POAC'15
Trondheim, Norway

Proceedings of the 23rd International Conference on
Port and Ocean Engineering under Arctic Conditions
June 14-18, 2015
Trondheim, Norway

PROBABILISTIC PROPERTIES OF FORMULA FOR GLOBAL ICE PRESSURE IN THE CASE OF LOGNORMAL DISTRIBUTION OF ICE REGIME PARAMETERS

Petr N. Zvyagin[1,2]
1. St. Petersburg State Polytechnical University, St. Petersburg, RUSSIA
2. Krylov State Research Centre, St. Petersburg, RUSSIA

ABSTRACT

In the paper the probabilistic properties of formula for global ice pressure, which is provided by the code ISO 19906, are considered. Ice regime parameters are taken as random variables with lognormal distribution. The relation between moments of ice regime parameters and ice pressure quantiles are derived theoretically for introduced probabilistic model. The example of quantiles calculation is provided. The influence of ice regime parameters mutual correlation on the calculated ice pressure value is investigated. The influence of powers in the formula on the mean of resulting pressure is discussed and the theorem about mean minimum is formulated and proved.

INTRODUCTION

Probabilistic modelling of ice pressure and ice loads can imply different approaches, depending on generalization of the model of ice-structure interaction. Common approach is to model random ice features, calculate loads caused by them on the structure according to the model of mechanical interaction, and then estimate the pressure. The more generalized approach leaves particular ice features behind the consideration, and has deal only with dynamics of ice loads time series (Zvyagin and Sazonov, 2013). From these positions, consideration of design formulas for maximal ice pressure and ice load seems to be even more generalized approach than mentioned above.

At the same time, design formulas recommended by national standards as well as by a number of investigators summarize the experience in this area. But probabilistic properties of these formulas when their components are random variables are not investigated at the moment. Certainly these probabilistic properties greatly depend on the type of components distribution. Also probabilistic properties depend on the construction of the formula, signs of the powers, etc.

All of these properties could be roughly investigated statistically by conducting statistical modelling with usage of appropriate software. But in some cases these properties could be derived analytically. Further such investigation is performed for effective global ice pressure formula recommended by standard ISO 19906 and lognormal distribution of ice regime parameters. This type of distribution is rather widely met in papers dedicated to analysis of ice conditions.

LOGNORMAL DISTRIBUTION OF ICE REGIME PARAMETERS

The lognormal distribution is not the only which is used to describe ice regime parameters distribution (Sinitsyna et al. 2013), a number of other continuous distributions like Gamma, Beta, Weibull and others are used for this purpose. Despite of this, in some of cases (Johnston et al. 2009; Sinitsyna et al. 2013; Masaki et al. 1996) lognormal distribution provides the best fit to the empirical histogram. At the same time, this type of distribution has a number of profitable properties.

In their overview of Arctic multi-year ice thickness, Johnston et al. (2009) proposed the lognormal distribution for the data. They claim that histogram of the data of multi-year ice thickness they collected from different sources is well enough described by lognormal distribution. Their data was aggregated from 4987 measurements of different investigators. However, the special statistical investigation of ice thickness distribution law is rather rare, usually only histograms without any investigation are presented. From histograms of ice thickness presented in the paper (Bourke and Garret, 1987) and in Workshop on ice thickness proceedings (Wadhams and Amanatidis (Editors), 2006) we can make conclusion that lognormal distribution seems to be plausible for this ice regime parameter.

For ice compression strength there are more statistical analysis can be found. In the paper of Masaki et al. (1996) the results of measurements made in Saroma Lagoon by Takeuchi et al. in March 1995 are discussed. In that paper it is stated that distribution of ice strength can be taken as lognormal. Truskov et. al. (1992) had fitted lognormal distribution to observations of ice compressive strength measured on the Northeastern part of Sakhalin shelf (Figure 1). The investigation of how different distributions fit compressive ice strength experimental histograms was conducted by Synitsina et al. (2013); among the others the lognormal distribution was investigated and for some of the compressive strength data this distribution had provided good fit.

