POAC'15
Trondheim, Norway

Proceedings of the 23rd International Conference on
Port and Ocean Engineering under Arctic Conditions
June 14-18, 2015
Trondheim, Norway

Preliminary results of 3D simulations of a failing ice sheet

Christiane Heinicke [1], Arttu Polojärvi [1,2], Jukka Tuhkuri [1,2]
[1]Aalto University, Espoo, Finland
[2] Sustainable Arctic Marine and Coastal Technology (SAMCoT), Centre for Research-based Innovation (CRI), Norwegian University of Science and Technology, Trondheim, Norway

ABSTRACT
We study the failure of an initially intact ice sheet against an inclined structure, where the most prevalent forces on the ice sheet are vertical. We describe the resulting vertical bending with the Reissner-Mindlin first-order shear deformation theory, implemented in a three-dimensional in-house finite element code. When certain stress criteria are met, the plate fractures according to a phantom node algorithm. The simulated fracture process is validated with analytical models and field experiments described in the literature. Focus is laid on cantilever beam tests and vertical breakthrough tests. Simulations agree with the literature data in both the distribution of stresses leading to fracture and the most common resulting fracture patterns. In future, our aim is to couple the finite element code with an existing discrete element code to simulate ice plate failure against a structure and the subsequent pile-up process.

INTRODUCTION

When an initially intact ice cover moves against an inclined structure, it fails by fragmenting into discrete ice blocks which then accumulate in front of the structure and form an increasingly large rubble pile. Understanding this fragmentation process and predicting the subsequent ice load on the structure are important for the design and operation of marine structures in ice-covered waters.

Ice failure in ice pile-up processes has previously been simulated with the discrete element method (DEM) by e.g. Hopkins (1997) and Paavilainen et al. (2009). In addition, e.g. Määttänen and Hoikkanen (1990) have used the finite element method (FEM) to determine the ice forces on structures. The most promising approach, however, seems to be a combination of both methods (FEM-DEM), as done by e.g. Munjiza (2004) or Paavilainen (2013).

The latter has conducted 2D FEM-DEM simulations on ice rubble formations, where the original ice sheet is modeled with FEM Timoshenko beams and the broken off fracture pieces are modeled using DEM. The work presented here is aimed at implementing the 3D continuation of the Paavilainen model, and we will present the model at its current stage, together with its momentary capabilities and preliminary simulation results.

This paper focuses entirely on the finite element modeling of the intact ice sheet subjected to a bending load. More specifically, for the purpose of this paper we will ignore horizontal loading of the ice sheet, rather the stationary ice sheet is deformed by constant vertical loads. At high stress levels the ice sheet starts to fail according to stress-based failure criteria.

In the following section we will describe the physical settings that we are going to focus on throughout this paper, then continue with the mathematical model underlying the numerical simulations in section 3. In section 4, we present some preliminary simulations and we will conclude the paper with an outlook on the next immediate steps of extending the model.

SETTING AND DEFINITIONS

We consider an isotropic and homogeneous, stationary ice plate of width W, length L, and height h, that is floating on water modeled as an elastic foundation with modulus k. The plate is subjected to a constant vertical downward force P acting over a circular area with radius r. The response of the plate is considered to be quasi-static.

For validation we chose two well defined geometries, a simply supported square plate and a cantilever test beam (see figure 1). The plate is 20 m × 20 m large and 0.5 m thick, and is loaded centrally with a total force of 1 kN across an area of radius 2 m. All four edges are simply supported, i.e. vertical displacements are zero, but the edges are free to rotate. The cantilever beam is chosen to be 4.5 m in length, 0.5 m in width and 0.5 m in thickness. One end is clamped over a length of 0.5 m in order to approximate the setup in actual field tests, while the other is loaded with a force of 9.0 kN that is distributed over a semi-circular area of 0.05 m radius. Material parameters are characteristic for sea ice (see table 1).

To initiate cracking, total loads are set to 100 kN and 9.0 kN, respectively. In both cases, the plates are initially intact and fail in bending due to the imposed high stress levels. Increasing the applied loads further leads to crack propagation up to the point where the plates fail entirely.

