STATISTICAL METHODS FOR APPLYING ICING ESTIMATES IN OFFSHORE DESIGN

E. S. Hansen [1], K. J. Eik [2] and S. H. Teigen[1]
[1]Statoil ASA, Bergen, Norway
[2] Statoil ASA, Oslo, Norway

ABSTRACT

While methods for establishing statistical extreme values of wind and wave loads for use in offshore design are well established, this is not the case for estimates of marine icing. Using estimates of marine icing from a numerical model, both existing and new statistical methods are applied for calculating extreme spatial ice distribution that may be applied in the design process. Statistics for global loads and directional dependency of icing are also described, as well as methods for assessing the expected duration of icing events. The methods are applied to a relevant offshore structure, and the process of developing a generalized extreme icing profile is described. Comparisons are made with the existing recommendations given in the offshore standards.

INTRODUCTION

There are several available methods for estimating the rate of marine ice accretion, given a set of meteorological and oceanographic parameters – both simple statistical algorithms (Overland, 1990) or more sophisticated numerical models (Kulyakhtin & Tsarau, 2014). If freezing rates and melting rates can be estimated from a given a set of metocean parameters, and time series for these metocean parameters are available, it is trivial to construct time series of ice thickness. However, it is not trivial how these time series may be used to estimate statistically extreme icing events, in order to be applied to offshore design.

This paper will assume that time series for icing loads, both global and local, are available. For the case study presented, these time series will be calculated using a numerical model described by Hansen & Teigen (2015). The paper will further suggest and describe methodology that may be used to develop statistical extreme estimates for global and local loads that are suitable for use in offshore design and design checks.

THEORY

The established method of estimating extreme values of loads or metocean parameters is as follows:

1. **Define the event and its associated value *x* that you wish to find the extreme value for**.
   Examples:
   a. Height of individual waves – the event is the time between two up-crossings, the value is the difference between the highest and lowest water height during the event.

b. Significant wave height – the event is e.g. a 20 minute time period, the value is four times the standard deviation of the wave height during the event.
c. Extreme cold temperatures – the event could be an entire year, the value is the coldest temperature registered during that year (in this case, it must be further specified if e.g. daily average temperature or instantaneous temperature recordings are to be used).

2. **Choose a statistical distribution suitable for the event and the associated value.**
   - The cumulative distribution $F(x; \alpha, \beta, \ldots)$ is dependent on the associated value x as well as a number of function-specific parameters (α, β, …).
   - For example, individual wave height can be approximated by a Rayleigh distribution, significant wave height can be approximated by a Weibull distribution, and extreme events (e.g. temperature) within a given time period can often be described by a Gumbel distribution (Kotz, et al., 2006).
3. **Find the parameters ($\alpha, \beta, \ldots$) of the chosen distribution that best fit the measured or estimated data *X***
   - This can be done by e.g. Maximum Likelihood Estimate (Kotz, et al., 2006).
4. **Find the value x from the cumulative distribution for a probability level equal to the inverse of the expected number of events during a characteristic time period.**
   - The characteristic time period for manned offshore structures is typically 100 years or 10,000 years, equal to an annual exceedance probability of 10^{-2} and 10^{-4}, respectively.

For offshore design in cold areas, it is of interest to find extreme estimates of ice thickness or total ice mass rather than icing rate. This provides a challenge in defining what an icing event is. One option is to define an icing event as the time period between when icing first occurs until the ice has been completely removed or melted, and that the associated value is the maximum ice thickness during this period. However, this definition means that if the ice does not completely melt down between two periods of ice build-up, the two periods will be counted as a single event even if they are largely independent. Similarly, if one defines an icing event as simply a local maximum in the time series, one may have a brief period of melting in a longer period of ice build-up that causes the same ice to be counted as several maxima, and therefore several events.