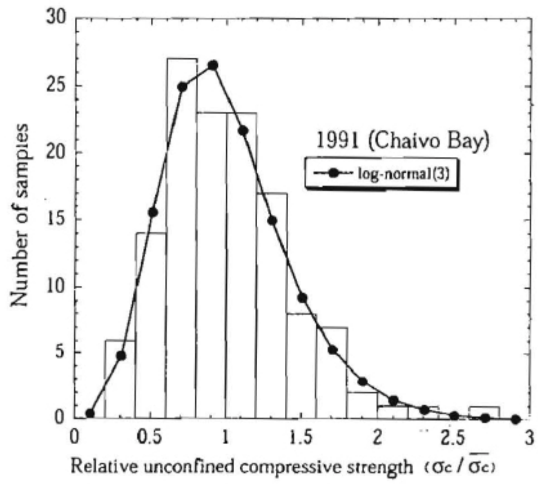

Figure 1. Frequency distribution of the unconfined compressive strength of ice in the Northeastern part of Sakhalin (Masaki et al., 1996).

In fact, lognormal distribution corresponds to the product of a number of independent random variables and it can provide good fit for distribution of other ice regime parameters, such as ice drift velocity. In (Nesterov et al, 2009) the histogram of ice drift velocity measured in the North-Eastern Barents Sea is presented, which corresponds lognormal distribution.

Adjusting the lognormal distribution curve to data we should agree that data points are independent observations of the same random variable. Such condition is not always met practically, and the distribution fitting is senseless in the case when this is not met.

MODEL OF GLOBAL ICE PRESSURE BASED ON DESIGN FORMULA

Global ice pressure, as well as global ice force, should somehow be dependent on such ice regime parameters as ice thickness and ice strength. National standards of different countries offer formulas for calculating of maximum possible ice pressure on the structure. Standards SNiP 1982, API RP-2N 1995 (Loset et al. 2010), ISO 19906 code (2010, equation A.8.21) imply the next formula to estimate effective ice pressure P:

$$P = \lambda h^{\alpha} R^{\beta} d^{\gamma}; \qquad (1)$$

Here h is ice thickness, d is contact area width, and R is compressive ice strength; λ, α, β, γ are some constants. In the earlier formula given by Korzhavin (1962) parameters α, β, γ were set by the value 1. In a number of formulas for ice pressure given by later authors (Loset et al. 1999, 2010) as well as in the ISO Standard's formula those parameters take on different values. Particularly in the ISO Standard (2010) it is recommended to estimate maximum effective global ice pressure according to the formula

$$P \approx d^{-0.16} h^{n+0.16} R \qquad (2)$$

where n is an empirical coefficient, taken as $n = -0.5 + h/5$ if $h < 1$ m and as $n = -0.3$ if $h \geq 1$ m (Palmer and Croasdale, 2013).

If we shall consider ice regime parameters h and R in the right part of (1) and (2) as random variables, then pressure P will be also random variable. Then, having the distribution laws for h and R we can estimate the histogram for P using probabilistic modeling or, in some cases, find theoretical distribution law for P.

Design formulas (1)-(2) are constructed to describe the general relation between maximum pressure on a structure caused by a plain ice sheet and the characteristics of this ice sheet. Parameters $\lambda, \alpha, \beta, \gamma$ in (1) are estimated by the authors according to the available field data, and at usual they provide best fit to the specific data set. The problems regarding the reliability of these formulas and their relevance to the physical reality are discussed by a number of authors, starting from Korzhavin (1962) through the present (Loset et al., 1999, 2010).

PROBABILISTIC MODEL OF PRESSURE WHEN ICE REGIME PARAMETERS ARE INDEPENDENT

Basing on the ISO formula (2) for effective pressure and on the assumption of parameters h and R lognormality we shall propose the probabilistic model for the ice pressure. Correlation of h and R is another important factor for the probabilistic model. However, there is a lack of publications about experiments where the statistics for ice thickness and ice strength were measured at the same time. Therefore, it is difficult to find univocal opinions about how these parameters can be statistically related.

