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Three Physical Mechanisms of Wave Energy Dissipation in Solid Ice

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ABSTRACT

In the present study we compare wave energy dissipation caused by three physical processes: anelastic and viscous deformations of ice due to the periodic bending, and migration of liquid brine in pore spaces, and friction on the ice-water interface. The wave damping due to anelastic and viscous deformations of ice is estimated using a model of ice behavior under cyclic loading developed by Cole (1995) and standard assumptions of thin plate theory. The wave damping due to ice-water friction is estimated using the theory developed by Liu and Mollo-Christensen (1988) and values of the eddy viscosity in ice adjacent layer calculated from field data (Marchenko et al., 2015). The wave damping due migration of liquid brine through the ice is estimated using a recently developed analytical model based on the Darcy's law that treats the ice as a permeable material and quantifies energy loss due to the periodic movement of liquid brine within the ice.

KEY WORDS: Wave, Sea ice, Cyclic loading, Deformations, Boundary Layer, Permeability

INTRODUCTION

Reduction of ice covered areas in the Arctic influences development of offshore activity, navigation and tourism in regions which were not considered earlier for these perspectives. Although the amount of sea ice in the Arctic becomes smaller it still exists in summer time and covers almost all Arctic Ocean in the winter. Depending on the weather conditions wind action on sea surface in ice free areas may influence an increase of storm activity there. Wind waves and swell may destroy drift ice over big areas in relatively short time and influence the formation of Marginal Ice Zone. Drift ice destroyed by waves into small floes becomes more dynamic, and increases energy exchange between the atmosphere and ocean. Therefore investigation of wave damping in ice covered areas is of interest for engineering applications and climate issues.

Wind action on sea surface creates waves in wide spectral range with periods from 5 s to 30 s. High frequency part of the wave spectrum is disappeared due to waves-floes interaction in the Marginal Ice Zone. Wave energy dissipates due to the waves scattering and water-ice friction. Longer waves penetrate below the ice consisting of big floes and solid ice. Measurements performed on solid drift ice in the Barents Sea demonstrate that waves with period about 10 sec penetrate on several tens of kilometers into the solid ice (Collins et al., 2015; Marchenko et al, 2015). Experimental data on the spectral composition of flexural–gravity waves in the Arctic were analyzed by Nagurny et al. (1994) and Wadhams et al. (1995) to show that a major portion of the wave energy corresponds to waves with periods greater than 16 s.

Capacity of waves to break up the ice depends on wave amplitude and wave length. Damping decreases the wave amplitude, and therefore influences ice breakup by waves. Wave damping is characterized by the ratio of energy dissipation rate due to some physical process to the wave energy. In the present paper we perform numerical estimates and compare energy dissipation rates for the three processes causing wave damping in solid ice including viscous and anelastic deforming of ice, migration of liquid brine through the ice, water-ice friction in ice adjacent boundary layer.

DISSIPATIVE PROCESSES IN SEA ICE AND ICE ADJUCENT BOUNDARY LAYER

Granular structure of sea ice is columnar with isotropic (S2) or anisotropic (S3) distribution of optical axes in the horizontal plane (see, e.g., Weeks and Hibler, 2010). Inclusions of liquid brine trapped inside the ice during sea water freezing are distributed between closed brine packets and permeable channels (see, e.g., Cole and Shapiro, 1998). Amount of liquid brine and its distribution between the packets and channels depends on the temperature and salinity of sea ice (Golden et al., 2007). The ice is permeable, and the brine is able to migrate through the matrix under the influence of pressure gradients. The permeability may be affected by thermal changes in the ice, as brine pockets separate and merge during freezing and melting cycles.

