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Ice Loads Dynamics for Model Scale Cylinders of Various Diameters

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ABSTRACT

The question, if the distribution law of model scale ice load depends on the diameter of a cylindrical indenter and its indentation speed, is still open. In new ice tank facility of Krylov Research Centre (St. Petersburg) a series of experiments with cylindrical indenters of diameters 30, 40 and 96 mm were conducted. In these experiments global ice loads were recorded with high sampling rate. Two ice fields were involved into experiments – with thickness of 55 and 72 mm. The indentation experiments considered in the paper were conducted at speeds of 0.025, 0.1, 0.15, 0.3 m/s.

Tests on distribution law with implementation of D'Agostino's method were performed. Results of statistical processing and ice load process' distribution law evaluation of obtained time series are presented in the paper. The distribution law of global load process appeared to be speed dependent. Probably it also depends on indenter's diameter. Model ice parameters for conducted experiments are provided in the paper. Comparison with results, which were obtained earlier, is performed.

KEY WORDS: Ice; Model; Load; Stochastic; Lognormal.

NOMENCLATURE

In the text all random variables are denoted with bold italic letters, constant values and non-random variables – with regular italic.

$\boldsymbol{X}(t)$ – Stochastic process of ice load.

X – Time series (single record) of process $\boldsymbol{X}(t)$

n – Number of data points in the time series X.

q – Sampling rate of the time series X.

$\boldsymbol{\xi}$ – Statistical population.

μ_i – i-th central moment of the statistical population $\boldsymbol{\xi}$.

γ_1 – Pearson's asymmetry coefficient of the statistical population $\boldsymbol{\xi}$.

$\{\boldsymbol{x}\}$, $\{\boldsymbol{z}\}$ – Random samples taken from statistical populations; random data points from these samples are denoted with $\boldsymbol{x}_i$ and $\boldsymbol{z}_i$. Particular samples with particular data points are denoted with $\{x\}$, $\{z\}$; particular data points are denoted with x_i and z_i.

N, k – Sizes of the samples $\{\boldsymbol{x}\}$ and $\{\boldsymbol{z}\}$ respectively.

l – distance, which separates two almost uncorrelated data points in time series X.

$\overline{\boldsymbol{x}}$ – Sample average. Value of $\overline{\boldsymbol{x}}$ found for the particular sample $\{x\}$ is denoted with $\bar{x}$.

$\boldsymbol{\sigma}_z$ – The point estimator of standard deviation of $\boldsymbol{Z}$. Its particular value is denoted σ_z.

$\boldsymbol{m}_i$ – i-th central moment of sample.

$\boldsymbol{g}_1$ – Pearson's asymmetry coefficient of sample. Value of it found for the particular sample $\{x\}$ is denoted with g_1.

$\boldsymbol{Z}$ – Statistics of D'Agostino's test.

$\boldsymbol{Y}, W, \alpha, \delta$ – Components of expression for $\boldsymbol{Z}$.

H_0 – Hypothesis "Population $\boldsymbol{\xi}$ is normally distributed".

$H_0^{'}$ – Hypothesis "Mean value of $\boldsymbol{Z}$ is 0".

$H_0^{''}$ – Hypothesis "Standard deviation of $\boldsymbol{Z}$ is 1".

$\chi_p^2(k-1)$ – p-th quantile of χ^2 distribution with $k-1$ dimensions of freedom.

$\boldsymbol{r}_{dat}(\tau)$ – Estimator of the autocorrelation function of the stationary process $X(t)$. Value of it found for the particular sample $\{x\}$ is denoted with $r_{dat}(\tau)$.

τ – Time lag in seconds.

INTRODUCTION

This paper continues the study started earlier by Zvyagin & Sazonov (2015), Zvyagin (2016) and Dobrodeev et al. (2016). Aim of this study is to make reliable stochastic model of ice load dynamics in the case of offshore structures with column supports. For that reason tests with cylindrical indenters of different diameters were performed in the ice tank of Krylov State Research Centre. Global loads caused by the model ice on these cylindrical indenters were recorded and studied.

The stochastic process model was chosen for ice load because of crushing process complexity. Such mathematical model is applied since the paper of Sundararajan and Reddy (1973). According to Bjerkas (2004) stationarity of ice loads is associated with high indentation speed and non-simultaneous brittle ice failure mode. Later Karna et al. (2007) emphasized that the stochastic process approach is applicable to describe loads when sheet of plain ice is failing by continuous crushing, and the structure has vertical walls.

Stationarity and normality or lognormality of ice load stochastic process provides multiple benefits for analysis and simulation. Sodhi (1998) have presented records of loads caused by freshwater ice on vertical T-shaped, 25-mm thick steel plates with 49.5-mm wide contact area at high speed (409.3 mm/s and 401.3 mm/s). These records visually are looking like as time series of stationary processes, though investigation on that was not presented. Example of normal stationary ice load process was presented by Guo (2012). For simulation of a stationary lognormal process the simulation of logarithm of it, which is a normal process, can be implemented. Autocorrelation function of lognormal process and autocorrelation function of its logarithm are related as described by Zvyagin and Sazonov (2014).