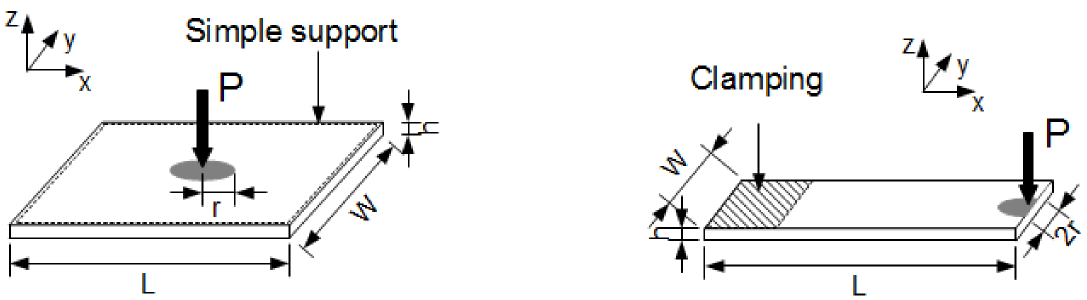

Figure 1. General settings. Left: A square plate is resting on an elastic foundation and subjected to a vertical force P which is applied over a circular area of radius r. The dimensions of the plate are length L, width W, and thickness h, and all four edges are simply supported. Right: A narrow plate resembling a cantilever beam is subjected to a force P on a semi-circular area at one end and clamped at the opposite end.

Table 1. Material parameters for the ice plate.

Material parameter	Symbol	Value
Young's modulus	E	5.0 GPa
Poisson's ratio	ν	0.35
Shear correction factor	κ	5/6
Modulus of the elastic foundation	k	10055 N/m^3
Tensional strength	s_f	0.6 MPa
Shear strength	t_f	0.6 MPa
Compressive strength	f_c	3.0 MPa

MODEL

We use a first-order shear deformation theory and linear elasticity to describe deformations of the homogeneous and isotropic plate before fracture. Cracking is purely stress-induced; the model presented here is quasi-static and ignores the effects of strain and strain history on the fracture process.

The shear deformation theory we employ is the Reissner-Mindlin theory (see e.g. Reddy 2007 or Szilard 2004) that includes the effect of transverse shear in bending deformations. Cross sections that are originally normal to the mid-plane of the plate may rotate with respect to it during deformation. Therefore three degrees of freedom are necessary to describe the deformation of the plate: vertical displacement w of the mid-plane $z=0$, and the two rotations $\varphi_x=\frac{\partial u}{\partial z}$ and $\varphi_y=\frac{\partial v}{\partial z}$ of the transverse normal about the y- and x-axes, respectively. The two horizontal displacements u and v are zero on the mid-plane for the static bending model employed here.

Our FEM model is based on Ferreira (2009) and Reddy (2007). Displacements $\boldsymbol{d}$ are found by solving the linear system of equations

$$\boldsymbol{K}\,\boldsymbol{d}=\boldsymbol{f}\ , \tag{1}$$

where $\boldsymbol{K}$ is the stiffness matrix and $\boldsymbol{f}$ the force vector. Both are computed with the help of Gaussian quadrature, but we use reduced integration for the shear contribution of the stiffness matrix in order to avoid shear locking. Strains and stresses of the plate are obtained from the calculated displacements during post-processing.

We use an unstructured mesh of T3 elements that has been generated with GMSH (Geuzaine 2009). Each node is assigned three degrees of freedom $(w, \varphi_x, \varphi_y)$. The total number of elements is 1000 for the square plate and 516 for the cantilever beam. It should be noted that even though the mesh itself is 2D, displacements can be recovered for the entire three-dimensional plate.

Failure in the plate occurs according to the criterion suggested by Schreyer et al. (2006) which employs a decohesion function based on the stress state of the plate material. At this stage, we do not yet include cohesiveness of cracks (Hillerborg 1976), effectively inducing a catastrophic failure through the entire thickness (not length) of the plate as soon as a crack is initiated at one location.

The decohesion function is defined as (Schreyer 2006):

$$F=\frac{\sigma_{nt}^2}{\tau_{sf}^2}+\frac{\sigma_{nn}}{\tau_{nf}}+\frac{\sigma_{tt}^2}{f_c^2}-1 \tag{2}$$

with the stresses $\boldsymbol{\sigma}=\begin{pmatrix}\sigma_{nn} & \sigma_{nt}\\ \sigma_{nt} & \sigma_{tt}\end{pmatrix}$, the shear strength τ_{sf}, the tensional strength τ_{nf} and the compressive strength f_c.