Cyclic bending deformation of ice influences cyclic tension/compression of horizontal layers in the ice, and creates cyclic changes of vertical pressure gradient inside the ice. In-plane deformation ε are of the order $h/2R$, where h is the ice thickness and R is the curvature radius of the ice surface. The curvature is approximated with the formula $1/R{\sim}ak^2$, when the ice is deformed by sinusoidal wave with amplitude a and wave length $\lambda=2\pi/k$. In-plane stresses are estimated as a product of the elastic modulus on the strain: $\sigma=E\varepsilon$. Dependence of the stress σ from the wave length λ is shown in Figure 1a, when E=1 GPa, h=1 m, and a=0.1 m. Period of gravity waves is greater 8 sec when the wave length is greater 100 m. From Figure 1a it follows that wave induced stresses in the ice are lower 0.2 MPa, and it is lower 0.4 MPa if the elastic modulus is equal to 2 GPa. In this case energy of anelastic and viscous deformation takes not more than 5% of the energy of elastic deformations (Cole, 1995). Anelastic and viscous deformations are related mainly with relative displacements of grain boundaries and density of dislocations is not growing. Anelastic and viscous deformations of ice dissipate the wave energy when wave propagates below the ice.

Vertical pressure gradient in the ice bended by waves is estimated with the formula ✰p/✰z~σ /h. It is smaller 0.2 MPa/m for the stresses shown in Figure 1a. The flux of the brine through the ice under the action of the pressure gradient is estimated with Darcy’s formula q=-κ✰p/✰z/μ, where κ is the permeability of ice by brine, and μ is the dynamic viscosity of water. Assuming that κ=10^{-12}m^2 and μ=1.5·10^{-3}kg/(s·m) we estimate q<1.3·10^{-4}m/s. Reynolds number is calculated with the formula Re=$\rho q d/\mu$, where ρ is the brine density and d is the grain size. Assuming ρ=10^3kg/m^3 and d~1mm we estimate Re<0.086. It is smaller the critical values when the linear Darcy’s law should be corrected (Polubarinova-Kochina, 1962). Thus we conclude that liquid brine inside ice will migrate in vertical direction through the ice when the ice is deformed by waves. Friction between moving brine and ice will dissipate the wave energy.

When wave propagate below the ice the surface particles of the water adhere to the ice bottom and lose their momentum. Effectivity of the momentum transfer from deeper particles to the surface particles depends on the turbulence in ice adjacent water level since fluctuations of wave induced water velocities are transported together with fluctuations induced by under ice turbulence. The process is characterized by the eddy viscosity which can be much greater the molecular viscosity of water. This mechanism of wave energy dissipation was considered by Liu and Mollo-Christensen (1988). The value of the eddy viscosity is most important for the wave damping.

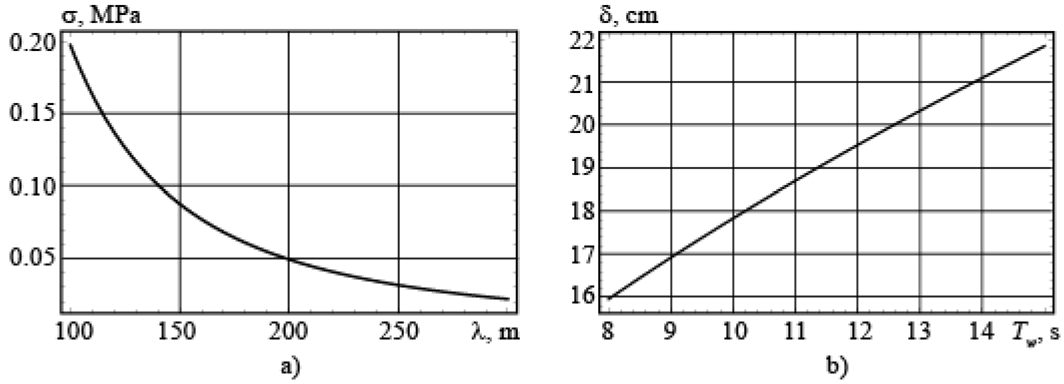


Figure 1. Bending stresses versus wave length (a). Thickness of wave induced boundary layer versus wave period (b).

WAVE DAMPING DUE TO VISCOUS AND ANELASTIC DEFORMATIONS OF ICE

Surface density of elastic energy of thin plate is calculated with the formula (see, e.g., Landau and Lifshitz, 1986)

$$F_{el} = D(\Delta\eta)^2/2,\quad D = \frac{Eh^3}{12(1-\nu_p^2)},\quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{1}$$

where E is the effective elastic modulus, h is the ice thickness, and ν_p is the Poisson's ratio, x and y are the spatial coordinates.