This article will suggest defining one year as one icing event, and that the associated value is the maximum ice thickness or ice mass during the entire year. However, these years cannot be calendar years, since the ice thickness 1st of January will not be independent from the ice thickness 31st of December the previous year. So for the purpose of icing estimate, one icing event will be defined as the period between 1st of July one year to the 1st of July the next year, and the associated value will be defined as the maximum ice thickness or ice mass during this period. For the remainder of this article, this time period will be referred to as a "winter year".

Since this value will be a maximum value from a given time period, it is appropriate to describe the ice thicknesses by a **Gumbel distribution**, given by the following cumulative distribution function:

$$F(x; \mu, \beta) = \exp\left(-\exp\left[-\frac{(x-\mu)}{\beta}\right]\right). \qquad (1)$$

It is assumed that both the global mass of ice due to marine icing, as well as the local ice thickness on all parts of the vessel, is reasonably well described by a Gumbel distribution.

Once the parameters μ and β are estimated from the time series, one may take the inverse of the Gumbel distribution and insert the desired probability level in order to find the desired extreme values.

IMPLEMENTATION & CASE STUDY

For the purpose of illustrating how the statistical methodology can be implemented, a case study has been performed for a simple cylinder, illustrated in Figure 1, at a location in the Barents Sea. The cylinder is 20m in diameter, and goes 20m above mean sea level. The NuMIS numerical model for marine icing (Hansen & Teigen, 2015) has been applied to the cylinder in order to calculate time series for global loads and local thickness. The meteorological data and wave data are taken from the Nora10 hindcast model (Reistad, et al., 2010) for the period 1957-2014 at the location 73.25°N, 25.10°E. The location is shown in Figure 2. The metocean data from Nora10 are sampled at 3 hour intervals, which is also therefore the sampling rate of the NuMIS model when applied to Nora10 data.

The resulting time series of the global ice load (total mass of all ice on the cylinder) is shown in Figure 3, including the maximum load each winter. There are a total of 57 winters in the time series, so there are 57 data points available to fit the distribution.

In order to estimate the 100-year load and 10,000-year load for accumulated ice mass on the cylinder, the 57 maximum loads are used to find the parameters μ_{glob} and β_{glob} of the Gumbel distribution, using the method of Maximum Likelihood. The resulting distribution, $F(x; \mu_{glob}, \beta_{glob})$ is plotted in Figure 4, along with the individual maximum loads and a 90% confidence interval for the distribution. The confidence interval is found by a bootstrapping method: The estimated distribution $F(x; \mu_{glob}, \beta_{glob})$ is used to draw a large number (10,000) of random sets of winter maxima, each set consisting of 57 points, and new Gumbel distributions for each set of data are estimated. The confidence interval is defined as the interval where 90% of all the new estimated Gumbel distributions will be within. This method of calculating a confidence interval for the distribution should be accurate so long as one assumes that the variation of parameter estimates when drawing from the estimated distribution is approximately the same as the variation of parameter estimates when drawing from the real distribution.

In order to calculate the 100-year and 10,000-year estimates of global load (more specifically, the loads with an annual exceedance probability (APE) of 10^{-2} and 10^{-4} respectively), one takes the inverse of the function described in equation (1),

$$x = \mu_{glob} - \beta_{glob}(\ln[-\ln F]), \tag{2}$$

and replaces F by the desired probability level of non-exceedance, i.e. $(1\text{-}10^{-2})$ or $(1\text{-}10^{-4})$ for 100-year and 10,000-year estimates respectively:

$$\begin{gathered} x_{100} = \mu_{glob} - \beta_{glob}(\ln[-\ln(1-10^{-2})]), \\ x_{10,000} = \mu_{glob} - \beta_{glob}(\ln[-\ln(1-10^{-4})]). \end{gathered} \tag{3}$$

A similar method is used to find the extreme estimates of the local thickness. The time series of ice thickness is found for each panel on the structure, and the winter maxima are found. Individually for each panel, the winter maxima are fitted to a Gumbel distribution and the

100-year and 10,000 year thicknesses are found from the method described by equation (2) and (3) . Figure 5 shows the spatial distribution of estimated 100-year thicknesses on the structure. It should be noted that, since the extreme loads are calculated independently, they are not expected to all happen at the same time. This means the spatial distribution shown in Figure 5 will be slightly conservative to use directly in design.

