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THE APPLICATION OF A NON-SMOOTH DISCRETE ELEMENT METHOD IN ICE RUBBLE MODELLING

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ABSTRACT
Ice ridges often govern the design of Arctic offshore structures. While the biggest load contribution arises usually from the consolidated layer of the ridge, a substantial part of the ridge load does come from its keel, i.e. the unconsolidated ice rubble. Therefore it is important to have an accurate numerical model for the simulation of keel-structure interaction. Up to now, discrete element models (DEM) of the ice rubble have utilised so-called smooth methods to resolve contacts. Non-smooth methods, on the other hand, can significantly reduce the calculation time. In non-smooth methods, a mixed linear complementarity problem is often formulated to resolve the contacts between interacting rigid bodies. This paper assesses the applicability of a non-smooth method in the modelling of ice rubble. Lab-scale experiments and a full-scale experiment are modelled. The results obtained with the non-smooth method are compared to the experimental results and to smooth DEM simulation results. The comparison indicates that the non-smooth method is well suited to model ice rubble and can lead to a significant improvement in calculation time.

INTRODUCTION
This paper investigates the application of a non-smooth method in discrete element modelling of ice rubble. An accurate representation of ice rubble is important in the modelling of ice ridges. The keel of a first year ridge consists of partially consolidated ice rubble. This rubble influences the ice load and can possibly interfere with moorings and other subsea structures (Serré, 2011a). Different modelling approaches can be considered when modelling ice rubble.

A distinction can be made between modelling ice rubble as a continuum and modelling the rubble as a large number of discrete ice blocks. In modelling of ice rubble as a continuum, finite element methods (FEM) are applied. When modelling the rubble as a large number of discrete blocks, one applies discrete element methods (DEM). FEM models of ice rubble have been developed and applied by Heinonen (2004), Serré (2011b) and Liferov (2005). Papers describing DEM modelling of ice rubble include Polojärvi et al. (2014), Polojärvi & Tuhkuri (2009), Polojärvi & Tuhkuri (2013), and Haase et al. (2010). To the author's knowledge, all current DEM models of ice rubble utilize 'smooth' discrete element (SDEM) methods. In the SDEM approach contacts are resembled with spring-damper elements or with penalty functions. The contacts are resolved in time. To ensure stability, the time steps get increasingly small for stiff materials. This causes fairly long calculation times

A subclass of the DEM in which the contacts between bodies are resolved as impulses can potentially give a significant improvement in computation time. Such methods are described in literature as non-smooth discrete element methods (NDEM). In granular material modelling this method is also referred to as Contact Dynamics (Radjai and Richefeu, 2009).

In this approach, contacts between rigid bodies are considered instantaneous events with an infinitely short duration. The contacts are resolved in the form of impulses.

NDEM methods were first used in ice mechanics by Konno & Mizuki (2006). They applied the NDEM method to model a ship in a broken ice field. The waterline processes and large scale failure in ice were incorporated into these methods for the first time by Lubbad and Løset (2011). An extensive review of the application of both SDEM and NDEM in ice mechanics can be found in Metrikin & Loset (2013).

A general comparison between NDEM and SDEM simulations can be found in Servin et al. (2014). Servin et al. found that NDEM is more beneficial for stiff materials, static or slow moving systems and with increasing error tolerance. Ice is a stiff material, and interaction with ice rubble may be described as slow moving in interaction scenarios with fixed or moored structures. Therefore it was expected that NDEM can potentially be beneficial in terms of calculation time in modelling ice rubble.

To investigate the applicability of a NDEM method in the modelling of ice rubble, NDEM simulation results were compared to SDEM simulation results and to measurements from a full-scale punch through test. NDEM results were also compared to results from lab-scale shear box experiments. The shear box experiments are described in Pustogvar et al. (2014), and the SDEM simulations of the shear box experiments are described in Polojärvi et al. (2014). The full-scale punch through test were conducted by Heinonen & Määttänen (2001). SDEM simulation of the punch through experiments are described in Polojärvi & Tuhkuri (2009).

Section 2 of the current paper describes NDEM as applied here. Section 3 summarizes the most important experimental and numerical parameters of the experiments that are used for benchmarking. Sections 4 and 5 show the results of the NDEM simulations in comparison to the experimental observations and the SDEM results. Section 6 concludes with a summary of the most important findings.