In this section we shall consider random ice regime parameters h and R as independent. To avoid randomness in the power of random variable h using formula (2), we shall take the power value α as constant.

It is well-known, that the product ζ_1 of two random variables h and R with lognormal distributions raised to the constant powers α and β, and multiplied by constant c,

$$\zeta_1 = c h^{\alpha} R^{\beta}, \qquad (3)$$

has a lognormal distribution irrespective of statistical dependence or independence of h and R. If ζ_1 has lognormal distribution, h and R are independent, then $\zeta_1 = c e^{\zeta}$ where ζ is distributed normally with next mean a and standard deviation σ:

$$a = \alpha \ln Mh + \beta \ln MR - \frac{\alpha}{2} \ln\left(\left(\frac{\sigma(h)}{Mh}\right)^2 + 1\right) - \frac{\beta}{2} \ln\left(\left(\frac{\sigma(R)}{MR}\right)^2 + 1\right), \quad (4)$$

$$\sigma = \sqrt{\alpha^2 \ln\left(\left(\frac{\sigma(h)}{Mh}\right)^2 + 1\right) + \beta^2 \ln\left(\left(\frac{\sigma(R)}{MR}\right)^2 + 1\right)}, \quad (5)$$

where Mh and MR are theoretical means of random h and R respectively; $\sigma(h)$ and $\sigma(R)$ are standard deviations of these random variables. Then probability of the event $l_1 \le \zeta_1 \le l_2$ where l_1, $l_2 > 0$, can be calculated by the formula:

$$Prob(l_1 \le \zeta_1 \le l_2) = \Phi\left(\frac{\ln l_2 - a - \ln c}{\sigma}\right) - \Phi\left(\frac{\ln l_1 - a - \ln c}{\sigma}\right). \quad (6)$$

If we shall denote p-quantile of ζ_1 with x_p, then

$$x_p = \exp\left(\sigma \Phi^{-1}(p - 0.5) + a + \ln c\right). \quad (7)$$

Here $\Phi(x)$ is Laplace function $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_0^x e^{-\frac{t^2}{2}} dt$, and $\Phi^{-1}(x)$ is inverse Laplace function.

The lognormality of the global pressure is a subject of discussion. However, the lognormal stochastic model for local ice loads time series already have been studied by Zvyagin and Sazonov in 2010, and histograms of ice pressure presented by Fransson and Olofsson (2005) and measured in field conditions do not contradict the supposition of their lognormal distribution.

To provide an example of calculations we shall consider a structure of width $d = 10$ m, a formula (2) and independent parameters of h and R, both with lognormal distribution. For practical calculations all theoretical moments of considered random variables will be replaced by their point estimators. For the ice thickness, we take data of the fast ice thickness in the Sea of Okhotsk made at Chaivo Station (Shirasawa et al, 2005). The average value of the fast ice thickness on the 15th of March was $Mh = 0.76$ m, and the standard deviation was $\sigma_h = 0.118$ m.

In this example we take the coefficient $n = -0.5 + 0.76/5 \approx -0.35$ using mean value 0.76 of ice thickness. Thus, the power of h in formula (2) is $\alpha = -0.19$.