GENERALIZED ICING PROFILE

For the purpose of offshore design, using a vessel specific extreme spatial icing distribution is not necessarily the best option. Firstly, the vessel geometry may not be available. Secondly, the vessel-specific spatial distributions will often imply an inappropriately high level of certainty in the icing estimates. Icing estimates are inherently uncertain, and should not be used for high-detail engineering. It is therefore of interest to develop generalized icing estimates that are largely independent the vessel geometry and mainly depend on the weather conditions at the location and height above the waterline. These icing estimates should be easy and intuitive to apply in preliminary design.

This article will explain the calculation of a generalized icing profile, where the extreme icing on any part or object on a structure may be approximated simply by knowing the height above mean sea level and which side of the structure the part or object in question is positioned.

This is done by estimating the ice thickness on a cylinder, with a diameter that should be representative of the characteristic length of the vessel in question. The reason for using a cylinder is that icing from all compass directions is estimated under symmetrical conditions, and that the normal vector of all panels will be directed along the distance from the cylinder centre to the panel. This means that e.g. the ice thickness on the northernmost part of the cylinder corresponds to an estimate of icing for parts of a vessel that are exposed towards the north.

One may then examine the panels at each height level of the geometrical model, and find the average and maximum extreme ice thickness for each height. More specifically, the code will go through a set of predetermined height intervals (e.g. from 0 to 20 meter above surface level with 0.5 m resolution) and find all panels with a centroid placed in the interval. The area-weighted average extreme ice thickness, and the maximal extreme ice thickness, of these panels are used as “directional average” value and “worst direction” value for a z value placed at the middle of the interval. By doing this for a number of z intervals, a vertical profile can be constructed. The resulting generalized icing profiles are plotted in Figure 6, along with the icing profile recommended from NORSOK N-003 north of 68°N.

DISCUSSION

As previously mentioned, the time series of ice thickness on each panel is fitted to a statistical distribution independent from the time series of other panels. This means that the spatial distribution shown in Figure 5 does not represent the spatial distribution at one single time. Similarly, the vertical profile in Figure 6 shows ice thicknesses for different heights that generally will not occur at the same time. For instance, large installations will not have icing near the waterline (<5m) in storms due to wave washing, while smaller vessels and lifeboats may experience the most severe icing in events with cold winds but small waves. In other words, the spatial ice distribution shown in Figure 5 and Figure 6 should be reasonable for the local extreme loads on all parts of the structure, but is not representative for the global extreme load.

This is also the reason why the effect of wave washing, which is implemented in the model, is not apparent in Figure 5 and Figure 6.

As shown in Figure 4, the 100 year accumulated total ice mass for the case study structure is estimated at 152 tonnes. However, the 10,000 year total ice mass (not shown in the figure) is estimated to be 255 tonnes, or 68 % higher than the 100 year mass. This represents a higher difference between 100 year and 10,000 year loads than what is common in offshore design. In offshore design, it is common practice to use **either** the 100 year load multiplied with a safety factor of 1.3, **or** the 10,000 year load with no safety factor, whichever is highest. It appears in the statistical extreme icing estimates that the 10,000 year load is typically more than 30 % higher than the 100 year load. This may indicate that calculating the 10,000 year icing load is more important to offshore design than the 100 year load. However, one should keep in mind that going from a 100 to a 10,000 year load is a purely statistical exercise. In reality, there will be other physical limitations of icing growth (such as ice falling off if it becomes too heavy) that will not be reflected in the statistical extremes.

It should also be mentioned that the estimates only takes into consideration marine icing, not atmospheric icing (snow and freezing rain). It is possible to develop extreme estimates for atmospheric icing as well, but using the statistical extremes in combination (i.e. assuming the total 100 year design load is the 100 year marine icing load **plus** the 100 year snow load) is overly conservative, since the two will generally not occur at the same time. A more appropriate method would be to add estimated snow loads into the marine icing model, and estimate time series of the combined load. This time series can then be used to calculate the extreme combined load.