THE NON-SMOOTH DISCRETE ELEMENT METHOD

The NDEM approach appeared around 1990 and was initially focused on graphical applications such as games and training simulators. Important early works include Hahn (1988) and Baraff (1992). There is a clear difference between the approaches by Baraff and Hahn, referred to as the constraint based approach and the impulse based approach. The impulse based approach as explained by Hahn is also referred to as event driven simulation. This distinction is no longer obvious in modern methods, but the distinction can still lead to some ambiguities in terminology. Therefore the difference is explained below.

The difference between the constraint based approach as used by Baraff and the impulse based approach as used by Hahn is explained in Mirtich (1996). In the constraint based approach as used by Baraff, a linear complementary problem (LCP) is formulated based on non-penetration constraints between rigid bodies. A direct or analytical solution to the LCP is found based on pivoting (Baraff 1994). The impulse based approach as described by Hahn only solves pair-wise contacts. It proceeds in time until a collision occurs. The collision is solved using impulses and the system proceeds until the next collision. In the impulse based approach it is assumed that multiple collisions never occur at exactly the same time. Impulse based methods are less efficient for stable and simultaneous contacts.

Modern physics engines use the Projected Gauss Seidel or blocked projected Gauss Seidel method. This method was first published by Moreau (1999). It is an iterative method to solve the LCP. The current paper uses a method referred to as sequential impulses. According to its creators, it is equivalent to the projected Gauss-Seidel method, but without having to formulate the LCP (Coumans, 2014). The model is advanced in time with a predefined time step. All collisions are detected. For every contact, the non-penetration constraint is solved while disregarding the constraints at other contacts. The solution of one constraint will invalidate the other constraints. However, over many iterations the system will converge to a global solution. This is equivalent to solving the LCP as posed by Baraff (1992) by iterating over all contacts and calculating the needed impulses. The advantage of iterative methods in comparison to the direct methods is that it always converges to a solution. Also it is possible to include Coulomb friction, which is problematic in the direct approach. An overview of solver methods and other aspects related to physics engines can be found in Bender et al. (2014). Here the projected Gauss-Seidel method and other solver methods are derived and explained.

The NDEM simulations in this paper are done using the Bullet physics engine (Coumans, 2015). This is an open source physics engine that is widely used in gaming and graphics applications and in scientific applications related to robotics. The use of Bullet in the modelling of granular materials is quite novel. A simulation by Izadi & Bezuijen (2015) demonstrated the use of Bullet in modelling the compaction of granular materials. They reported promising results. Bullet is compared to Physx and ODE in Roennau et al. (2013). Bullet, Physx and ODE are some of the most used physics engines. Although the comparison is made with another field of application in mind, the benchmarking tests that are described are quite general and the results provide a useful comparison of some aspects of the physics engines that are of importance in the modelling of ice rubble. The physics engines are compared on several aspects. Comparisons related to collision detection, contact resolution, execution time and friction are considered most relevant in the modelling of ice rubble. Results from the comparison by Roennau et al. (2013) are summarized here;

- **Collision detection:** Roennau et al. found that Bullet fails to detect some collisions when objects become too small. This problem can be resolved by using a scaling parameter, by which you effectively change the units of the problem, for instance from *m* to *cm*. If objects are bigger, bullet shows better performance than PhysX and ODE.
- **Contact resolution:** Contact resolution is the accuracy with which contact forces are resolved. This can be measured by checking stacking stability. Boxes are stacked on top of each other and it is checked when the stack becomes unstable. Bullet performs worst in this test. It is unclear why Bullet performs worst.
- **Execution time:** In terms of execution time, Bullet is slightly slower than PhysX. ODE is much slower than Bullet and PhysX. The calculation time of ODE increases exponentially with the number of blocks, while the execution time Bullet and PhysX increases linearly.
- **Friction:** The friction is resolved much more accurate in Bullet than in the other two solvers. This is considered to be a major advantage, since friction has a big influence on the unconsolidated rubble strength.

Inspection of the Bullet source code by the authors shows that the reported problems with collision detection are caused by certain collision detection parameters that are not scaled with body size in the unmodified source code. Scaling these parameters has the same results as applying a scaling parameter to the body size.

The application of a scaling parameter is a more convenient way of achieving the same result, and does not involve changes in the source code. Bullet is chosen for simulations because it is open source, the low execution time, and the accurate friction model. The fact that Bullet is open source allows us to verify the code and make changes were needed in order to improve the accuracy of the results, possibly sacrificing some execution speed.