Main aims of the previous studies performed by the authors were: verification of ice load process stationarity in wide sense, evaluation of parameters of process and analysis of its autocorrelation function (ACF). Study of loads on indenters of 20 mm diameter at 0.01 m/s indentation speed against 15 and 19 mm thick ice fields have revealed lognormal distribution and features of stationarity (Dobrodeev et. al, 2016). At the same time record of load on 100 mm width cylinder against 30 mm thick ice made with speed of 0.084 m/s has performed normal behavior (Zvyagin and Sazonov, 2015). Appearance of such differences was not explained in previous studies and needs more detailed investigation.

In recent study new records of global ice loads were investigated on their normality/lognormality. Connection of indentation velocity and indenter diameter with load process distribution law is proposed.

The structure of the paper is as following. First Section is introductive. In the second Section parameters of experiments in ice tank are listed. In the third Section details of application of D'Agostino's method to load process normality test are described. In the next three Sections analysis of ice load records measured in experiments in the ice tank is made. After that discussion and conclusions are made.

DESCRIPTION OF EXPERIMENTS IN ICE TANK

Experiments with cylindrical indenter were performed in the new ice tank of Krylov State Research Centre. Detailed description of this facility was earlier made by Timofeev et. al (2015). Three different indenters of 30 mm, 40 mm and 96 mm diameter were used for tests in two ice sheets with thickness 55 mm and 72 mm. Speeds of indentation were 0.025, 0.1, 0.15 and 0.3 m/s.

Tests were performed using FG (Fine Grain) grained model ice. Parameters of ice sheets used in experiments are provided in the Table 1. Ice load on indenter was measured by the dynamometer. Experiments were performed using the principle of inversed motion. The method of indentation experiments is fully described in Dobrodeev et al. (2016). The picture of facility and the example of time history of load measured in test with cylindrical indenter of 30 mm diameter against level ice with thickness of 72 mm are presented in Figure 1.

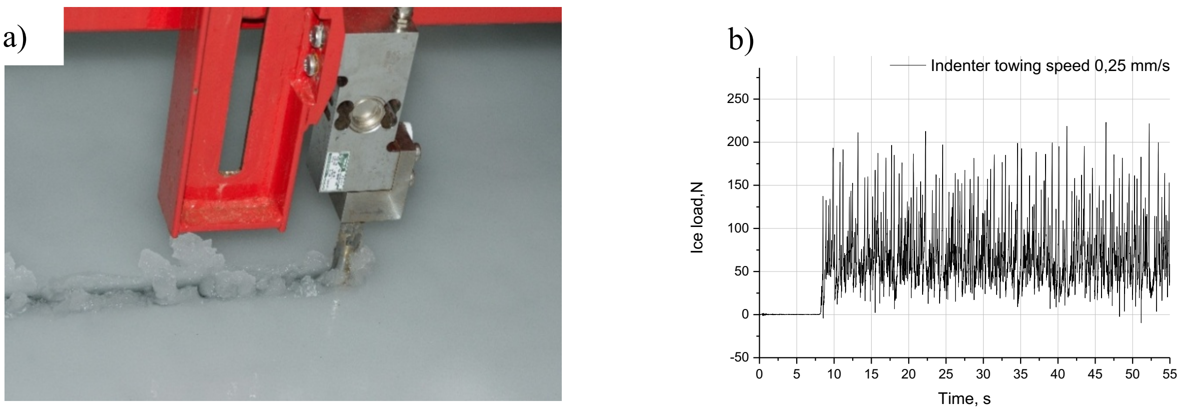

Figure 1. a) Indentation test equipment; b) Time history of ice load measured in indentation test against 72 mm thick ice, diameter of indenter is 30 mm.

Table 1. Parameters of ice fields used in indentation tests (average values)

Number	Thickness, mm	Flexural strength, kPa	Compressive strength, kPa	Elastic modulus, MPa
1	55	22.45	29.1	31.75
2	72	25	47.1	39.25

AN APPLICATION OF D'AGOSTINO METHOD TO THE ICE LOAD RECORD NORMALITY TEST

In this Section we will follow statistical notations accepted in Kendall and Stuart (1977) and D'Agostino and Ralph (1970). As it was said in the Section "Nomenclature", all random variables will be denoted with bold italic letters, and values of those random variables along with constants and non-random variables – with regular italic letters.

Let us consider the model of stochastic process $\boldsymbol{X}(t)$ for the ice load. The particular record of ice load made in the indentation experiment will be denoted with X and considered as time series of the process $\boldsymbol{X}(t)$. To conduct statistical tests we need to get somehow the sample $\{x\}$ of data points from X. Most of statistical tests are designed for samples, elements in which are statistically independent.