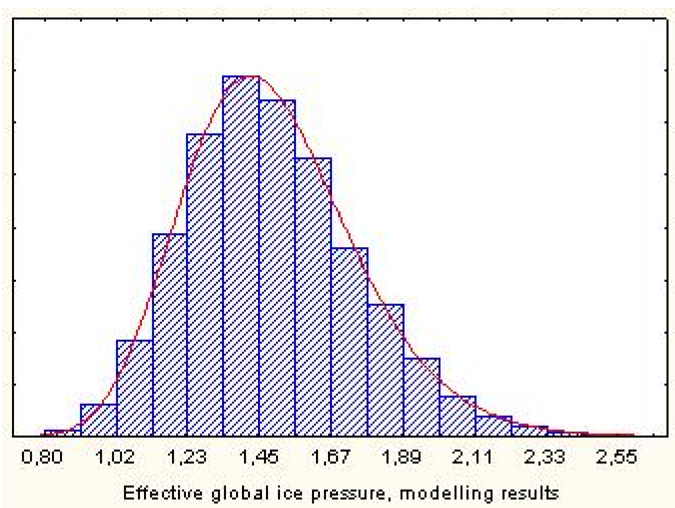

Figure 2. Histogram of lognormally distributed effective global ice pressure modelling results.

For the parameter of the ice strength R we shall take statistics for the one-year ice strength observations in Okhotsk Sea made by Truskov and presented also by Sinitsyna et al. (2013): $MR = 2.04$ MPa, $\sigma_R = 0.36$ MPa. As a distribution law for R, we take the lognormal law, which agrees with the data described in the mentioned paper and with Figure 1 in which the histogram of ice strength for nearby region is presented.

Thus, if we denote $P = d^{-0.16}\exp(\zeta)$ then we can calculate parameters of normally distributed variable ζ according to (4) and (5): $a_\zeta = 0.752$, $\sigma_\zeta = 0.175$, and for constant cofactor we have: $d^{-0.16} \approx 0.692$. Quantiles of global pressure, calculated according formula (7) using mentioned parameters are provided in the Table 1. These results can be compared with results of software probabilistic modelling of effective global ice pressure (Figure 2) according the formula (2) with the same parameters. In the Figure 2 the corresponding probability density curve is presented as well.

Table 1. Quantiles of random global effective ice pressure given by ISO formula (2).

p	p-quantile x_p, MPa	p	p-quantile x_p, MPa
0,005	0,935	0,95	1,96
0,01	0,978	0,975	2,07
0,025	1,044	0,99	2,2
0,05	1,1	0,995	2,306

PROBABILISTIC MODEL OF PRESSURE WHEN ICE REGIME PARAMETERS ARE CORRELATED

As it was said earlier, there is no univocal opinion, how ice thickness h and ice compressive strength R are related statistically. For probabilistic modelling it is profitable to take these parameters as independent. At the same time, Timco and Frederking (1990) summarised results of the plain ice sheet strength and made a conclusion that the ice strength for plain ice normally increases with increasing ice thickness. This strengthening occurs because ice salinity decreases with increasing ice thickness. Therefore we can suppose positive correlation between h and R.

Let denote in relation (3) $h = \exp(\xi)$, $R = \exp(\eta)$ and $h^\alpha R^\beta = \exp(\zeta)$, where ξ, η and ζ are normally distributed random variables with means a_ξ, a_η and a_ζ and standard deviations σ_ξ, σ_η and σ_ζ respectively, and correlation coefficient of variables h and R is ρ. Then

$$a_\zeta = \alpha a_\xi + \beta a_\eta,\quad \sigma_\zeta = \sqrt{(\alpha\sigma_\xi)^2 + 2r\alpha\beta\sigma_\xi\sigma_\eta + (\beta\sigma_\eta)^2}\,, \tag{8}$$

where r is correlation coefficient of variables ξ and η. The relation between r and ρ is given by the next formula (Johnson and Kotz, 1972):

$$r = \frac{\ln\left(1 + \rho\sqrt{\left(\exp(\sigma_\xi^2) - 1\right)\left(\exp(\sigma_\eta^2) - 1\right)}\right)}{\sigma_\xi\sigma_\eta}. \tag{9}$$

This relation allows us to simulate global effective ice pressure $P = d^{-0.16}\exp(\zeta)$ knowing enough information about h and R and correlation of them. In addition we can provide one profitable relation, which we can easily get applying infinitesimal functions properties. If σ_ξ^2 and σ_η^2 are very small (to say, each of them is less than 0.05), then we can use the next relation:

$$r \approx \frac{\ln\left(1+\rho\sigma_\xi\sigma_\eta\right)}{\sigma_\xi\sigma_\eta}.$$

If the product $\sigma_\xi\sigma_\eta$ is also very small, then

$$r \approx \rho.$$

Introducing positive correlation of ice regime parameters to formula (2) we should be ready to one contradictory result in probabilistic modelling. The combination of positive correlation between *h* and *R*, and negative power α of *h* will provide us more likely smaller values of global pressure, than in the case of independent *h* and *R*.