SIMULATED EXPERIMENTS

The NDEM simulation results are compared to lab-scale and full-scale experimental results and SDEM simulations. It was chosen to compare to both lab scale and full-scale because both scales pose their own modelling challenges. Lab-scale experiments have better defined experimental conditions and therefore offer a more reliable validation. Because it is not yet clear how lab-scale rubble properties translate to full-scale, it was chosen to also verify the model against full-scale experiments. The lab-scale shear box experiments are described in Pustogvar et al. (2014), and the SDEM simulations of the shear box experiments are described in Polojärvi et al. (2014). The full-scale punch through tests were conducted by Heinonen & Määttänen (2001). SDEM simulation of the punch through experiments are described in Polojärvi & Tuhkuri (2009).

Shear box experiments

The shear box experiments that are used as test case were carried out for two different block sizes and two different confining pressures. Figure 1 shows the experimental set up. All relevant parameters are shown in Table 1.

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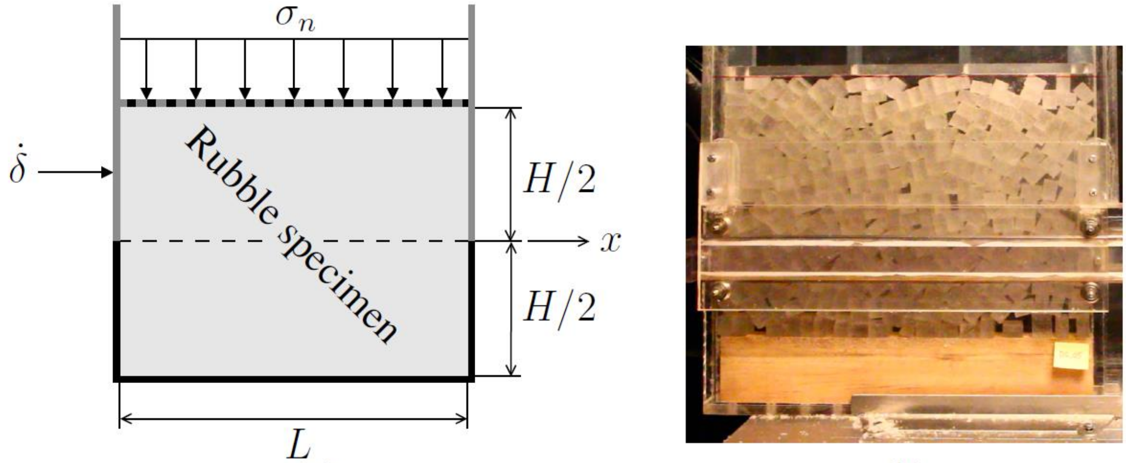

(a) (b)

Figure 1: Experimental set-up (Polojärvi et al., 2014).

Table 1: Shear box experiment and simulation parameters.

	Parameter	Unit	Value
NDEM simulation parameters	Restitution	-	0.1
	Time step	s	$5 \cdot 10^{-4}$
	Iterations	-	200
	Collision margin	mm	0.3
SDEM simulation parameters	Contact stiffness normal	Pa	$4.0 \cdot 10^{8}$
	Contact stiffness tangential	Pa	$1.5 \cdot 10^{8}$
	Plastic limit	Pa	$2.0 \cdot 10^{6}$
Experimental parameters	Block dimensions	m	0.02×0.03
	Ice-wall friction	-	0.3
	Ice-ice friction	-	0.5
	Rubble length	m	0.6
	Rubble height	m	0.4
	Shearing velocity	ms^{-1}	0.02
	Confining pressure	kPa	5.76, 11.03

The shape, orientation and position of rubble blocks in the initial configuration is equal in the NDEM and SDEM simulations. The confining pressure is applied by placing a beam on the blocks with a mass such that the experimental confining pressure is resembled. Shearing is started when all rubble blocks are at rest. The time step and the number of iterations are chosen such that the average value of the results is not influenced by further refinement of the time step or a higher number of iterations. The collision margin in the NDEM simulation is used to prevent penetration of rigid bodies. This is beneficial because recovering from penetration increases calculation time and can cause instability in the simulation. The collision margin should be chosen based on the average velocity of interactions and the timestep that is used. It should be chosen such that the movement of one body generally does not exceed the margin within a timestep. A side effect of the collision margin is that the corners of rigid bodies will appear slightly rounded. In the current simulation, the rounded corners caused by the addition of a collision margin are not necessarily a bad thing. After all, actual ice blocks neither have completely sharp corners.