Only stress states that result in $F\leq 0$ are admissible. The state $F=0$ indicates the onset of fracture or unloading. In the case $F<0$, the material is intact or the crack is not propagating.

During simulation, the maximum of F is calculated for a given load P. If it is found to produce a stress state with $F<0$, the load is increased and the displacements and corresponding stress state recalculated until $max(F)=0$ is reached.

The node at which this maximum occurs is duplicated and both the original and the new node are assigned to elements on either side of the crack. In order to prevent the plate from failing catastrophically, only one or two nodes at a time may fail. That is, if the load P is found to be too high, it is reduced by half of the load increment of the previous step. Then $\boldsymbol{f}$ and $\boldsymbol{K}$ are updated, together with the corresponding displacements and stresses. In other words, we use a bisecting algorithm for finding the load that cracks the plate at either one or two nodes.

The direction of the crack is determined from the critical angle found in Schreyer (2009) and can be either at an angle of $\theta_c=0$ or $\theta_c=\pm\alpha$ to the principal direction, where α is given by

$$\tan^2\alpha=\left(\frac{(\sigma_1-\sigma_2)}{\tau_{sf}^2}-\frac{1}{\tau_{nf}}+\frac{2\sigma_2}{f_c^2}\right)\left(\frac{(\sigma_1-\sigma_2)}{\tau_{sf}^2}+\frac{1}{\tau_{nf}}-\frac{2\sigma_1}{f_c^2}\right)^{-1}. \tag{3}$$

The element boundary ending at the cracking node that has its angle θ closest to either one of the critical angles θ_c will be the one along which the crack propagates.

PRELIMINARY RESULTS

1. Square plate

Our square plate setup resembles that of a loaded floating ice sheet before vertical breakthrough. Due to its importance, the deformation due to vertical loading has been studied widely, and analytical solutions for the vertical displacement w of a simply supported plate are readily available, e.g. from Szilard (2004):

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{P_{mn}\sin(m\pi x/L)\sin(n\pi y/W)}{D\pi^4[(m^2/L^2)+(n^2/W^2)]^2+k} \tag{4}$$

with the Fourier coefficients

$$P_{mn}=\frac{4P}{WL}\sin\frac{m\pi x_p}{L}\sin\frac{n\pi y_p}{W}\ . \tag{5}$$

The circular load P is composed of point loads at the individual points (x_p, y_p) equally distributed over the circular cross section.

For the square plate, the maximum deflection predicted by equations (4) and (5) for a load of 1 kN is 69.7 µm. Both our FEM model and a comparable COMSOL agree remarkably well with both magnitude and shape of the deflection, as can be seen in figure 2.

If the load is increased to 9 kN, the deflection increases to 7.0 mm and cracks start forming at the bottom surface of the plate, as shown in figure Error: Reference source not found. The first to appear is an approximately straight crack, which is soon joined by a second crack almost perpendicular to the first. A further increase in the load leads to the formation of cracks at various angles that continue growing until the edge of the plate is reached. This cracking pattern mimics the early stages of an ice sheet breakthrough found in numerous field observations (Beltaos 2001, Masterson 2009).

While the load that leads to the first crack is found to be 100 kN, it has to increase gradually in order to propagate the crack. When the load reaches almost three times its original value, the crack reaches the edge of the simulated plate. For comparison, breakthrough in the field may occur with loads as low as $350h^2$ = 87 kN, but typically is expected for 440 kN (Sodhi 1995). Note that our model plate is too small to exhibit circumferential cracks and actual breakthrough.

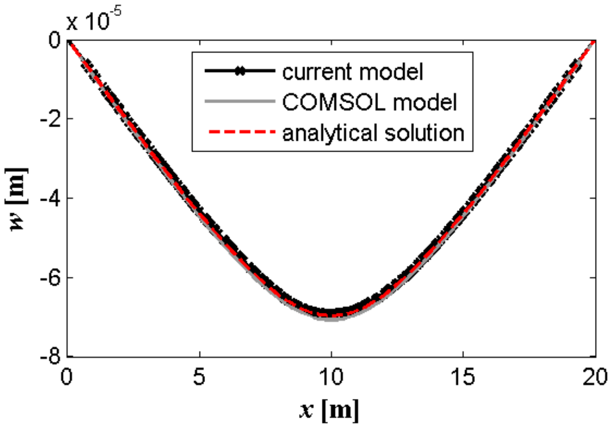

Figure 2.Comparison between the model described in this paper, a COMSOL model and the corresponding analytical solution. For all three models a plate of 20 m × 20 m × 0.5 m is resting on an elastic foundation and a total force of 1kN is applied over a circular cross section of 2 m radius at the center of the plate. Predicted deflections are 69.5 µm, 70.7 µm, and 69.7 µm, respectively.