To decrease statistical dependency of data in the sample $\{x\}$ we shall take distant points from X for it. The particular method of data points taking will be described later. Then, if probabilistic properties of $\boldsymbol{X}(t)$ are preserved in time, we can consider data points in $\{x\}$ as taken from single statistical population $\boldsymbol{\xi}$.

Let us consider statistical population $\boldsymbol{\xi}$. Let us denote the Pearson's asymmetry coefficient of it with γ_1:

$$\gamma_1 = \mu_3/\mu_2^{3/2}. \qquad (1)$$

Here μ_3 and μ_2 are third and second central moments of population $\boldsymbol{\xi}$. Then corresponding estimator of γ_1, calculated with presence of sample $\{\boldsymbol{x}\}$ of size N taken from that population will be $\boldsymbol{g}_1$:

$$\boldsymbol{g}_1 = \boldsymbol{m}_3/\boldsymbol{m}_2^{3/2} \qquad (2)$$

Value of $\boldsymbol{g}_1$ obtained from the sample $\{x\}$ we shall denote with g_1:

$$g_1 = \frac{\sum_{i=1}^{N}(x_i-\bar{x})^3}{\left(\sum_{i=1}^{N}(x_i-\bar{x})^2\right)^{3/2}}\sqrt{N} \qquad (3)$$

where $\bar{x}$ is the average value of $\{x\}$ and x_i are data points of this sample.

Null hypothesis to test will be the following: $H_0 =$ "Population $\boldsymbol{\xi}$ is normally distributed". To approve or reject H_0 the skewness of sample can be investigated with respect of zero values of μ_3 and γ_1 (1) under the valid hypothesis H_0. In fact, even if H_0 is valid the particular value of random variable (2) calculated with using of particular $\{x\}$ by chance ca n fall rather far from the expected zero value. How large the discrepancy of (3) wi th 0 should be to say, that H_0 is invalid? To answer this question the exact distribution of (2) under the valid hypothesis H_0 should be known.

The problem is that the distribution of $\boldsymbol{g}_1$ with validity of H_0 is depending on the sample size N. With respect of this, D'Agostino (1970) have suggested next transformation of $\boldsymbol{g}_1$, that allows to consider statistics $\boldsymbol{Z}$, which distribution under H_0 is not depending on N:

$$\boldsymbol{Y} = \boldsymbol{g}_1\sqrt{\frac{(N+1)(N+3)}{6(N-2)}}$$

$$W^2 = -1 + \sqrt{2\left(\frac{3(N^2+27N-70)(N+1)(N+3)}{(N-2)(N+5)(N+7)(N+9)} - 1\right)}$$

$$\delta = 1/\sqrt{\ln W} \qquad (4)$$

$$\alpha = \sqrt{2/(W^2-1)}$$

$$\boldsymbol{Z} = \delta ln\left[\boldsymbol{Y}/\alpha + \sqrt{(\boldsymbol{Y}/\alpha)^2 + 1}\right].$$

Under the valid hypothesis H_0 the statistics $\boldsymbol{Z}$ is distributed approximately normally with parameters $(0,1)$ (D'Agostino & Ralph, 1970). Transformation provided above is applicable when $N \geq 8$. Critical area for $\boldsymbol{Z}$, which leads to rejection of H_0, can be found by using of any table of standard normal distribution percentiles.

In our study we shall not rely on single value of $\boldsymbol{Z}$ obtained from single $\{x\}$: simulations have shown that the probability of type II error in the test of H_0 can be rather significant.

Taking in account assumption of $\boldsymbol{X}(t)$ stationarity, we shall take k samples $\{x\}$ and calculate k values of $\boldsymbol{Z}$ according to (4), which will make sample $\{z\}$. For that sample we shall perform Chi-squared goodness-of-fit test on normality and also test two hypotheses: $H_0' =$ "Mean value of $\boldsymbol{Z}$ is 0" and $H_0'' =$ "Standard deviation of $\boldsymbol{Z}$ is 1" using corresponding well known confident intervals for normally distributed random variable. If both hypotheses H_0' and H_0'' are not rejected, then there are not strong objections against symmetry of distribution of $\boldsymbol{\xi}$. As soon as an alternative to H_0 from our point of view is mostly connected with asymmetric distributions, e.g. lognormal, it means that there are no strong objections against H_0. This way we shall accept hypothesis H_0.

Let us take confidence level 0.95, then significance level will be 0.05. Let the critical area will be two-sided. The critical area for $\bar{\boldsymbol{z}} = \sum_{i=1}^{k} \boldsymbol{z}_i/n$, falling in which leads to rejection of H_0', is the next: $\bar{z} > 1.96/\sqrt{k}$ or $\bar{z} < -1.96/\sqrt{k}$.