Full-scale punch through experiments

The full-scale punch through experiments to which the results are compared were carried out on unconsolidated rubble. Unconsolidated rubble was obtained by first conducting a punch through test on consolidated rubble, and then conducting another test at the same location. Table 2 shows the experimental parameters and input parameters of the NDEM and SDEM simulations. Figure 2 shows a schematic representation of the punch through experiments.

The number of ice blocks in the NDEM simulation was chosen such that the rubble thickness resembles the rubble thickness from the SDEM simulations. The time step was varied to investigate its influence. Note that the timesteps used in the punch through simulation are much bigger than in the shear box simulation. The maximum allowable timestep is related to the interaction velocity in comparison to the size of the bodies in the simulation. Because the bodies in the punch through experiment are much larger, while the velocity is comparable, a bigger timestep can be used. Additionally, the shear box simulation is much more prone to sudden changes in force because of the formation and buckling of force chains. This necessitates a further reduction on timestep in case of the shear box simulation. Because of the low indentor velocity, the damping value and ice restitution did not influence the results.

These values were chosen such that the simulation time was optimized. The ice-ice friction value is chosen similar to the value used in the SDEM experiments.

In the experiments, a circular cut was made in the consolidated layer. This circular piece was pushed down into the rubble. Naturally, the buoyancy of this piece contributed to the indentor force. The SDEM results do not account for the buoyancy of the consolidated part. The experimental results shown here were corrected for this buoyancy, meaning that the force contribution due to buoyancy of the consolidated layer is subtracted.

Table 2: Punch through experiment and simulation parameters.

	Parameter	Unit	Value
NDEM simulation parameters	Number of ice blocks	-	1740 - 2375
	Restitution	-	0.0
	Time step	s	$5 \cdot 10^{-2} - 5 \cdot 10^{-3}$
	Iterations	-	10 - 500
	Collision margin	m	0.04
	Ice-ice friction	-	0.3
	Damping	-	0.2*
SDEM simulation parameters	Rubble thickness	m	2.21 – 2.85
	Ice-ice friction	-	0.3
	Penalty term	-	$1 \cdot 10^{-6} - 1 \cdot 10^{-8}$
	Damping constant	$Ns\, m^{-3}$	$5 \cdot 10^{-4} - 8 \cdot 10^{-4}$
	Time Step	s	$1 \cdot 10^{-4} - 2 \cdot 10^{-4}$
Experimental parameters	Block width	m	1.1 ± 0.4
	Block thickness	m	0.2
	Water density	$kg\, m^{-3}$	1010
	Ice density	$kg\, m^{-3}$	920
	Pool length	m	20
	Pool width	m	20
	Indentor velocity	ms^{-1}	0.025
	Indentor diameter	m	4.7

* Damping is defined in Bullet as the percentage of velocity lost per second.

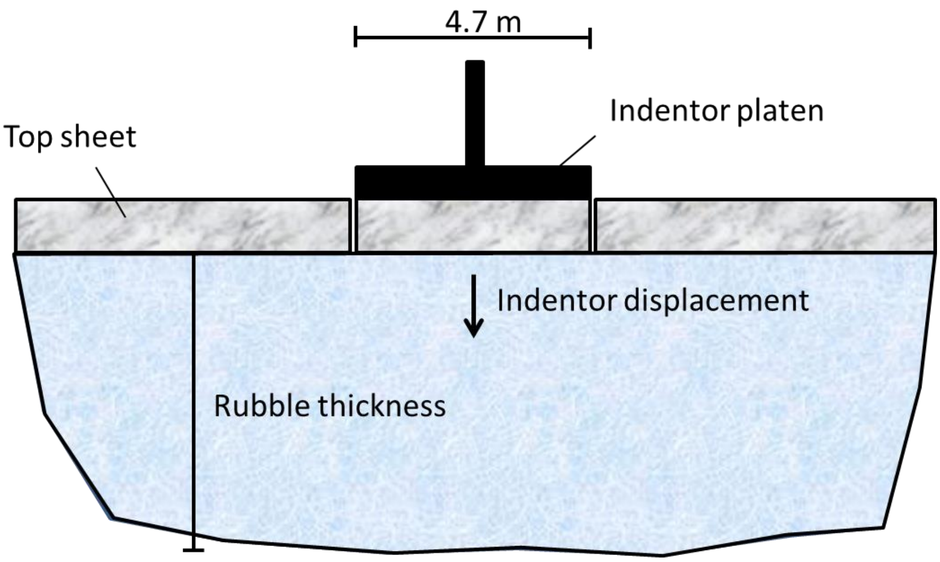

Figure 2: Schematic representation of punch through experiments (based on information from Polojärvi and Tuhkuri, 2009).