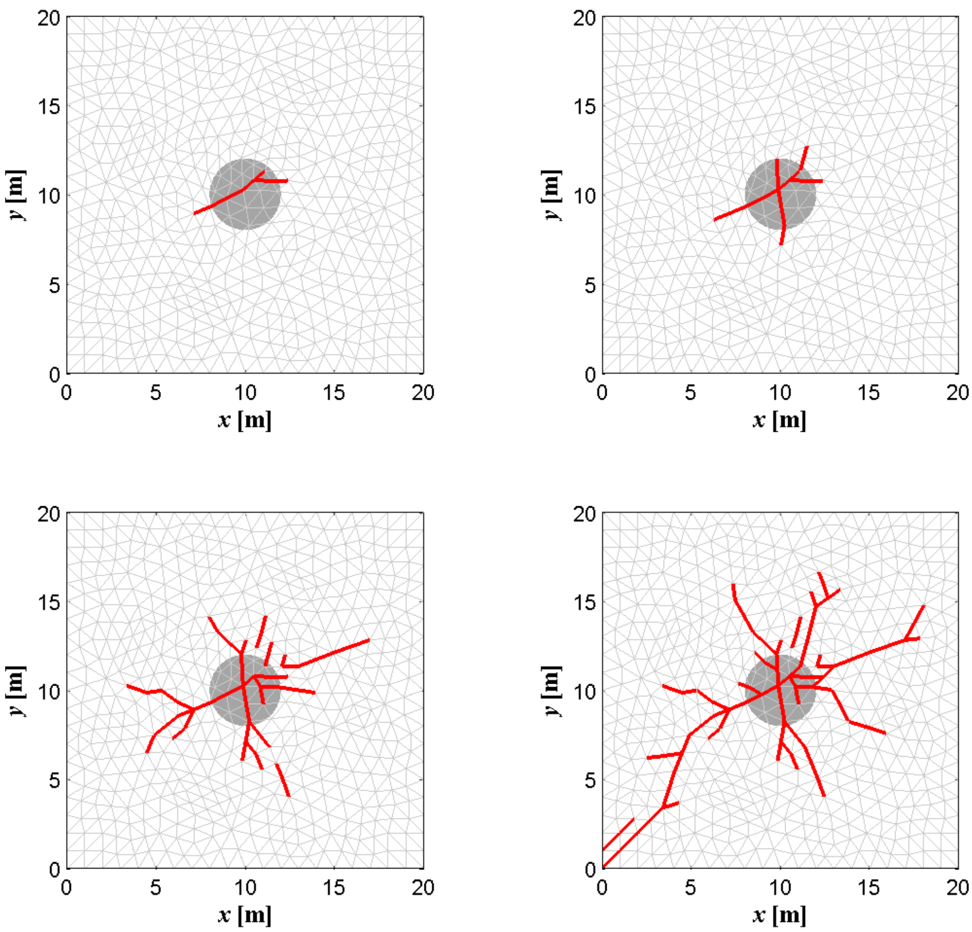

Figure 3. Square plate loaded vertically (into the plane of the sketch). The grey circle marks the area over which the applied force is spread. Red lines mark nodes and element boundaries that have been duplicated, that is they represent position and direction of cracks. All cracks are formed at the bottom surface of the plate. Applied loads are 120 kN (top left), 150 kN (top right), 245 kN (bottom left), and 285 kN (bottom right), respectively.