On using a Gumbel distribution

As mentioned in the Theory section, the Gumbel distribution is assumed to be a reasonable good fit for both global and local accumulated ice load. The main reason for using a Gumbel distribution is due to the difficulty of specifying statistically independent icing events otherwise, also mentioned in the Theory section. If one were to use i.e. a Weibull distribution, one would have to define what constitutes an icing event and which associated ice load to use. If an icing event was defined as the time between the start of icing and the time when all ice was melted away, then it is possible that the time period contained several mostly independent times of ice build-up, but where the ice did not have the time to melt completely in between. Similarly, if each local maximum of ice thickness was defined as an event, the same ice may be counted several times and the events would not be independent.

By using an entire winter as one icing event, this problem is circumvented. The icing during each winter will be independent from neighbouring winters (disregarding long-term climate trends), since the ice will always melt completely during the summer. As long as each winter contains a multitude of potential icing events (a reasonable assumption in the Barents Sea), then the statistical problem is reduced to finding the maximum value from a number of samples. This suggests a generalized extreme value distribution, of which the Gumbel is the most simple. This argument is valid both for global and local ice load.

Figure 4 seems to indicate that the global ice load fits the Gumbel distribution reasonably well.

On the generalized icing profile

The idea of a generalized icing profile is to have a thickness profile that can be used in preliminary design. If one is to design a production vessel/structure for a specific field, it can often be assumed that time series of weather and wave data are known for the field location (through e.g. hindcast data). The generalized profile should therefore be a vessel-independent estimate, which can be used to obtain estimates for exposed surfaces.

The reason for using a cylinder is two-fold. First of all, the icing estimates will be independent from the orientation of the structure. Second, the orientation of each panel on the structure is given trivially by its location on the structure, which gives an intuitive view of which compass direction the bulk of the icing will come from. For instance, if the extreme thicknesses are highest at the northeast part of the cylinder, this means that for any structure the parts that are exposed towards the northeast will face the most icing.

Finally, the generalized icing profile allows for simple and structure-independent comparison of the severity of icing conditions for separate locations.

The generalized icing profile uses a characteristic length of the structure. For platform legs, this can be assumed to be equal to leg width or diameter. For cylindrical platforms, the diameter of the platform should be used. For ship-shaped structures, the ship width should be used. Keep in mind that for large structures, the icing is not very sensitive to characteristic length, so the characteristic length does not need to be determined with high precision.

Also note that the generalized icing profile applies to exposed surfaces – for surfaces that are only partially exposed, the icing will be less severe. If icing for partially exposed surfaces needs to be determined accurately, it is recommended to apply the numerical icing model on the specific vessel geometry rather than use the generalized profile.

SUMMARY

Methodology for using icing estimates to develop statistical extremes used in preliminary offshore design has been developed and described. The method assumes the existence of a long-lasting time series of ice thickness estimates, which in the case of this article comes from a previously developed numerical model for icing. Methods for fitting ice thickness estimates to statistical distributions and extrapolate these to common extreme levels used in design are described, as is a suggested method of calculating a generalized structure-independent vertical extreme ice thickness profile.