Comparison criteria

The NDEM simulations are compared to the SDEM simulations and the experiments on the following criteria;

- Visual rubble behaviour
- Execution time
- Accuracy of results

Visual rubble behaviour is specifically related to the formation of force chains in the shear box experiments and the deformed shape of the rubble in the punch through experiments. The execution time is monitored in the punch through simulation. Because a reduced execution time is the biggest potential advantage of using the non-smooth method, it is important to compare this. The force-displacement plots are compared with the simulation and experimental results. Finally, the influence of the time step on the NDEM results is investigated for the full-scale punch through experiments.

SHEAR BOX RESULTS AND DISCUSSION

Figure 3 and Figure 4 show the NDEM results as an overlay to the SDEM and experimental results. To eliminate the influence of the initial configuration, the SDEM and NDEM simulations were run with identical initial block configurations. Although the force signals of the SDEM and NDEM simulation are distinctly different, the average and maximum values correspond fairly well. The unfiltered NDEM signal shows high frequency variations which do not seem to severely influence the rubble behaviour. Such variations are most likely not physical but a numerical artefact which has to be further investigated. The initial block configurations in the experiments were different from the initial configurations in the numerical analyses. Because the initial block configuration has a high influence on the result, a direct comparison between the experimental load signal and the simulated load signals should not be expected. The average and peak values of the simulated load signals are in fairly good accordance with the average and peak values of the experimental signal.

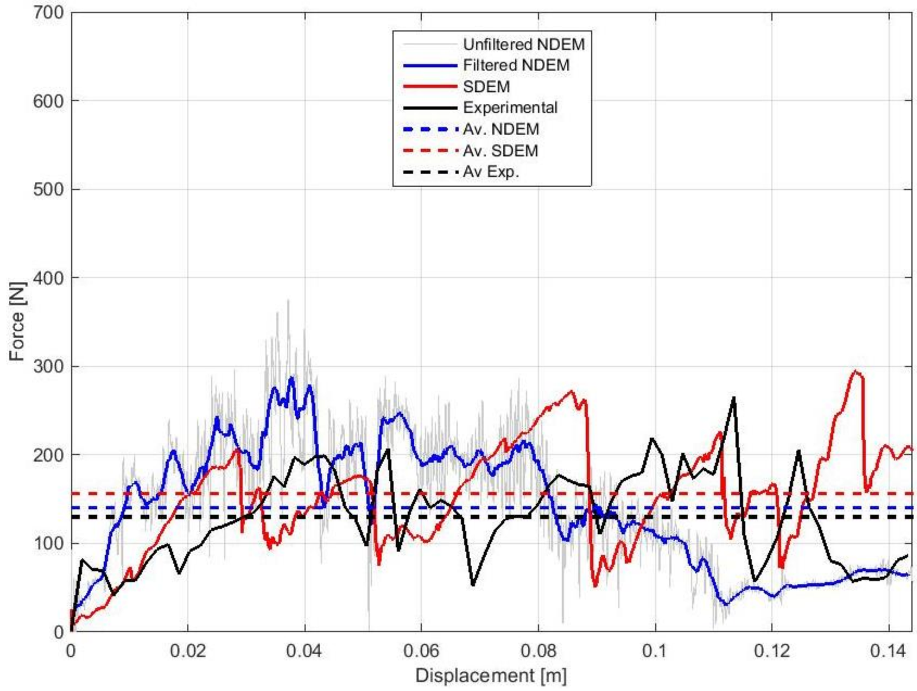

Figure 3: Comparison between NDEM, SDEM and experimental results for a confining pressure of 5.76 kPa, for an equal initial block configuration.