Let us denote with $\varepsilon_1 = \chi^2_{0.025}(k-1)$ and $\varepsilon_2 = \chi^2_{0.975}(k-1)$ values of 2.5 and 97.5 percentiles of $\chi^2(k-1)$ distribution. Then the critical area for $\boldsymbol{\sigma}_z = \sqrt{\sum_{i=1}^{k}(\boldsymbol{z}_i - \bar{\boldsymbol{z}})^2/k}$ with significant level 0.05 is $\sigma_z < \sqrt{\varepsilon_1/k}$ or $\sigma_z > \sqrt{\varepsilon_2/k}$. Falling in that critical area leads to rejection of H_0''.

In the case of testing of $\boldsymbol{X}(t)$ on lognormality, logarithmic time series should be considered instead of original time series.

LOGNORMAL DISTRIBUTION FOR TESTS AT SPEED 0.025 m/s

At first we shall consider records of global load measured in experiments with cylinders with 30 mm and 40 mm diameter at speed 0.025 m/s against 55 and 72 mm thick model ice. These records were 40 seconds long, which provides $n = 4000$ data points at 100 Hz sampling rate.

Let us study how strong the correlation of neighboring data points in load time series is. For that purpose the autocorrelation values $r_{dat}(\tau)$ were calculated according to the formula:

$$r_{dat}(\tau) = \frac{\sum_{i=1}^{n-q\tau}(x_i-\bar{x})(x_{i+q\tau}-\bar{x})/(n-q\tau)}{\sum_{i=1}^{n}(x_i-\bar{x})^2/n}\ , \qquad (5)$$

here x_i are sequential data points from ice load time series X of size n, $\bar{x} = \sum_{i=1}^{n} x_i/n$ is the average of all data from X, q $[1/s]$ is the sampling rate and τ $[s]$ is the time lag in seconds. This way, $r_{dat}(\tau)$ for every τ is the value which random autocorrelation function estimator $\boldsymbol{r}_{dat}(\tau)$ took on for particular τ. Plots of these values for considered load time series are presented in Figure 2. From Figure 2 it follows that two data points of X separated by the time lag $\tau \approx 0.1\ s$ might be almost uncorrelated at least for the observed time series.

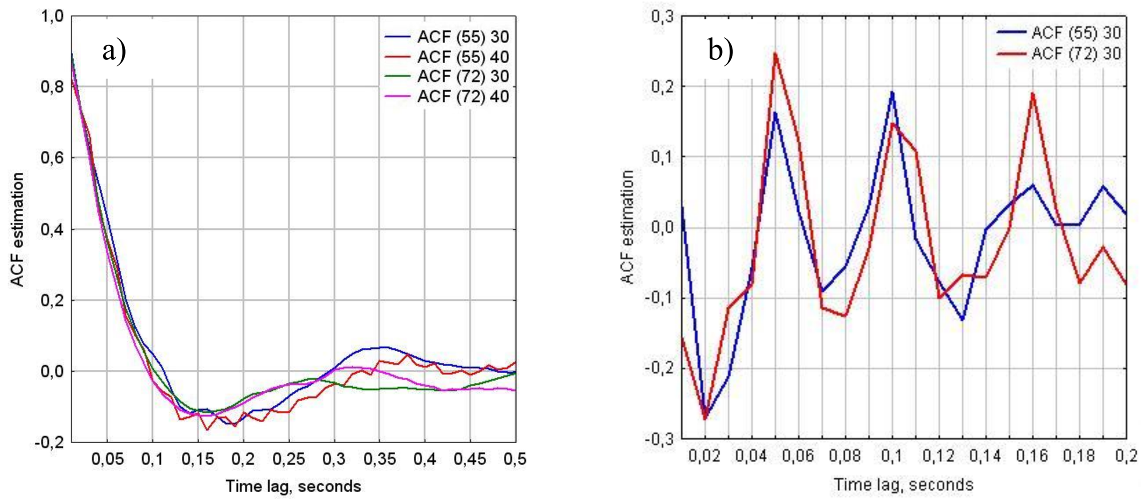

Figure 2. Autocorrelation estimation: a) of global ice loads on 30 and 40 mm wide cylinders at speed 0.025 m/s against 55 mm and 72 mm thick model ice; b) of global ice loads on 30 mm wide cylinder at speed 0.3 m/s against 55 mm and 72 mm thick model ice.

Let us assume that considered ice load records are the time series of lognormally distributed stationary process $\boldsymbol{X}(t)$. The validity of this assumption will be confirmed later. Let us denote the logarithm of time series X with lnX: this way data points of lnX are logarithms of corresponding data points of X.

To study parameters of lnX let us compose a sample $\{x\}$ in the following way: we shall take N data points from lnX separated by l =10 data points from each other. With respect of $q = 100\,Hz$ the distance of 10 data points corresponds 0.1 s, and from Figure 2a it follows that therefore the data in $\{x\}$ might be almost uncorrelated. Taking in account our assumption on lognormal distribution law the selected data points can be assumed as independent.