This contradiction can be theoretically justified for lognormally distributed components of formula (1). To avoid this contradiction it could be recommended to use powers of formula (1) components of the same sign, and to introduce auxiliary factors instead of rational powers for taking in account so-called "size-effect" of global pressure.

In general, mean value MP of effective global ice pressure given by (1) depends on first two theoretical moments of *h* and *R*, including mixed second moment (covariance) as well as on distributions of *h* and *R*. But at the same time, values of powers α and β are of great importance for determining MP. Whether these values are large or small by their absolute values, whether they are positive or negative – all of this influences the resulting MP. The next theorem allows to find minimum of MP in depend of values α and β in relation (1), knowing parameters of random *h* and *R*.

Theorem. Let h and R in (3) be correlated lognormal random variables, $h=\exp(\xi)$ with parameters (a_ξ,σ_ξ), $R=\exp(\eta)$ with parameters (a_η,σ_η), and correlation coefficient of ξ and η is r, $-1<r<1$. Then, the global minimum of the ice pressure mean MP is provided by the next values of powers:

$$\alpha_m = \frac{a_\eta\sigma_\xi\sigma_\eta r - \sigma_\eta^2 a_\xi}{\sigma_\eta^2\sigma_\xi^2\left(1-r^2\right)}; \quad \beta_m = \frac{a_\xi\sigma_\xi\sigma_\eta r - \sigma_\xi^2 a_\eta}{\sigma_\eta^2\sigma_\xi^2\left(1-r^2\right)}. \qquad (10)$$

And if h and R are independent lognormal variables, then $\alpha_m = -\frac{a_\xi}{\sigma_\xi^2}$, $\beta_m = -\frac{a_\eta}{\sigma_\eta^2}$.

Proof of Theorem. The natural logarithm of the ice pressure mathematical estimation, $M\xi_1 = \lambda\exp\left(a+\sigma^2/2\right)$ using equations (8) is

$$\ln Mp = \ln c + \alpha^2\frac{\sigma_\xi^2}{2} + \alpha a_\xi + \beta a_\eta + \beta^2\frac{\sigma_\eta^2}{2} + r\alpha\beta\sigma_\xi\sigma_\eta = f(\alpha,\beta)$$

The solution of the system

$$\begin{cases} \dfrac{\partial f}{\partial\alpha} = \alpha\sigma_\xi^2 + a_\xi + r\beta\sigma_\xi\sigma_\eta = 0 \\ \dfrac{\partial f}{\partial\beta} = \beta\sigma_\eta^2 + a_\eta + r\alpha\sigma_\xi\sigma_\eta = 0 \end{cases}$$

is:

$$\alpha_m = \frac{a_\eta\sigma_\xi\sigma_\eta r - \sigma_\eta^2 a_\xi}{\sigma_\eta^2\sigma_\xi^2\left(1-r^2\right)}; \quad \beta_m = \frac{a_\xi\sigma_\xi\sigma_\eta r - \sigma_\xi^2 a_\eta}{\sigma_\eta^2\sigma_\xi^2\left(1-r^2\right)}.$$

It is true that

$$\frac{\partial^2 f}{\partial\alpha^2}\frac{\partial^2 f}{\partial\beta^2} - \left(\frac{\partial^2 f}{\partial\alpha\partial\beta}\right)^2 = \sigma_\xi^2\sigma_\eta^2 - r^2\sigma_\xi^2\sigma_\eta^2 = \sigma_\xi^2\sigma_\eta^2\left(1-r^2\right) > 0 \text{ if } -1<r<1, \text{ and } \frac{\partial^2 f}{\partial\alpha^2} = \sigma_\xi^2 > 0$$

which means that point (α_m,β_m) is the global minimum of function $f(\alpha,\beta)$, when $-1<r<1$.

