



KINETIC FRICTION OF ICE SLIDING SLOWLY OVER ITSELF: PHYSICAL MECHANISMS

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ABSTRACT

This paper addresses the coefficient of kinetic friction of relatively warm ice ($-50\text{ }^{\circ}\text{C}$) sliding slowly ($<1\text{ m s}^{-1}$) over itself and presents physical models to account for two characteristics: velocity-strengthening at lower speeds and velocity-weakening at higher speeds. The models are based upon a combination of creep deformation within a near-surface zone and frictional heating cum localized melting of asperities that protrude from opposing surfaces. Both freshwater ice and saltwater ice are addressed. The paper closes with a comment on the role of friction on brittle compressive failure.

1. Introduction

Frictional sliding plays a fundamental role in the mechanics of ice, on scales small and large. Whether across opposing faces of microcracks within laboratory specimens or across strike-slip like features within the sea ice cover on the Arctic Ocean, sliding is central to the process of brittle compressive failure.

Recent experiments on the dynamic or kinetic coefficient of friction of relatively warm ice ($>-50\text{ }^{\circ}\text{C}$) sliding slowly ($<0.1\text{ m s}^{-1}$) over itself have revealed two characteristics. At lower speeds, the coefficient increases with increasing velocity and reaches a maximum at $\sim 10^{-5}$ - 10^{-4} m s^{-1} (Kennedy et al., 2000; Schulson and Fortt, 2012). At higher speeds, the opposite behavior is seen and the coefficient decreases with increasing velocity (Kennedy et al., 2000; Maeno et al., 2003; Lishman et al., 2011; Schulson and Fortt, 2012; Sukharakov and Loset, 2013), reaching a minimum at $\sim 10^{-1}\text{ m s}^{-1}$. At still higher speeds the coefficient again increases with increasing velocity (Oksanen and Keinonen, 1982), owing to hydrodynamic effects that result from interface melting related to frictional heating. Figure 1 illustrates the lower-velocity behavior. Velocity-strengthening and velocity-weakening are observed in both freshwater ice and saltwater ice sliding across both smooth (~ 1 micrometer) and rough (~ 1 mm) interfaces, prepared either mechanically by milling or naturally through the generation of Coulombic shear faults (Fortt and Schulson, 2011), implying that the characteristics are fundamental to ice and are not the result of surface topography. The coefficient of friction of rougher surfaces, however, is somewhat greater, scaling as $\mu_k \propto R_a^{0.08}$ (Schulson and Fortt, 2012). Needless to say, velocity-strengthening and velocity-weakening have practical consequences in relation to ice mechanics, the former leading to stable slip and the latter to unstable slip, for instance, as ice floes and other features slide past each other.

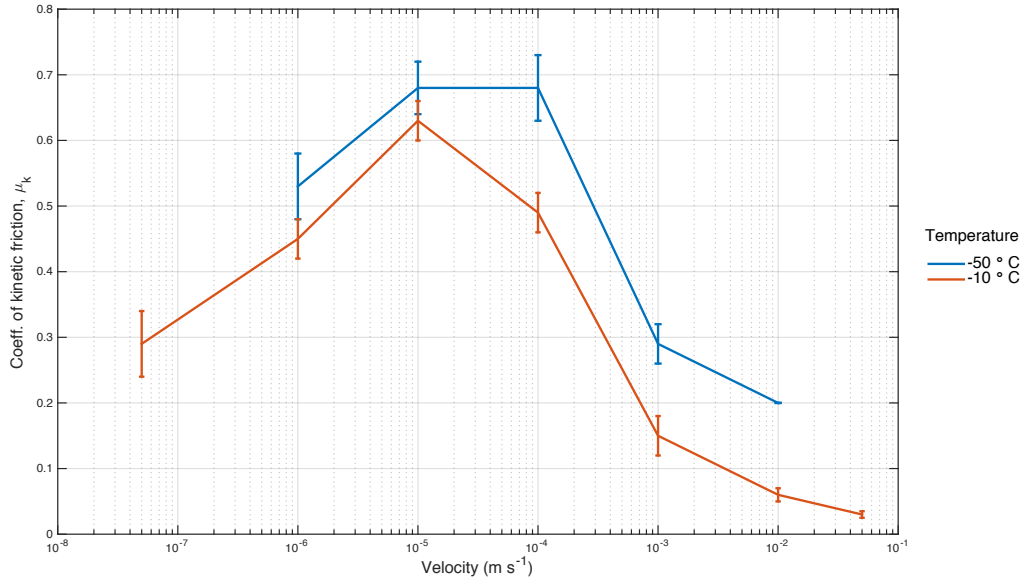


Figure 1: Graphs of the coefficient of kinetic friction at -50°C and -10°C vs. sliding velocity (5×10^{-8} to $5 \times 10^{-2} \text{ m s}^{-1}$), for freshwater ice of ~ 1 micrometer surface roughness; the bar through each point denotes the standard deviation. (From Kennedy et al., 2000 and Schulson and Fortt, 2012)

The question we address in this paper is: what physical processes account for strengthening and weakening?