Having $\{x\}$ let us implement D'Agostino's method described above to calculate one value of $\boldsymbol{Z}$ according to (4). Let us then put this value to the sample $\{z\}$. Let us repeat the procedure of making $\{x\}$ and obtaining values of $\boldsymbol{Z}$ by taking further N data points from lnX separated by l from each other. This way we shall obtain sample $\{z\}$ of size $\left\lfloor\frac{n}{N\cdot l}\right\rfloor$, which is floor of ratio $n/(Nl)$ and equal to 20 in our particular case. This $\{z\}$ is finally should be tested on the centered standard normal distribution law to check validity of our assumption on lognormal distribution of $\boldsymbol{X}(t)$. To perform such a test simple goodness-of-fit Chi squared test was used along with testing hypotheses $H_0^{'}$ and $H_0^{''}$ described in the previous section.

To use D'Agostino's transform the N should be greater than 8. Generally the larger N, the better. But with increasing N and with constant l and constant time series length n, the size of $\{z\}$ can become too small to make confident outcomes. The influence of taken value of N on the properties of sample $\{z\}$ under the valid H_0 should be studied in the future in more details.

For the study described in this section $N = 20$ was taken. Thus with time series of size $n = 4000$ and $l = 10$ size of $\{z\}$ was k =20. For normality goodness-of-fit Chi squared test of $\{z\}$ number of intervals was taken as 4. Here parameters of population are defined as $(0; 1)$, so the number of dimensions of freedom of Chi squared test statistic under the null hypothesis is 3. Resulting parameters of $\{z\}$ obtained for considered ice load time series are presented in Table 2 along with values of Chi squared test statistics $\chi^2_{experim}$.

Table 2. Results of test of global ice load time series on the distribution law.

Ice thickness	55 mm			72 mm				
Trolley speed, m/s	0.025		0.3	0.025		0.1	0.15	0.3
Indenter's diameter	30 mm	40 mm	30 mm	30 mm	40 mm	96 mm	96 mm	30 mm
Type of test	LNormal	LNormal	Normal	LNormal	LNormal	Normal	Normal	Normal
$\bar{z}$	0.18	0.05	-0.01	0.93	0.06	0.17	-0.28	0.22
$\sigma_{\{z\}}$	0.72	1.01	1.01	0.52	1.24	0.99	0.91	0.91
min $\{z\}$	-1.23	-2.15	-1.54	0.16	-2.63	-1.36	-2.39	-0.73
max $\{z\}$	1.61	1.78	1.41	1.86	1.92	2.42	0.82	1.82
$\chi^2_{experim}$	2.1	0.94	0.85	20.39	0.99	0.98	2.57	1.83
N	20	20	10	20	20	10	10	10
k	20	20	7	20	20	13	13	7
Result	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes

From the Table 2 one can see that records of load made for cylinders with diameters of 30 and 40 mm against ice with thickness of 55 mm with indentation speed 0.025 m/s can be considered as of lognormal origin. For the ice thickness of 72 mm, indenter diameter of 30 mm and the same speed, the result of the test on lognormal distribution was negative. For ice thickness of 72 mm, indenter diameter of 40 mm and speed 0.025 m/s the result was positive.

All of 20 samples $\{x\}$ for each of time series X were united to make the sample $\{\{x\}\}$. This way data in $\{\{x\}\}$ can also thought as almost uncorrelated. Histograms of the $\{\{x\}\}$ along with fitted lognormal curves are provided in the Figure 3 a-d. Parameters of process $\boldsymbol{X}(t)$

estimated from samples $\{\{x\}\}$ in those cases when lognormal distribution was confirmed, are provided in corresponding columns of the Table 3.

Table 3. Estimations of ice load process' moments obtained from samples $\{\{x\}\}$.

Ice thickness	55 mm			72 mm			
Trolley speed, m/s	0.025		0.3	0.025	0.1	0.15	0.3
Indenter's diameter	30 mm	40 mm	30 mm	40 mm	96 mm	96 mm	30 mm
Mean	33.21	42.33	58.8	75.7	267	303.4	145.9
Std. deviation (unbiased)	12.04	14.59	24.6	33.1	91.3	113.2	83.2
Number of data points in $\{y\}$	400	400	70	400	130	130	70