FIGURES

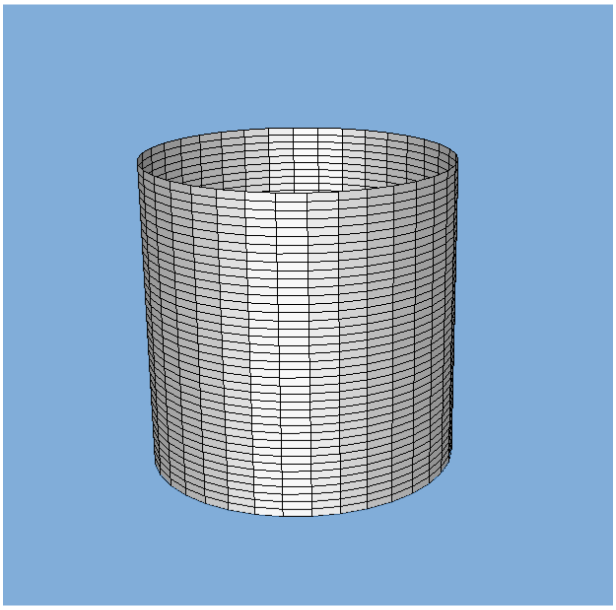

Figure 1. Illustration of the 20m diameter and 20m high cylinder used in the icing calculations, as well as the numerical grid. The geometry totals 1440 panels.

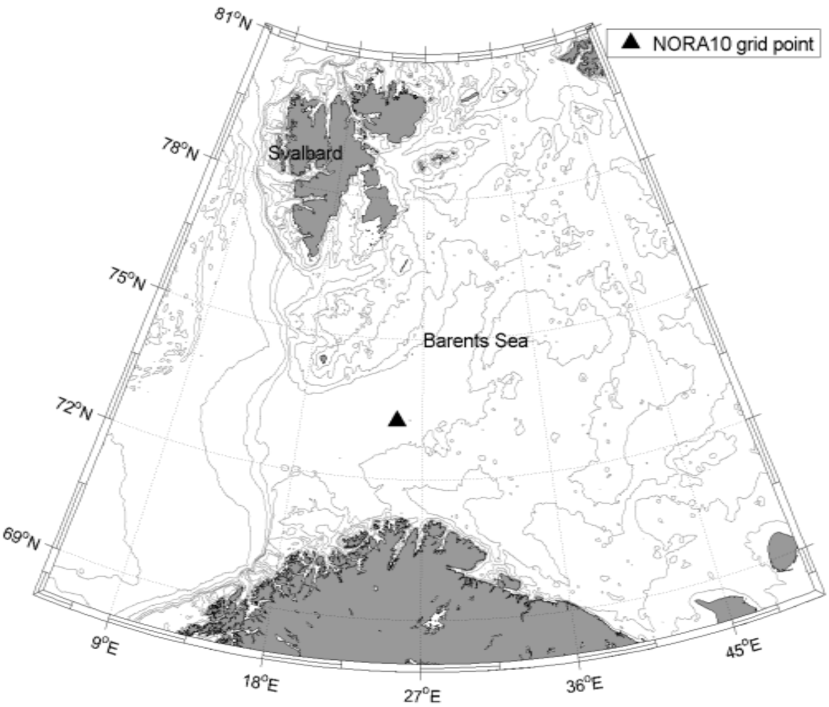

Figure 2. Map showing the location of the NORA10 hindcast grid point used in the case study.