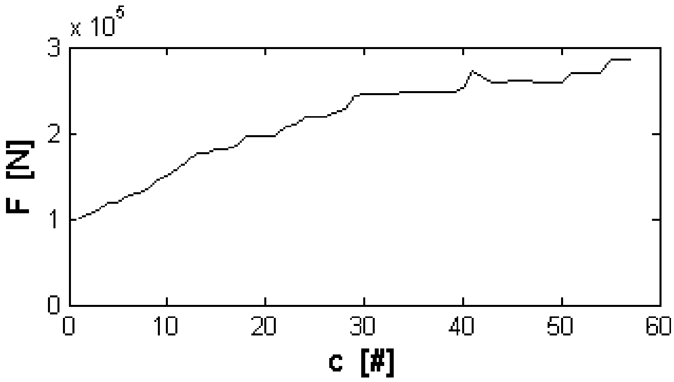

Figure 4. Load history of the square plate versus total length of all cracks combined, measured in the number of nodes that have been duplicated. The first crack appears at 100 kN; the first crack reaches the edge of the plate at 285 kN.

2. Cantilever Beam

We compare the cantilever beam simulations with analytical solutions, COMSOL simulations and field observations. In our simulations, the maximum deflection at the end of the beam amounts to 2.6 mm and the highest stresses of 2.4 MPa are on the horizontal surfaces at some distance from the clamping.

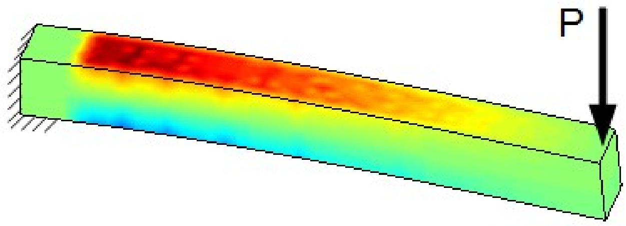

Figure 5. Tensional stress distribution (σ_{xx}) in cantilever test beam. Downward deflection amounts to 2.6 mm. Maximum of tensional stress (2.4 MPa) is located at the top of the beam at some distance from the clamping, which is consistent with the COMSOL model. Shearing becomes apparent due to the rotation of the end surface of the beam at x = 4.5 m.

The analytical solution predicts the following vertical deflection at the end of the beam (Ervik 2013):

$$w(L)=\frac{2F\beta}{k}\frac{\sinh 2\beta L-\sin 2\beta L}{\cosh^2\beta L+\cos^2\beta L} = 5.3\text{ mm}$$

and a stress at the crack root of

$$\sigma(x=0)=\frac{6}{Wh^2}\frac{F}{\beta}\frac{\sinh\beta L\cos\beta L+\cosh\beta L\sin\beta L}{\cosh^2\beta L+\cos^2\beta L} = 1.9\text{ MPa}$$

where

$$\beta=\sqrt[4]{\frac{k}{4\mathrm{EI}}}\ \text{ and }\ I=\frac{Wh^3}{3}.$$

The COMSOL model predicts a similar deflection of 3.6 mm but a lower stress of 1 MPa. Considering a reasonable flexural strength similar to the tensile strength of 0.6 MPa (see table 1), one should expect from Marchenko et al. (2014) that the beam can withstand a maximum of about 3 kN, according to the following equation:

$$F_{max}=\frac{\sigma_b W h^2}{6L}.$$

However, our model requires a force of 9 kN to initiate cracking. Nevertheless, the deflection and stress distribution agree well qualitatively. This is particularly reflected by the fact that the cracks in our model initiate at a small distance from the clamping at the upper surface of the beam, and are directed across the beam.

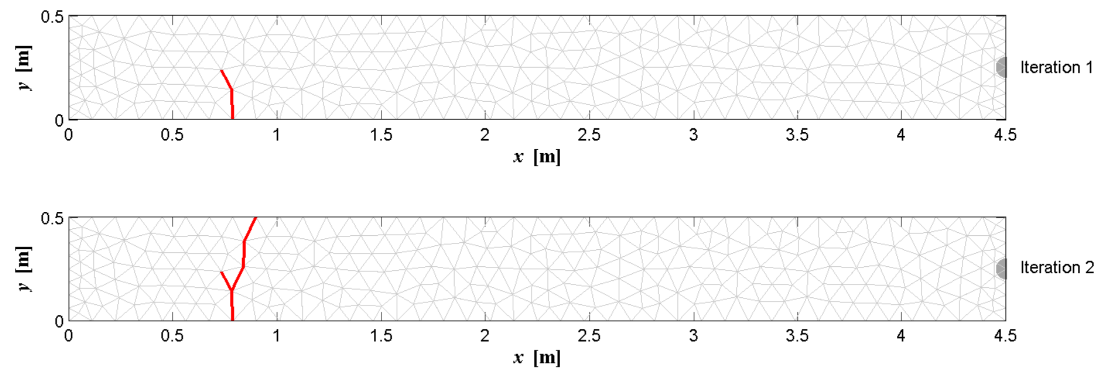

Figure 6. Crack initiation for the cantilever test beam at the top surface in the region of maximal tensional stress. The crack crosses the beam almost in a straight line. Location of the vertical load is marked by the grey semi-circular area at the right end of the beam.