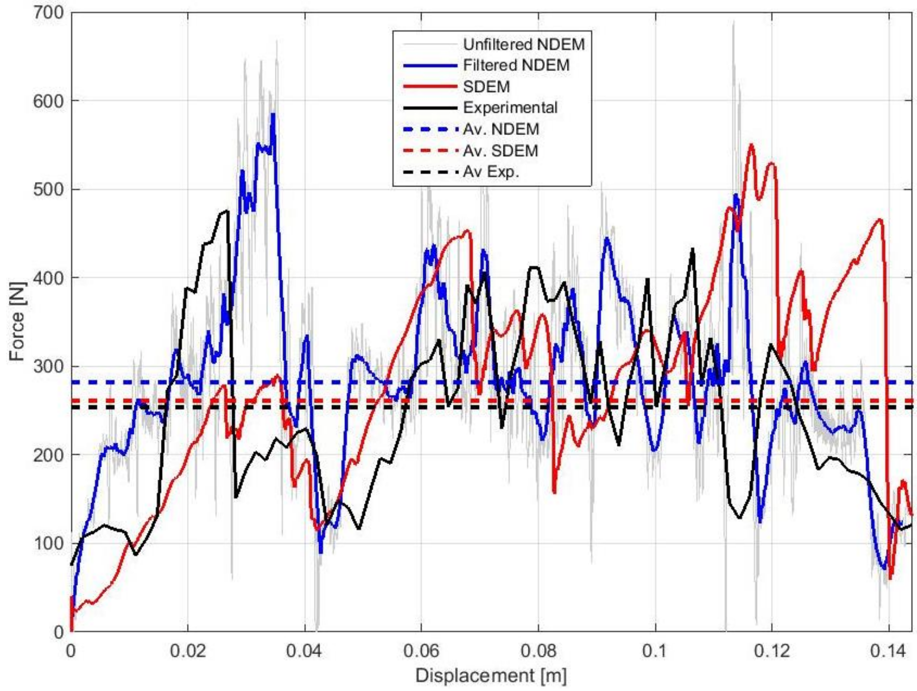

Figure 4: Comparison between NDEM and SDEM results for a confining pressure of 11.03 kPa, for an equal initial block configuration.

Force chains, as observed in the SDEM simulations of Polojärvi, were also observed in the NDEM simulations. Figure 5 shows the force chains as observed in the NDEM simulations in comparison to the force chains as observed in the SDEM simulations.

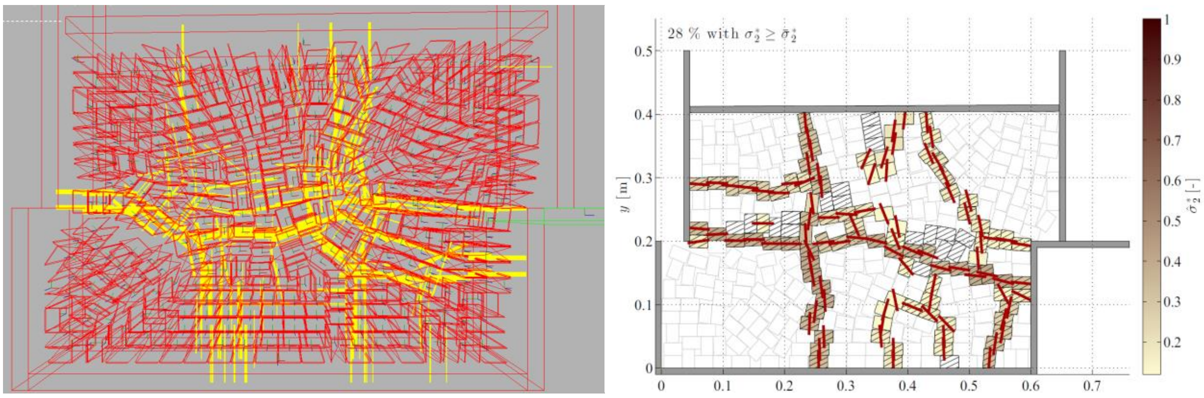

Figure 5: Force chains in NDEM experiments (left) compared with force chains as reported in Polojärvi et al. (2014)(right).