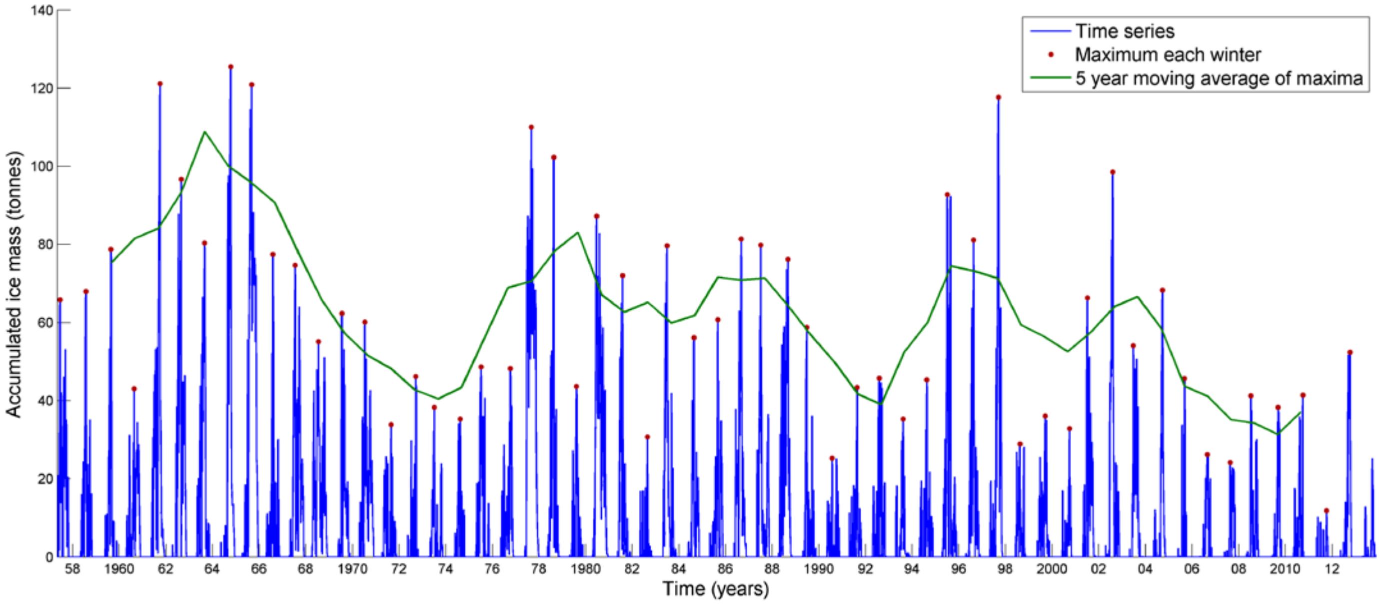

Figure 3. Time series of the total accumulated mass of ice on the structure shown in Figure 1, positioned at 73.25°N, 25.10°E. The maximum value during each winter is also plotted, along with the moving average of the maxima. The mass is calculated using the NuMIS numerical model for icing *(Hansen & Teigen, 2015).*

