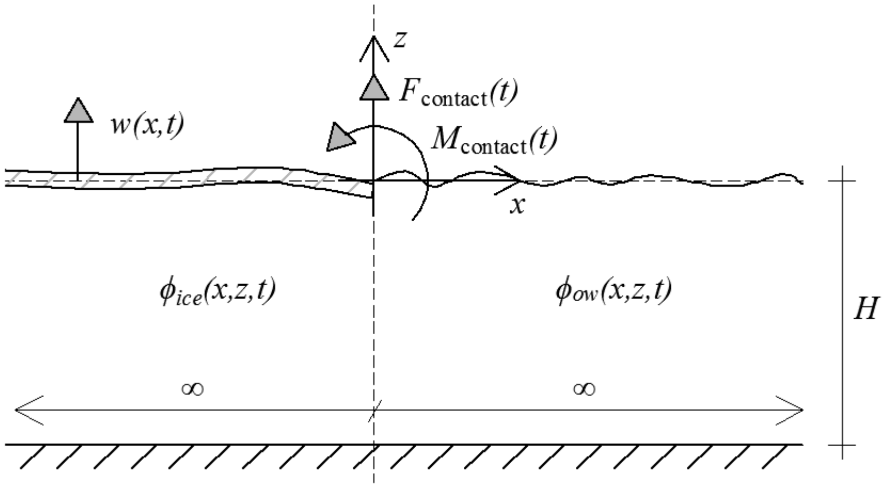

Figure 2. The solution is found by splitting the problem into two subdomains.

Using this model the breaking length of the level ice is computed for a range of velocities. The breaking length is defined by the location where the bending stress first exceeds the flexural strength σ_{fl} of the ice. The results are studied next. The parameters used for the model are the same as those used by Valanto and are: ice thickness $h = 1/33.33$ m, $\rho_i = 916$ kg/m^3, $\rho_w = 1025$ kg/m^3, $g = 9.81$ m/s^2, $H = 1$ m, $\sigma_c = 10.67$ kN/m^2, $\sigma_{fl} = 23.78$ kN/m^2, ice-steel friction coefficient $\xi = 0.1$ [-] and hull angle $\alpha = 15°$.

COMPARISON OF THE RESULTS

The graph below shows a comparison between the dynamic breaking length predicted by the herein proposed model and Valanto's model described in (Valanto 1992):