When the plate has sinusoidal shape, $\eta = a\sin(kx-\omega t)$, the plate energy averaged over the wave length equals

$$\langle F_{el}\rangle = Da^2k^4/4\,. \qquad (2)$$

Here a is the wave amplitude, k is the wave number, ω is the wave frequency, and t is the time.

According to the model of cyclic loading (Coal, 1995) the dissipated energy takes about 5% of the elastic energy when maximal load doesn't exceed 0.5 MPa and the loading frequency is around 0.1 Hz or higher. Thus the rate of energy dissipation due to the periodical bending of the plate is specified by the formula

$$\left\langle \dot{F}_v \right\rangle = \frac{0.05}{8\pi} D\omega a^2 k^4\,. \qquad (3)$$

Energy of surface gravity wave is determined by the formula (Lamb, 1975),

$$\langle F_w\rangle = \frac{\rho\omega^2 a^2}{2k\tanh(kH)}\,, \qquad (4)$$

where H is the water depth. The attenuation rate of the wave amplitude due to the viscous and anelastic deformations of ice is equal to

$$\gamma_v = \frac{\langle \dot{F}_v\rangle}{\langle F_w\rangle} = \frac{0.05}{4\pi}\frac{Dk^5\tanh(kH)}{\rho\omega}\,. \qquad (5)$$

WAVE DAMPING DUE TO WAVE INDUCED MOTION OF BRINE

Vertical momentum balance of liquid brine in permeable channels inside the ice is expressed by the equation (Polubarinova-Kochina, 1962)

$$\frac{\rho}{\phi}\frac{\partial q}{\partial t} = -\frac{\partial p}{\partial z} - \frac{\mu}{\kappa} q\,. \qquad (6)$$

where ϕ is the ice porosity. Pressure gradient is estimated from the theory of thin elastic plate

$$\frac{\partial p}{\partial z} \approx -\frac{1}{2}\frac{\partial \sigma_{xx}}{\partial z} = j_0\cos\theta\,, \quad j_0 = \frac{Eak^2}{2(1-\nu_p^2)}\,, \quad \theta = kx - \omega t\,. \qquad (7)$$

In case when the pressure gradient is expressed by formula (7) the solution of (6) has the form

$$q_z = \frac{j_0}{\Delta}\sin(\theta - \theta_0), \quad \Delta^2 = \left(\frac{\rho\omega}{\phi}\right)^2 + \left(\frac{\mu}{\kappa}\right)^2, \quad \theta_0 = \arccos\left(\frac{\rho\omega}{\phi\Delta}\right). \tag{8}$$

The permeability of saline ice by brine is estimated by the formula

$$\kappa = \kappa_0 e^{15\sqrt{\nu_b}} \quad , \tag{9}$$

where ν_b is the liquid brine content in ice, the coefficient κ_0=$10^{-13}$$m^2$ characterizes ice permeability at low values of the liquid brine content. Formula (9) approximates the data of the laboratory and numerical simulations shown in Figure 2 in Zhu et al (2006). Further we assume that ϕ= ν_b and estimated with the formula of Frankenstein and Garner (1967)

$$\nu_b = \sigma_{si}\left(\frac{49.185}{|T|} + 0.532\right), \quad -22.9^\circ\mathrm{C} < T < -0.5^\circ\mathrm{C}, \tag{10}$$

where σ_{si} and T_i are the sea ice temperature and salinity.

Assuming ν_b=0.12 (warm sea ice with temperature -2C and salinity 5ppt) we find κ~1.8·10^{-11} m2, and Δ~μ/κ>>1. Therefore solution (5) can be simplified to the formula

$$q \approx \frac{j_0\kappa}{\mu}\sin\theta\,. \tag{11}$$

Energy dissipation rate over unit area of the ice is calculated with the formula

$$\dot{F}_b = \frac{\mu h}{\kappa\phi} q^2\,. \tag{12}$$

Substituting formula (11) into formula (12) we find after the averaging over the wave period

$$\left\langle \dot{F}_b \right\rangle = \frac{\kappa h}{2\mu\phi} j_0^2\,. \tag{13}$$