CONCLUSION AND OUTLOOK

We have presented an FEM plate bending model that describes an initially intact plate developing cracks when stress levels are high. The employed model includes first order shear deformation and allows for fracture due to tension, compression or shear. The unstructured T3 mesh yields very good qualitative and quantitative agreement for the plate bending case presented. For the cantilever beam, quantitative agreement depends on the means of comparison. In both cases qualitative agreement of the cracking process with empirical findings is very promising.

The presented FEM model will be combined with an existing 3D DEM solver that will handle the pieces broken off the original plate. Before this, we will incorporate the cohesive theory and thus add the third dimension into our cracking model by allowing different levels of cohesion throughout the thickness of the plate. This will also require an adaptation of the current 2D fracture criterion. Later, we will add dynamical effects to our model and include lateral movement.

REFERENCES

Beltaos, S., 2001. Bearing Capacity of Floating Ice Covers: Theory versus Fact. 11th Workshop on the Hydraulics of Ice Covered Rivers, Ottawa, Ontario.

Ervik, Å., 2013. Experimental and Numerical Investigations of Cantilever Beam Tests in Floating Ice Covers. Department of Structural Engineering, Norwegian University of Science and Technology, Master's Thesis.

Ferreira, A.J.M., 2009. MATLAB Codes for Finite Element Analysis. Solids and Structures, Solid Mechanics and Its Applications 157, Springer Science+Business Media B.V., pp. 161-164.

Geuzaine, C. and Remacle, J.-F., 2009. Gmsh: a three-dimensional finite element mesh generator with built-in pre- and post-processing facilities. Int J Numer Meth Eng 79(11), 1309.

Hillerborg, A., Modéer, M., and Petersson, P.-E., 1976. Analysis of crack formation and crack growth in concrete by means of fracture mechanics and finite elements. Cement and Concrete Research 6, p. 773.

Hopkins, M., 1992. Numerical simulation of systems of multitudinous polygonal blocks. Technical Report 92-22, Cold Regions Research and Engineering Laboratory, CRREL, Hanover, NH, p. 69.

Marchenko, A., Karulin, E., Chistyakov, P., Sodhi, D., Karulina, M., and Sakharov, A., 2014. Three dimensional fracture effects in tests with cantilever and fixed ends beams. 22nd IAHR International Symposium on Ice, Singapore.

Masterson, D.M., 2009. State of the art of ice bearing capacity and ice construction. Cold Regions Science and Technology 58, p. 99.

Munjiza, A., 2004. The combined finite-discrete element method. John Wiley & Sons Ltd.

Määttänen, M., Hoikkanen, J., 1990. The effect of ice pile up on the ice force of a conical structure. Proceedings of the 10th International Symposium on Ice, IAHR, Espoo, Finland, p. 1010.

Paavilainen, J., 2013. Factors affecting ice loads during the rubbling process using a 2D FE-DE Approach. Aalto University publication series, Doctoral Dissertations.

Paavilainen, J., Tuhkuri, J., and Arttu Polojärvi, 2009. 2D combined finite-discrete element method to model multi-fracture of beam structures. Eng Computation 26, 6, p. 578.

Reddy, J.N., 2007. Theory and Analysis of Elastic Plates and Shells. CRC Press.

Schreyer, H. L., Sulsky, D. L., Munday, L. B., Coon, M. D., and Kwok, R., 2006. Elastic-decohesive constitutive model for sea ice, J. Geophys. Res., 111, C11S26
Sodhi, D.S., 1995. Breakthrough Loads of Floating Ice Sheets. Journal of Cold Regions Engineering 9, p. 4.
Szilard, R., 2004. Theories and Applications of Plate Analysis. John Wiley & Sons, Inc.