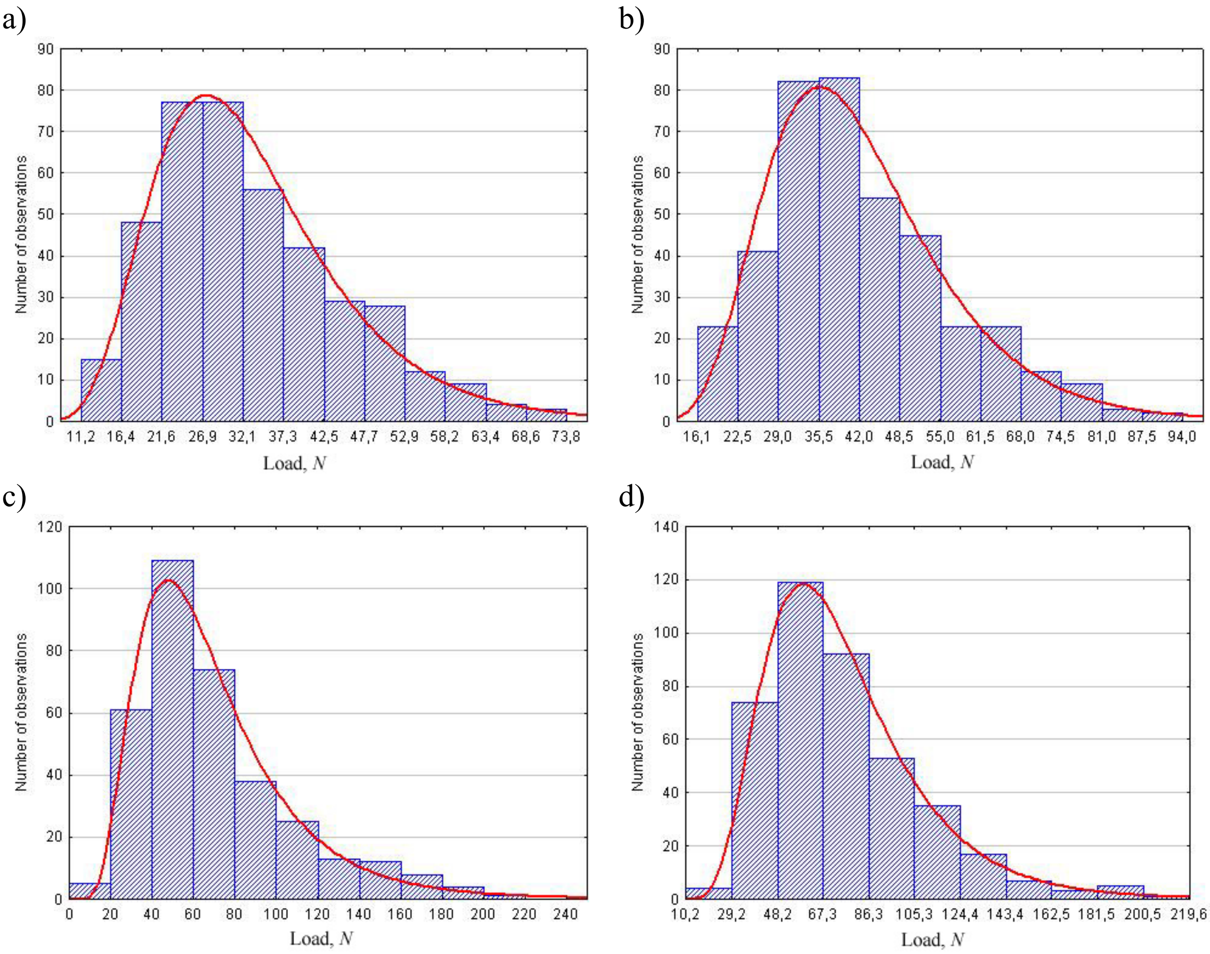

Figure 3. Histograms of uncorrelated data representing time series of global ice loads on cylindrical indenters with diameter: a) 30 mm against ice with thickness of 55 mm; b) 40 mm against ice with thickness of 55 mm; c) 30 mm against ice with thickness of 72 mm; d) 40 mm against ice with thickness of 72 mm. Indentation speed is 0.025 m/s.

Concluding this section we can say, that lognormal distribution of the global load signal has appeared not only in experiments with indenters of 20 mm in diameter, as was found earlier (Dobrodeev et. al., 2016) but also in experiments with indenters of 30 and 40 mm in diameter at moderate speed.

NORMAL DISTRIBUTION OF LOAD AT HIGH INDENTATION SPEED

At high indentation speed the load signal behaves in the other way, than at moderate speed. Case of moderate speed was considered in the previous Section, for 30-40 mm wide indenters lognormal distribution was confirmed.

In opposite, at indentation speed of 0.3 m/s for 30 mm wide indenter normal distribution was discovered for both of ice field thicknesses: 55 and 72 mm. With that speed only 3 seconds of load record were available, thus $n = 300$ data points with $= 100\ 1/s$, but it was quite enough. The plot of autocorrelation function estimation (5) is presented in Figure 2b. From Figure 2b it is clear, that we can take every 4^{th} data point from load time series X to compose the sample $\{x\}$ with low correlation of data points in it. Because of small n the following meaning was taken for N: $N = 10$.

This way with $l = 4$ and $N = 10$, we have for $\{z\}$ the size $k = 7$, which is rather small for conducting Chi squared test. Nevertheless, values of $\chi^2_{experim}$ were found for 3 intervals, and they are provided in the Table 2 along with other parameters for $\{z\}$. Here parameters of population ξ are defined as $(0; 1)$, so the number of dimensions of freedom of Chi squared test statistic under the null hypothesis is 2. From parameters provided in Table 2 it is clear that hypothesis on normal distribution of the process $\boldsymbol{X}(t)$ can be accepted.