The attenuation rate of the wave amplitude due to the vertical brine migration of brine through the ice is calculated with the formula

$$\gamma_b = \frac{\langle \dot{F}_b \rangle}{\langle F_w \rangle} = \frac{\kappa h}{4\rho\mu\phi}\frac{E^2 k^5 \tanh(kH)}{\omega^2(1-\nu_p^2)^2}\,. \tag{14}$$

WAVE DAMPING DUE TO WATER-ICE FRICTION

For the estimate of wave damping due to the friction on the ice-water interface we use the formula derived for the mean energy dissipation rate in oscillating water flow near a wall (formula (24.14) from Landau and Lifshitz, 1988),

$$\left\langle \dot{F}_f \right\rangle = \frac{\rho V^2}{2\sqrt{2}} \sqrt{\nu\omega}\,, \tag{15}$$

where V is the amplitude of the wave induced water velocity outside the wave induced boundary layer below the ice, and ν is the kinematic eddy viscosity below the ice. The thickness δ of wave induced boundary layer is specified by the formula

$$\delta = \sqrt{2\nu / \omega}\,. \tag{16}$$

The eddy viscosity below the drift ice is estimated from 100 cm^2/s to 200 cm^2/s (McPhee and Martinson, 1994). The thickness of the boundary layer calculated with ν =100 cm^2/s is shown versus wave period $T_w=2\pi/\omega$ in Figure 1b. High frequency measurements of under ice water velocity in the Barents Sea and data processing shown highest values of the eddy viscosity 140 cm^2/s. Locations of field measurements are shown in Figure 2. User setups of the deployments of the Acoustic Doppler Velocimeter SonTek 5 MHz are shown in Table 1. Calculated values of the eddy viscosities are shown in Table 2. Methodic of the eddy viscosity calculation is described by Marchenko et al (2015).

The amplitude of surface water velocity induced by wave is expressed via wave amplitude by the formula

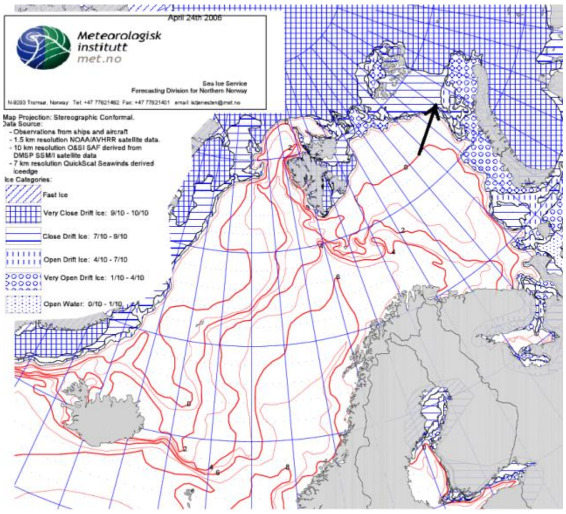
$$V = a\omega / \tanh(kH)\,. \tag{17}$$

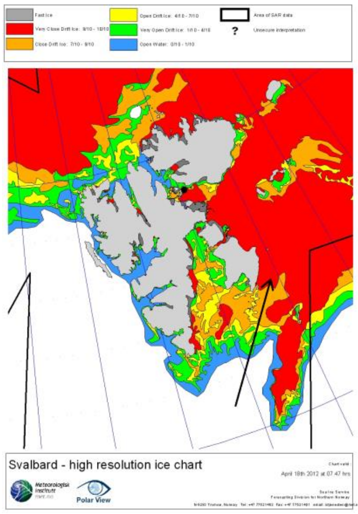
The attenuation rate of the wave amplitude due the water-ice friction is calculated with the formula

$$\gamma_f = \frac{\left\langle \dot{F}_f \right\rangle}{\left\langle F_w \right\rangle} = \frac{k\sqrt{\nu\omega}}{2\sqrt{2}\tanh(kH)}\,. \tag{18}$$

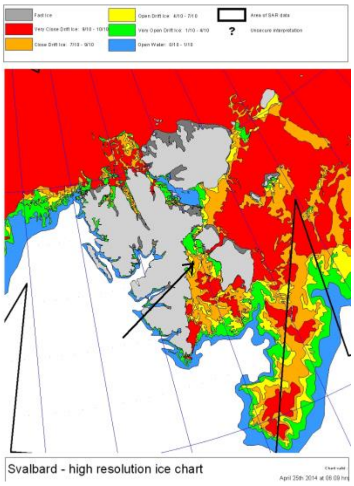
Table 1. User setups of the ADV SonTek 5 MHz Ocean probe deployments in the Barents Sea.