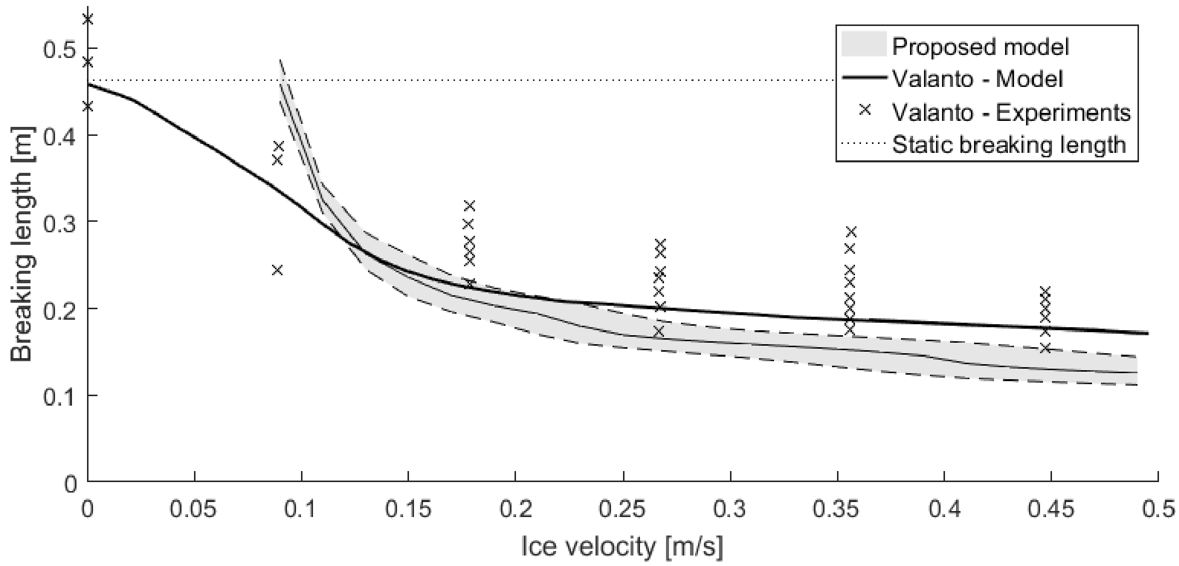

Figure 3. A comparison between the predicted dynamic breaking lengths. The Valanto data was directly extracted from figure 20 in the paper.

The figures shows the breaking lengths predicted by the proposed model, indicated by the gray area. A single value for the breaking length does not accurately portray the behavior seen by the model because at the time of failure a large portion of the ice is very close to failing. The gray area indicates the portion of the ice where the bending stress is higher than 99% of the yield stress at the moment of failure.

The graph shows there is a significant difference between the two models, especially at low velocities. This indicates that Valanto's model accounts for more forms of resistance causing the ice to fail with smaller breaking lengths and therefore earlier. The extra resistance even acts at very low velocities. This discrepancy must be caused by the different assumptions made by both models. Below is a list of the major items that are present in Valanto's model but are not present in the proposed model:

- the dynamic fluid pressure (see Eq. (4))
- rotational inertia of the ice
- steady-flow of the fluid around the vessel
- the effect of the vessel geometry on the fluid flow
- axial deformations of the ice
- ventilation effects

Of this list only two items were considered as a possible source of significant resistance: the rotational inertia and the dynamic pressure. To assess if rotational inertia can generate significant resistance it was incorporated by adding $\rho_i J \ddot{w}'$ to the equation of motion of the ice in Eq. (4), where J is the mass moment of inertia of the ice's cross-section. However, the rotational inertia has an insignificant effect $(< 1\%)$ on the breaking length and based on this was deemed not to be the source of the extra resistance.

Next the dynamic pressure was checked. Because of its non-linear nature incorporating it into the model is a non-trivial step. To gain insight into its effect without have to do a complete upgrade of the model to accommodate the nonlinearity, the dynamic pressure was evaluated using the solution of the linear model. To this end the transfer functions $\tilde{\boldsymbol{T}}$ were analyzed and it was found that the magnitude of the dynamic pressure can be as much as 60% compared to the magnitude of all other forces acting on the ice combined (these are the ice inertia, bending stresses, hydrostatic pressure and linear hydrodynamic pressure). Additionally it was found that the dynamic pressure behaves very much like a rapidly decaying exponential, with a maximum magnitude at the contact and then rapidly decaying in space.

Based on this analysis it was concluded that the dynamic pressure is the most likely cause of the discrepancy given the that it can take on such large magnitudes in the vicinity of the contact. To verify this hypothesis the model is currently being upgraded to include the dynamic pressure. Future work based on the upgraded model will give insight into whether the dynamic pressure is indeed the source of the discrepancy and thus an essential component of dynamic ice-structure interaction.

CONCLUSIONS

This paper showed that a 2D ice-structure interaction model based on a linear description of the fluid, i.e. incompressible, inviscid and irrotational fluid together with the *linearized* Bernoulli equation for the fluid pressure, gives a significant error in its prediction of the dependence of the breaking length of level ice on the interaction velocity. The most likely cause for this discrepancy was identified as the omission of the dynamic pressure term that was neglected when the Bernoulli equation was linearized. Future work aims to verify whether this term is indeed the source of the discrepancy and whether this terms is essential for the correct prediction of the dynamic breaking length of level ice and should be included in ice-structure interaction models.

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