The results presented above show that the NDEM method is capable of predicting the forces that occur during shear box experiments on ice rubble. The mean and peak forces predicted by the NDEM simulations are in fairly good accordance with the values predicted by SDEM simulations and with the experimental results. Similar rubble behaviour is observed in the NDEM and the SDEM simulation. In both simulations the formation of force chains was observed. Although the NDEM and SDEM simulations had the same initial block configurations, the shape of the force signals show little resemblance. This is not surprising, considering the substantial differences between both calculation methods. A minor difference in block rearrangements in the beginning of the simulation will lead to different results later on.

PUNCH TEST RESULTS AND DISCUSSION

A comparison between the SDEM simulations, NDEM simulations and full-scale experimental results of punch through experiments is shown in Figure 6. The NDEM results are in good agreement with the SDEM modelling results and with the experimental results. In most NDEM cases the used modelling parameters were equal to the parameters used in the SDEM simulations (red lines). In one simulation the consolidated layer was included in the model (green line). The force contribution of the consolidated layer buoyancy was later excluded, as was also done with the experimental results shown in Figure 6.

Apart from the comparison of NDEM results to SDEM and experimental results, the influence of timestep on the NDEM results was also investigated. Kleinert et al. (2013) describes a large timestep dependence for the NDEM method in their comparison with SDEM. The timestep dependence is related to the fact that fast dynamics, like successive collisions and rearrangement events, are partially filtered out (Radjai and Richefeu, 2009). Such timestep dependence is only observed in the current analysis for timesteps larger than $2.5 \cdot 10^{-2}\ s$. Because the current simulation resembles a slow moving test case, the timestep can be taken quite large before the filtering starts having a significant effect. This is shown in Figure 7. The timestep in this figure was varied between $5 \cdot 10^{-2}\ s$ and $5 \cdot 10^{-3}\ s$. Only for a timestep of $5 \cdot 10^{-2}\ s$ the simulated indentor force is consistently lower than for higher timesteps.

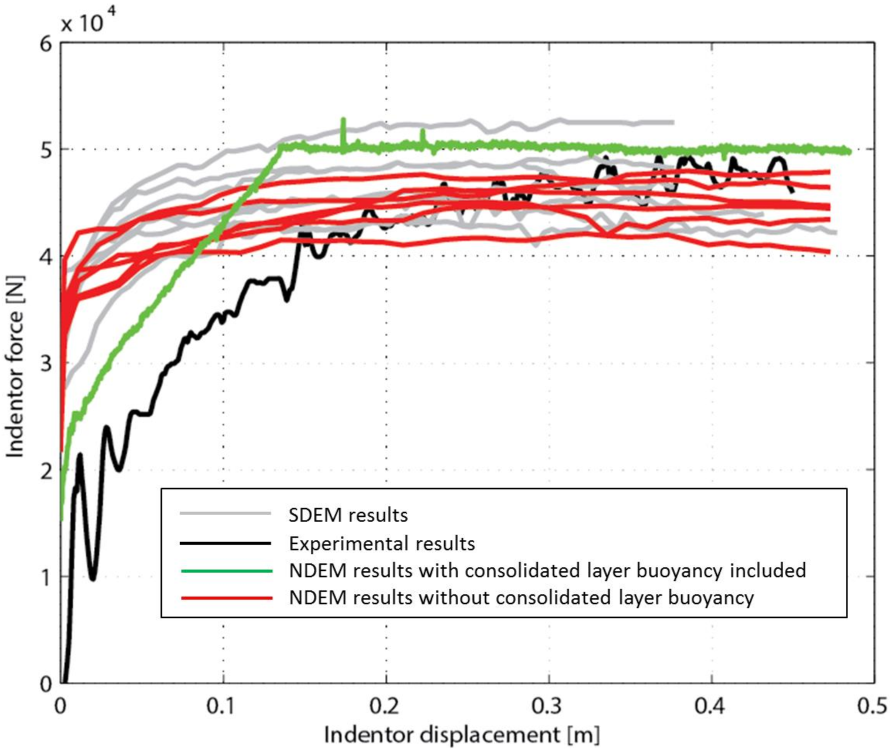

Figure 6: NDEM and SDEM simulation results compared to full-scale measurements. SDEM results from Polojärvi & Tuhkuri (2009). In the NDEM simulations the rubble thickness was between 2.19m and 2.36 m. In the SDEM simulations the rubble thickness was between 2.21 and 2.85 m. Ice-ice friction coefficient of 0.3.