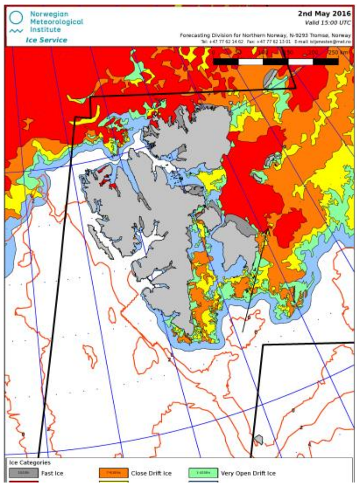
N	Data	Sampling frequency, Hz	Burst interval, s	Samples per burst	Number of bursts	Depth, cm
1	April 24, 2006	5	1800	8000	2	190
2	April 25, 2006	10	1200	900	3	180
3	April 18-19, 2012	10	1200	1200	50	112
4	April 25, 2014	10	1200	3000	50	41
5	May 1-2, 2016	10	360	2400	94	80





a) b)





c) d)

Figure 2. Locations of measurements of water velocity in ice adjacent boundary layer on April 24-25, 2006 (a); April 18-19, 2012 (b); April 25, 2014 (c); May 1-2, 2016 (d).

Table 2. Eddy viscosity below drift ice in the Barents Sea, cm^2/s.

1	2	3	4	5
0.25	0.002	96.07	0.1	140

DISCUSSION

Figure 3 shows the dependencies of the attenuation rates of the wave amplitude due the viscous and anelastic deformations of ice, due to the brine migration through ice, and due to the water-ice friction versus the wave length. Examples the sea ice salinity is 5 ppt. Figure 3a is constructed with sea ice temperature -5C and different ice thicknesses. Figures 3b,c are constructed with ice thickness equals 1 m. Figure 3c is constructed with ice temperature equals -2 C. The dependence of graphics shown in Figure 3 from the water depth is very small when H>20 m.

From Figure 3a and 3c it follows that the attenuation rates due the viscous and anelastic deformations of ice and due to the water-ice friction are of the same order when the eddy viscosity is relatively big. The attenuation rate due the viscous and anelastic deformations of ice dominates in rest water. Figure 3b demonstrates that the attenuation rate due to the brine migration through ice is much higher than the attenuation rates due the viscous and anelastic deformations of ice and due to the water-ice friction when the ice permeability is greater $10^{-13}m^2$. From Figure 3b it follows that waves with wave length 100-200 m practically don't penetrate into the ice covered regions. This effect is needed in future study. Probably wave induced brine migration influences micro-cracking in the ice. It can influence the reduction of filtration velocity and dissipated energy due to the friction between brine and ice. The micro-cracking may influence the increase of wave damping due to due the viscous and anelastic deformations of ice.