All of 7 samples $\{x\}$ for each time series X were united to make the sample $\{\{x\}\}$. The histograms of $\{\{x\}\}$ along with fitted Normal PDF curves are presented in Figure 4a and 4b. Characteristics of $\{\{x\}\}$, which can be considered as estimations of process $\boldsymbol{X}(t)$ parameters are given in the Table 3.

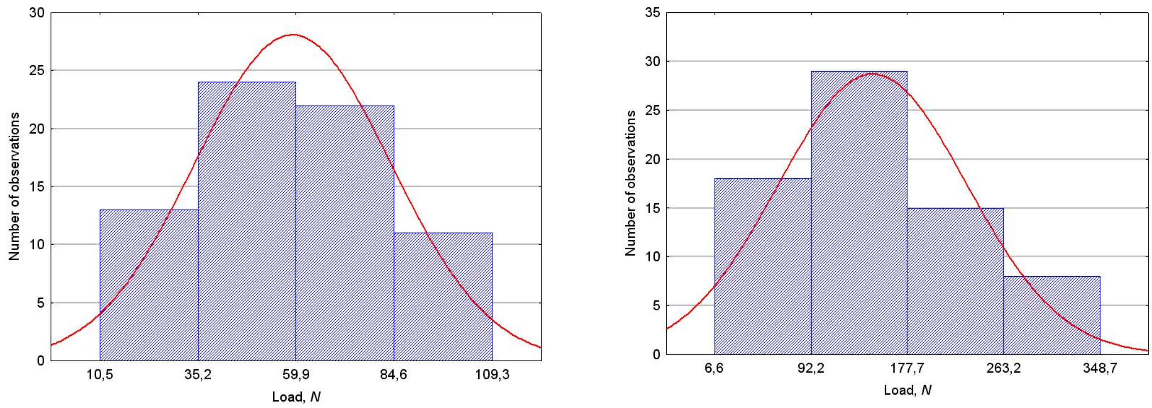

Figure 4. Histograms of uncorrelated data representing time series of global ice loads on cylindrical indenter with diameter of 30 mm at speed 0.3 m/s against ice with thickness a) 55 mm; b) 72 mm.

NORMAL DISTRIBUTION OF LOAD ON INDENTER WITH 96 mm DIAMETER

Another case when normal distribution of the signal was revealed was experiment with relatively wide indenter – cylinder of 96 mm in diameter. Normal distribution appeared in tests with it against 72 mm thick ice at speeds of 0.1 and 0.15 m/s. Records with duration of 16.5 and 9.5 seconds were obtained in these two experiments. Autocorrelation function estimations (4) for these cases are provided in Figure 5. From this Figure it follows that lags l =12 and l =7 respectively seem to be appropriate. For N= 10 this provides $k = 13$ for both of cases.

Parameters of sample $\{z\}$ for both of cases are provided in the Table 2 along with values of $\chi^2_{experim}$ found for 3 intervals. Here the number of dimensions of freedom of Chi squared test statistic under the null hypothesis is 2. From parameters provided in Table 2 it is clear that hypothesis on normal distribution of the process $\boldsymbol{X}(t)$ can be accepted in both of cases.

Histograms of sample $\{\{x\}\}$ are provided in the Figure 6a and 6b. Parameters of $\{\{x\}\}$ are presented in the Table 3.

Results provided in this Section are in agreement with the normal distribution obtained earlier (Zvyagin and Sazonov, 2015) for process of ice load on cylinder with diameter of 100 mm against 30 mm thick ice measured at speed of 0.084 m/s.

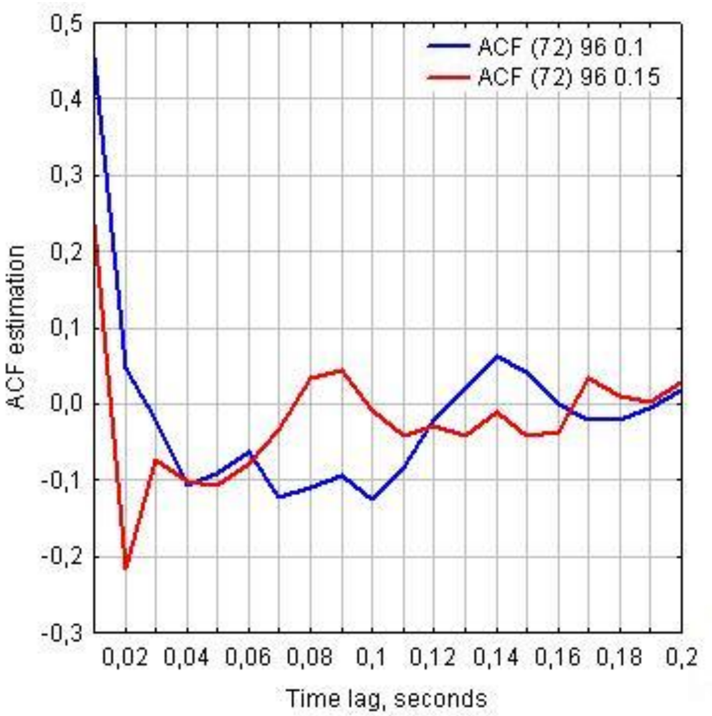

Figure 5. Estimation (5) of autocorrelation of global ice load on cylindrical indenter with diameter of 96 mm against model ice with thickness of 72 mm at speeds 0.1 m/s (blue curve) and 0.15 m/s (red curve).