If parameter β is fixed, then

$$\alpha_m = \frac{-a_\xi - r\beta\sigma_\xi\sigma_\eta}{\sigma_\xi^2}.$$

If parameter α is fixed, then

$$\beta_m = \frac{-a_\eta - r\alpha\sigma_\xi\sigma_\eta}{\sigma_\eta^2}.$$

It is evident that the values of α and β that provide the global minimum of variance $\sigma^2(P)$ are $\alpha = 0$, $\beta = 0$.

Let us fix parameters α and β with values used in the example above, particularly $\alpha = -0{,}19$, $\beta = 1$. Let the distribution law of ice thickness and strength here remains lognormal with statistical characteristics considered in the previous chapter, as well as structure width remains the same. The dependence of the mathematical estimation MP of pressure (2) and its standard deviation $\sigma(P)$ on the parameter r is presented in Figure 3. Here r is related with the h and R correlation coefficient ρ according to the equation (9). From this figure we can see the feature mentioned above: MP and $\sigma(P)$ decrease when r increases (and ρ as well). This happens because of the negative power $\alpha = -0{,}19$ of factor h, and vice versa, the negative correlation between h and R increases the mean and variance of pressure P.

From this we can make outcome that formula (2) is not intended for using correlated parameters.

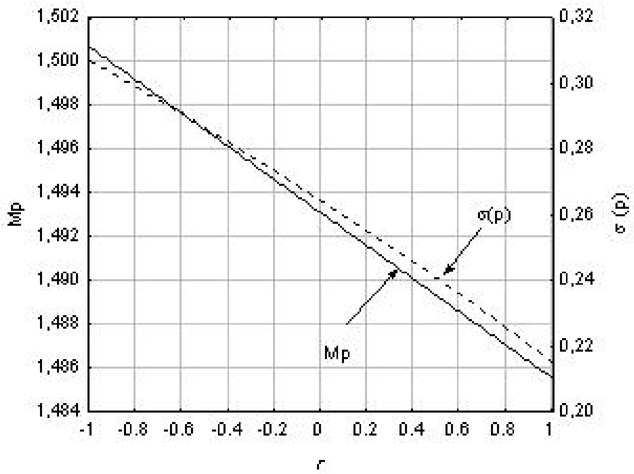

Figure 3. Dependence of MP (solid line, left axis) and $\sigma(P)$ (dashed line, right axis) on parameter r, when $\alpha = -0.19$ and $\beta = 1$.

We should note that some empiric formulas offered for pressure (Loset et al. 1999) contain factor h in a positive power as well as factor R, and for those models the positive correlation of lognormally distributed h and R will increase the mean and variance of pressure p.

OUTCOMES

Having the reliable and verified model which connects ice regime parameters and ice pressure is very important for easy probabilistic modeling of this pressure as well as reliable estimating of the specific bound exceeding probability.

Design formula provided by standard ISO 19906 (equation A.8.21) allows to estimate maximum effective pressure on the structure. It can be used for a stochastic model by substituting corresponding random variables instead of fixed maximum ice thickness and ice compressive strength values.

When these ice regime parameters are supposed to have rather common lognormal distribution, the distribution law of pressure is also lognormal and he model has some profitable properties, like easy quantile calculating.

The influence of introduced correlation between ice thickness and ice compressive strength is investigated in the paper. The outcome was made that ISO 19906 model of global effective pressure is not intended for using correlated ice regime parameters because of contadictory behaviour of pressure statistical properties in this case.

However, a number of other design formulas exist, which are free of the named feature.