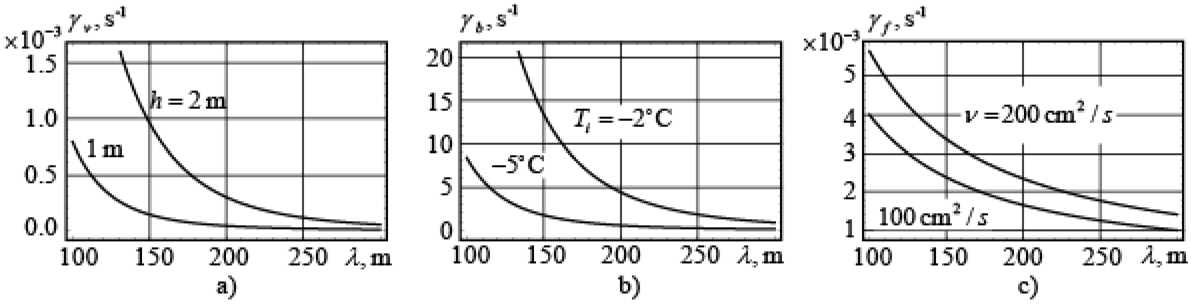


Figure 3. The attenuation rates of the wave amplitude due the viscous and anelastic deformations of ice (a), due to the brine migration through ice (b), and due to the water-ice friction (c) versus the wave length.

CONCLUSIONS

Three processes causing wave damping in solid ice are investigated. The attenuation rates of the wave amplitude due the viscous and anelastic deformations of ice, due to the brine migration through ice, and due to the water-ice friction are derived, analyzed and compared. It is discovered that the attenuation rates of the wave amplitude due the viscous and anelastic deformations of ice and due to the water-ice friction are of the same order when the eddy viscosity in ice adjacent water boundary layer is relatively high. In the rest water the attenuation rate of the wave amplitude due the viscous and anelastic deformations of ice dominates. The attenuation rate due to the brine migration through the ice is discovered very big. The estimates are based on Darcy's law and known values of sea ice permeability. The effect should be further investigated.

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REFERENCES

Cole, D.M.,1995. A model for the anelastic straining of saline ice subjected to cyclic loading. Philosophical Magazine A, 72(1), 231-248.

Cole, D.M., Shapiro, L.H., 1998. Observations of brine drainage networks and microstructure of first-year sea ice. Journal of Geophysical Research, 103(NC10), 21,739-21,750.

Collins, C,O., Rogers, W.A., Marchenko, A., Babanin, A.V., 2015. In situ measurements of an energetic waves event in the Arctic marginal ice zone. Geoph. Res. Letters, 42, 6, 1863-1870.

Frankenstein, G.E., Garner, R., 1967. Equations for determining the brine volume of sea ice from -0.5 to -22.9ºC., J. Glaciol., 6(48), 943-944.

Golden, K.M., Eicken, H., Heaton, A.L., Miner, J., Pringle, D.J., Zhu, J. 2007. Thermal evolution of permeability and microstructure in sea ice. Geophysical Research Letters, 34, L 16501.

Landau LD and Lifshitz EM (1986) Theory of elasticity. Volume 7:
Theoretical physics, 3rd edn. Butterworth-Heinemann, Oxford

Landau, L.D., Lifshitz,E.M., 1988. Fluid Mechanics, Second edition: Vol. 6 (Course of Theoretical Physics S), Butterworth-Heinemann, Oxford.

Liu, A. K. and E. Mollo-Christensen (1988), Wave propagation in a solid ice pack, J. Phys. Oceanogr., 18(11), 1702-1712.

Marchenko, A.V., Gorbatsky, V.V., Turnbull, I.D., 2015. Characteristics of under-ice ocean currents measured during wave propagation events in the Barents Sea. POAC15-00171.

McPhee, M.G., Martinson, D.G., 1994. Turbulent mixing under drifting pack ice in the Weddell Sea. Science, 263, 218-222.

Nagurny, A. P., V. G. Korostelev, and V. P. Abaza, 1994: Wave method for evaluating the effective thickness of sea ice in climate monitoring. (BRAS Physics/Supple), Phys. Vibrations, 58(3), 168–174.

Weeks, W.F., Hibler, W.D.III., 2010. On sea ice. University of Alaska Press, Fairbanks.

Wadhams, P., and S. C. S Wells, 1995: Ice surface oscillation measurements on SIMI using strain, heave and tilt sensors. Proc. Sea Ice Mechanics and Arctic Modeling Workshop, Anchorage, AK, Office of Naval Research, 176–189.

Polubarinova-Kochina, P.Ya., 1962. Theory of ground-water movement. Princeton Legacy Library.

Zhu, J., Jabini, A., Golden, K.M., Eicken, H., Morris, M. 2006. A network model for fluid transport through sea ice. Annals of Glaciology, 44, 129-133).