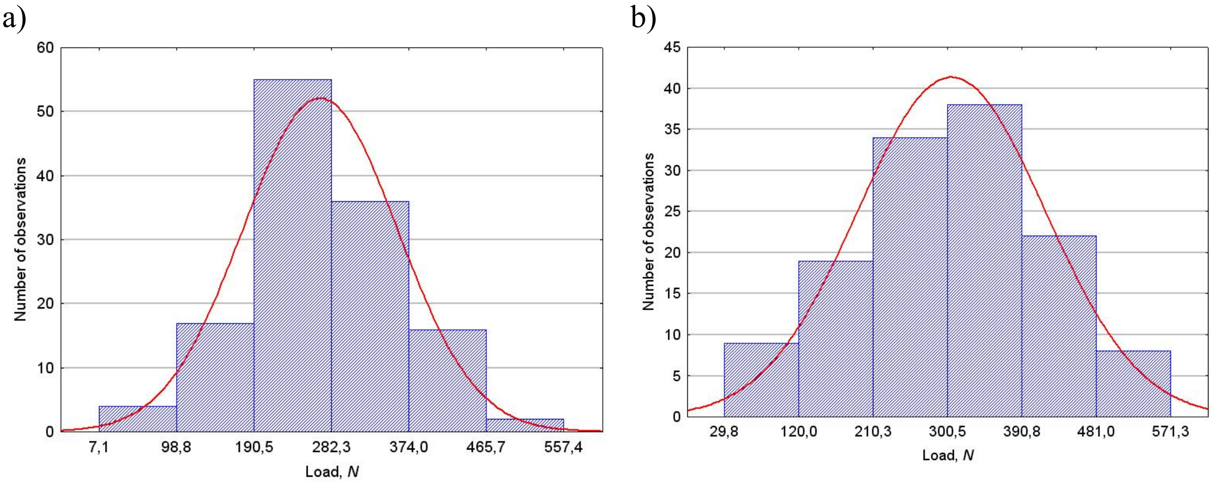

Figure 6. Histograms of uncorrelated data representing time series of ice loads on 96 mm wide indenter against 72 mm model ice at speeds a) 0.1 m/s and b) 0.15 m/s.

DISCUSSION

Dependency of ice load distribution law on indenter's diameter in the case of cylindrical indenters was expected due to previous studies conducted by the authors. But transition of distribution law from lognormal to normal with increasing speed was not revealed before. This phenomenon needs further study. Available length of signals registered at 0.3 m/s was rather short, but due to low autocorrelation of data points in those signals the amount of data was enough to make confident outcomes on the distribution law.

Stationarity, normality (or lognormality) of the distribution law, knowing of autocorrelation function, mean and variance, – these are sufficient conditions for performing successful simulations. As far as authors are aware, there is no software exist by now for simulation of ice loads time series with using of stochastic processes background. For this study authors have used self-written software for data processing and automation of testing procedure. Results in Table 2 were obtained with help of that software.

CONCLUSIONS

Findings of the conducted study are the following:

- The lognormal distribution law was revealed for some of global load signals measured in experiments with cylindrical indenters of 30 and 40 mm diameter at moderate indentation speed of 0.025 m/s and for both studied types of ice thickness – 55 mm and 72 mm. Earlier this distribution law was found for 20 mm wide indenters against ice 15-19 mm at 0.01 m/s.
- Autocorrelation functions of stationary load on cylinders with diameters of 30 and 40 mm at speed 0.025 m/s appeared to be similar for both studied types of ice thickness.
- Distribution law of the load signal is speed dependent. At high speed of 0.3 m/s normal distribution law has appeared.
- Distribution law of the load signal probably also depends on the indenter's diameter. For indenter of 96 mm in diameter the normal distribution law was observed both at indentation speed of 0.1 m/s and of 0.15 m/s. Earlier this distribution law was found for 100 mm wide cylinder against ice 30 mm at indentation speed of 0.084 m/s. However, more detailed investigation is needed on this issue.

The study opens several issues for future investigation. It is important to determine more precisely the indentation speed range and indenter's diameter range at which lognormal or normal distribution is most likely observed. It is important to reveal, how ice thickness influence the distribution law and the stationarity of the load signal. Also further study on the autocorrelation and/or spectral characteristics of the global load signal is desired.

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