2. Physical mechanisms

At the outset, it is important to appreciate that the real area of contact is considerably smaller than the nominal or apparent area (Bowden and Tabor, 1950; 1964). Contact occurs not globally across the entire interface, but locally at asperities that protrude from the opposing surfaces. It is these local sites that constitute the real area of contact. The interaction of asperities is then a major contributor to friction.

As the opposing surfaces slide over each other, surface traction develops as the asperities interact, augmented perhaps by dynamic cohesion in warmer ice sliding at lower velocities. The surface traction, in concert with the normal load and other contact forces, activate inelastic deformation within the near-surface regions (Kennedy et al., 2000), evident from microstructural and other changes (microcracking, fragmentation, dynamic recrystallization) that border the interface within $\sim 1 \text{ mm}$ (Barnes et al. 1971; Kennedy et al., 2000; Montagnat and Schulson 2003). The asperities per se also deform inelastically. This irreversible deformation, plus the deformation of fragments of ice created by fracture are important contributors to the kinetic friction of ice (Kennedy et al., 2000). So, too, is frictional heat which, at higher velocities within warm ice, causes localized melting.

2.1 Velocity-strengthening

Velocity-strengthening is a manifestation of rate-dependent, dry sliding and can be understood largely in terms of creep deformation (Barnes et al., 1971; Kennedy et al., 2000). Accordingly, upon defining the coefficient of kinetic friction as the ratio of the shear stress τ_c to maintain a given creep rate $\dot{\epsilon}$ to the normal stress supported by asperities, and taking the normal stress acting across the sliding interface to be limited by the hardness of the ice (Barnes et al., 1971), then:

$$\mu_{k,d} = \frac{\tau_c(\dot{\epsilon}, T)}{H(\dot{\epsilon}, T)} \quad (1)$$

where the subscript d implies dry sliding. Both creep strength and hardness depend upon strain rate $\dot{\epsilon}$ and temperature T . From Barnes et al.(1971):

$$\tau_c(\dot{\epsilon}, T) = B^{-1/n} \dot{\epsilon}^{1/n} \exp(Q / nkT) \quad (2)$$

and

$$H = Ct^{-1/n'} \exp(Q' / n'kT) \quad (3)$$

where t denotes time of contact, Q and Q' , respectively, denote apparent activation energies for the mechanisms governing creep and hardness under the imposed sliding conditions, k is Boltzmann's constant and B , C , n and n' are materials constants. Creep (strain) rate is related to the average sliding velocity (i.e., the velocity imposed, V_s) and to the thickness h of the near-surface inelastic zone through the relationship $\dot{\epsilon} = V_s / h$; time of contact is related to velocity and asperity diameter, $t = 2a / V_s$. The contribution from near-surface creep to the coefficient of kinetic friction may then be expressed by the relationship:

$$\mu_{k,d} = \frac{B^{-1/n} (V_s / h)^{1/n} \exp(Q / nkT)}{C (V_s / 2a)^{1/n'} \exp(Q' / n'kT)} \quad (4)$$

Equation (4) may be viewed as the coefficient of friction for dry sliding (vs wet sliding described below).

To assess this model, we perform the following exercise: First, we limit our attention to strengthening at -10°C . We take the value $h = 0.1 \text{ mm}$, based upon the extent of the deformation damage noted by Montagnat and Schulson (2003) and upon the value suggested by Barnes et al. (1971). Then for ice sliding at a reference velocity of $V_s = 10^{-7} \text{ m s}^{-1}$ the corresponding creep rate is $\dot{\epsilon} = 10^{-3} \text{ s}^{-1}$. To maintain that rate of creep at -10°C , a shear stress of $\tau_c = 4.2 \text{ MPa}$ is required, based upon the creep rate-stress data for freshwater ice shown in Figure 2 of Barnes et al. (1971), taking care to divide the applied stress given in that figure by the factor $\sqrt{3}$ to convert normal stress to shear stress. To obtain a contact time and hence a reference hardness, we take the average contact diameter to be $2a = 30 \mu\text{m}$, derived from an analysis of static strengthening or ageing (Schulson and Fortt, 2013) and again use $V_s = 10^{-7} \text{ m s}^{-1}$. That gives $t = 300 \text{ s}$ for which the corresponding hardness for the same kind of ice is $H = 15 \text{ MPa}$, from Figure 4 of Barnes et al. (1971). The model then predicts $\mu_k(10^{-7}, -10) = 0.28$, in good agreement with the measured value (Figure 1) of $\mu_k = 0.33 \pm 0.05$ under the same conditions for relatively smooth ($R_a \sim 1 \mu\text{m}$) surfaces sliding across each other.