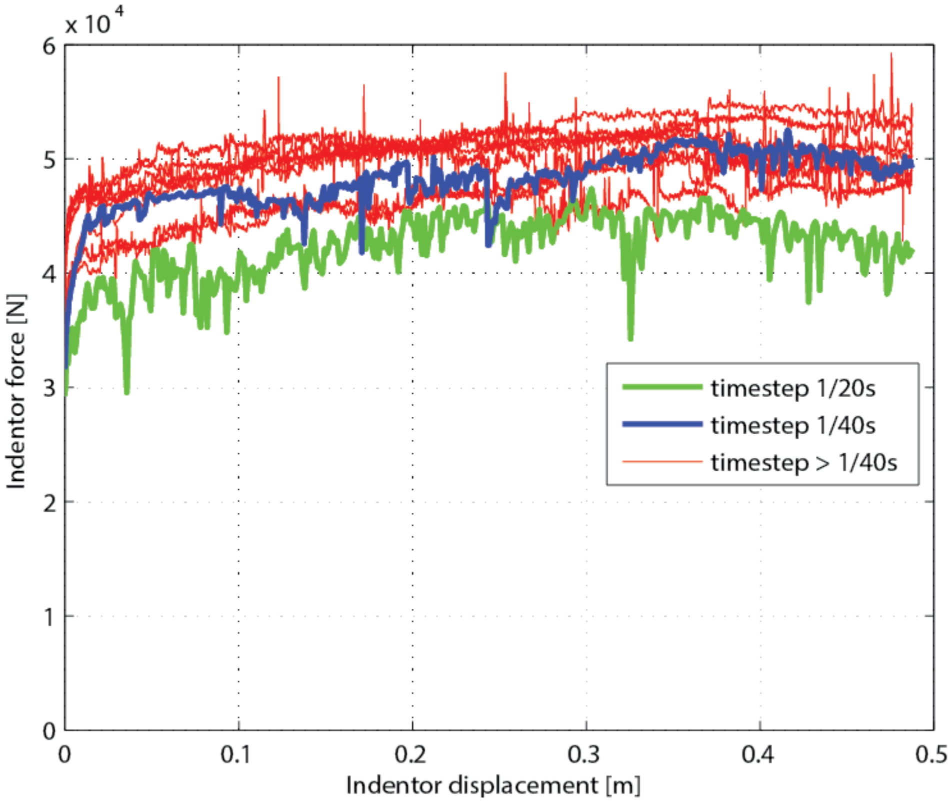

Figure 7: Timestep effect on indentor force.

The NDEM simulation already gives good results with time steps as large as 1/100 seconds and with only 50 iterations. Using these settings, the simulation of 1 second costs 4 seconds with 2000 rubble blocks. The simulation time increases linearly with the number of iterations, the number of ice blocks, and with a decreasing time step. Only one core was used in the presented simulations. By using multiple cores or a GPU, real time simulations with a much larger number of rubble bodies should be possible. The SDEM simulations to which the results were compared, on the other hand, could take several days.

It is interesting to consider what factors cause this substantial reduction in calculation time. Surely, part of the reduction is caused by the highly optimized nature of the used physics engine. The SDEM simulations used for comparison were obtained with a more scientifically oriented SDEM implementation in which optimization was not the main focus. However, it seems likely that the biggest part of the improvement comes from the inherent differences between the methods, which allow the timesteps to be a factor 100 higher when using the NDEM method in case of the punch through simulations. Note that the improvement in calculation time is also in accordance with the findings of Servin et al. (2014), as mentioned in the introduction.

CONCLUSIONS

NDEM simulations were compared to lab-scale shear box experiments and full-scale punch through tests. Results were also compared to SDEM simulations of the shear box and punch through experiments. The comparison results shown in this paper indicate that the NDEM method is well suited to model ice rubble.

For the shear box experiment, the mean and peak force predicted by the NDEM simulation is in accordance with the values predicted by the SDEM simulation and with the experimental values.

The NDEM results show high frequency variation, which is believed to be a numerical artefact. What causes this variation must be further investigated. Nevertheless, it seems that the high frequency variation does not influence the rubble behaviour significantly.

In the NDEM simulation of punch through tests, a big timestep and low number of iterations can be chosen without any apparent influence on the results. A 1 second simulation of 2000 blocks costs 4 seconds with a large timestep and low number of iterations. The execution time can potentially be further decreased by optimizing the code and the use of parallel computing. High frequency variation in the NDEM results was not observed in the simulation of punch through tests.

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