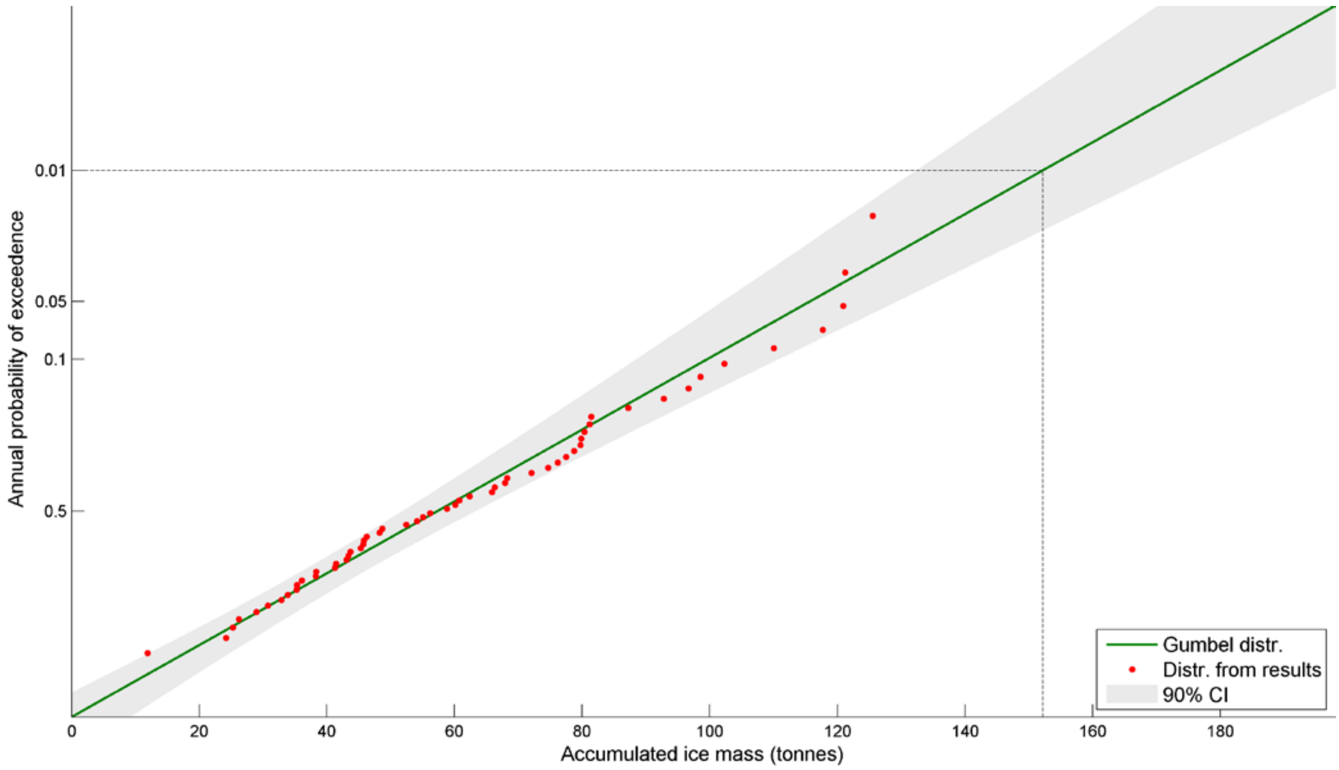

Figure 4. Probability paper of the maxima shown in Figure 2 and its estimated Gumbel distribution. The Y axis is chosen so that the Gumbel distribution becomes a straight line. A 90% confidence interval of the estimated distribution is shown in grey. The dotted line indicates the 100 year level of the accumulated ice mass.