The theorem of power coefficients, which provide minimum values for pressure mean in the case of ice regime parameters lognormal distribution is formulated and proved.

ACKNOWLEDGMENTS

Author wish to thank Prof. Oleg Timofeev from St. Petersburg Krylov Research Center for his support. Author is grateful to anonymous reviewers of this paper.

REFERENCES

Bourke, R.H., Garrett, P.R., 1987. Sea ice thickness distribution in the Arctic Ocean. Journal Cold Regions Science and Technology, Vol. 13, p.259-280.

International Standards Organization, 2010. Petroleum and natural gas industries – Arctic offshore structures. ISO 19906:2010.

Fransson, L., Olofsson, T., 2005. On Scaling of the maximum ice pressure on the vertical foundation. Proc. 18th POAC Conf., Potsdam, USA, p. 341-351.

Johnson, N. L. and Kotz, S., 1972. Distributions in Statistics: Continuous Multivariate Distributions. New York, Wiley.

Johnston, M. Masterson, D., Wright, B., 2009. Multi-year ice thickness: knowns and unknowns. Proc. 20th POAC Conf. Lulea, Sweden, POAC09-120 paper, 13 pp.

Korzhavin, K.N., 1962. Action of Ice on Engineering structures (in Russian - Siberian Branch of USSR Academy of Science publishing, Novosibirsk, 1962, 203 pp.; in English – Books for Business, 2002, 328 pp.)

Loset, S., Shkhinek, K., Uvarova, E., 1999. An overview of the influence of structure width and ice thickness on the global ice load. Proc. 15th POAC Conf. Espoo, Finland, p.425-434.

Loset, S., Shkhinek, K., Gudmestad, O., Hoyland, K., 2010. Actions from ice on arctic offshore and coastal structures (in Russian). “Lan” publishing company, St. Petersburg, 270 pp.

Nesterov, A., Sagerup, T. Gudoshnikov, Yu., Liferov, P., 2009. Ice drift in the North-Eastern Barents Sea: measurements and simulations. Proc. 20th POAC Conf. Lulea, Sweden, POAC09-143 paper, 12 pp.

Masaki, T., Okubo, Y., Honda, H., Otsuka, N., Saeki, H., 1996. Characteristics of unconfined compressive strength of the sea ice in the sea of Okhotsk. Proc. 13th Int. Symp. on Ice (IAHR), Beijing, China, pp. 72-79.

Palmer, A., Croasdale, K., 2013 Arctic offshore engineering. World Scientific. 350 pp.

Shirasawa, K., Leppäranta, M., Saloranta, T., Kawamura, T., Polomoshnov, A., Surkov, G., 2005. The thickness of coastal fast ice in the Sea of Okhotsk. Journal Cold Regions Science and Technology, Vol.42, 1, p.25-40.

Sinitsyna, T., Shkhinek, K., Hoyland, K., 2013. A statistical analysis of spatial strength heterogeneity of sea ice cover. Proc. 22th POAC Conf. Espoo, Finland, 10 pp.

Timco, G.W., Frederking, R.M.W., 1990. Compressive strength of sea ice sheets. Journal Cold Regions Science and Technology, vol. 17, p. 227-240.

Truskov, P. A., Astafiev, V. M., Surkov, G. A., 1992. Problems of choice of sea ice cover parameters design criteria. Proc. of the 7th International Symposium on Okhotsk Sea and Sea Ice, p 21-25.

Wadhams, P., Amanatidis, G. (Editors), 2006. Arctic Sea ice thickness: past, present and future. European Commission, Scientific report on International Workshop in Rungstedgaard, Denmark, 8-9 November, 2005, 293 pp.

Zvyagin, P. Sazonov, K., 2014. Analysis and Probabilistic Modeling of the Stationary Ice Loads Stochastic Process with Lognormal Distribution. Proc. 33th OMAE Conf., San Francisco, USA, OMAE2014-24713, 6 pp.