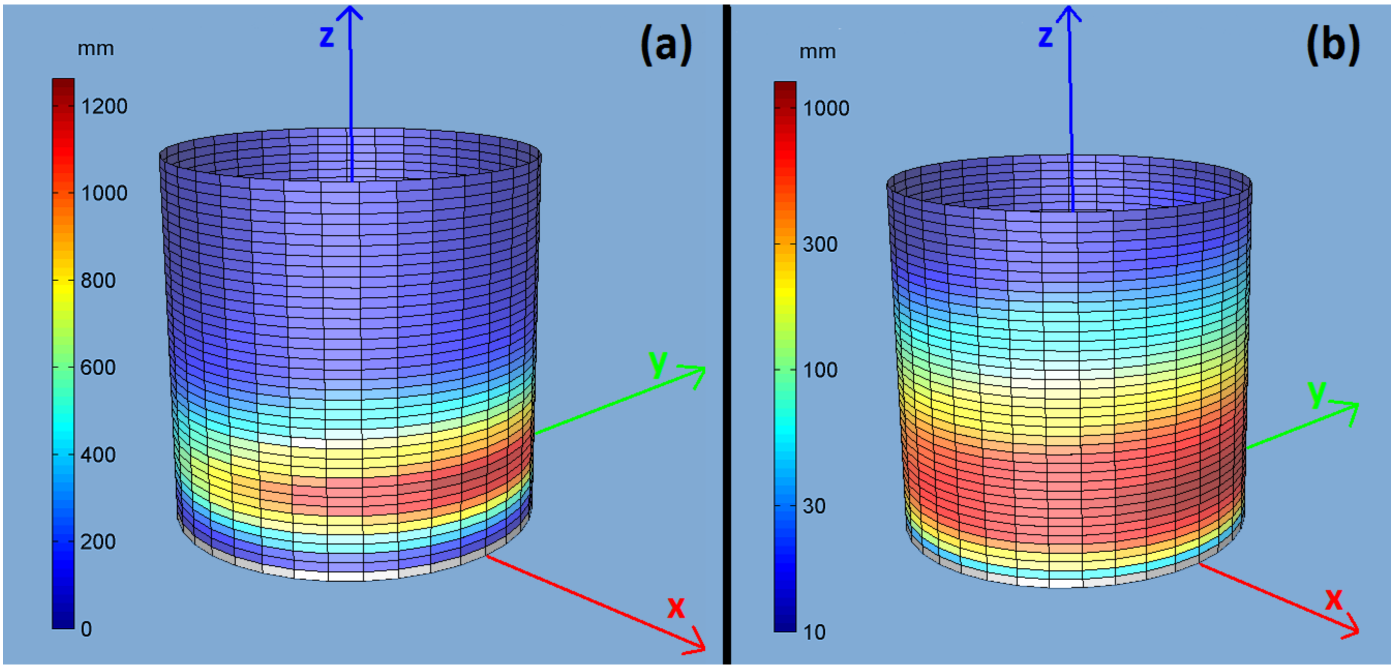

Figure 5. Spatial distribution of statistical extreme thicknesses on the cylindrical structure shown in Figure 1, using (a) linear and (b) logarithmic colour scaling. The x axis is oriented northwards. The statistical extremes are calculated independently on each panel; thus, they may not occur at the same time.

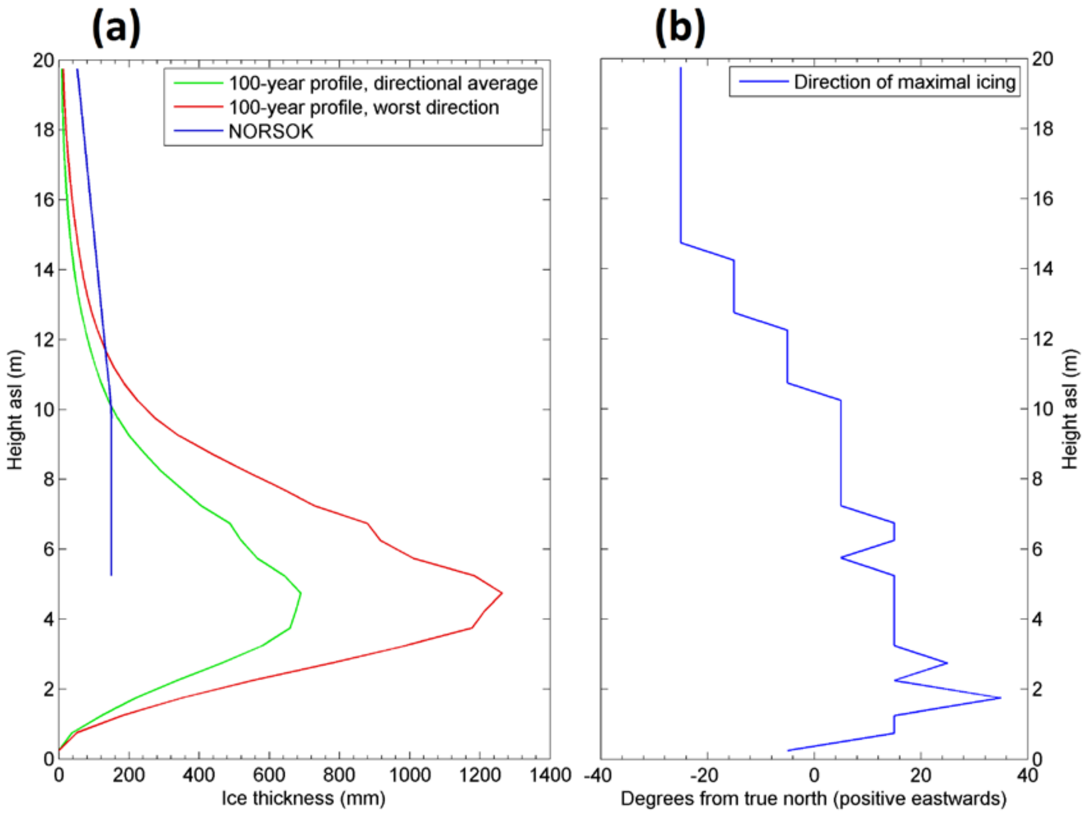

Figure 6. (a) Generalized icing profile for the location 73.25°N, 25.10°E, showing the 100-year profile for the directional average as well as the worst direction. The profiles are compared to the extreme ice thickness profile from the existing NORSOK N-003 standard. (b) The direction from which the maximal estimated icing conditions come from, for different height intervals.

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