To obtain the velocity dependence for the same kind of ice at the same temperature, we scale the shear stress and hardness noted above using the values $n = 3.0$ and $n' = 4.4$, derived from the experiments of Barnes et al. (1971). Then, at the higher velocities of 10^{-6} and 10^{-5} m s^{-1} the model dictates that $\mu_k(10^{-6}, -10) = 0.36$ and $\mu_k(10^{-5}, -10) = 0.46$, somewhat lower than the measured values under those conditions of 0.45 ± 0.06 and 0.63 ± 0.06 (Figure 1). In other words, the creep-based model indicates that for warm ice sliding over relatively smooth interfaces $\mu_k \propto V_s^{0.10}$, while experiment shows that $\mu_k \propto V_s^{0.13 \pm 0.01}$. Should this difference

($\Delta\mu_k = \mu_{k,measure} - \mu_{k,model} = 0.05$ at 10^{-7} m s^{-1} ; $\Delta\mu_k = 0.09$ at 10^{-6} m s^{-1} ; $\Delta\mu_k = 0.17$ at 10^{-5} m s^{-1}) be significant, it could indicate that in addition to creep ploughing or other inelastic processes contribute to frictional resistance.

Concerning the sliding of colder ice, we imagine that asperities continue to interact and that near-surface regions continue to deform plastically under the combined influence of surface traction and normal loads. Within the creep-based model, the influence of temperature is embodied in the apparent activation energies whose values have been reported (Barnes et al, 1971) to be $Q \sim 0.81 \text{ eV atom}^{-1}$ (78 kJ mol^{-1}) and $Q' \sim 0.75 \text{ eV atom}^{-1}$ (72 kJ mol^{-1}) for temperatures between about -10 and -50 °C. Then, using again the values $n = 3.0$ and $n' = 4.4$, the model predicts that upon lowering temperature from -10 °C to -50 °C, for instance, the friction coefficient is expected to increase by a factor of ~ 2.2 . Experiment, on the other hand, shows that over this range the coefficient increases by a smaller factor of ~ 1.3 (Figure 1). Although the predicted trend is similar to the one observed, the magnitude of the thermal effect is considerably lower than predicted. Perhaps, albeit tentative in the absence of experimental data, the reason for the difference is that the activation for creep in colder ice is lower than noted above, given that in very cold ice Durham et al. (1997) report the value $Q = 0.41 \text{ eV/atom}$ (38 kJ mol^{-1}).

A different interpretation of dry sliding is offered by Makkonen (2012) and Makkonen and Tikanmaki (2014). They present a model based upon thermodynamics in which the sliding resistance is governed not by the shear strength of interacting asperities, but by surface energy, specifically by the energy of surface steps or edges that are created at the nanoscale. For ice sliding upon itself under conditions where frictional heat is insufficient to melt a thin layer at the interface—i.e., dry sliding—they express the coefficient of friction as $\mu_k = \gamma / H\lambda$ where γ denotes the surface energy of ice, H , hardness and λ , a nanoscale contact length. Although data for λ are not available, they assume that this parameter is equal to the size of the smallest cluster of H_2O molecules that is stable in the solid state which, based upon calculations by Pradzynski et al. (2012), is of the order of $\lambda = 2 \text{ nm}$. Taking this value and taking from Table 1 and Figure 2 of Makkonen and Tikanmaki (2014) the values $\gamma = 73 \text{ mJ}^{-2}$ and $H = 60 \text{ MPa}$ (where 60 MPa corresponds to the hardness for an indentation time of $\sim 10^{-4} \text{ s}$ (Barnes et al., 1971) which is about the amount of contact time $t = \lambda / V_c$ to be expected for sliding at $\sim 10^{-5} \text{ m s}^{-1}$ across an asperity $\sim 2 \text{ nm}$ in diameter), the thermodynamic model dictates $\mu_{k,d} = 0.6$ at -10 °C. This prediction is in close agreement with the value measured for sliding across smooth surfaces at $\sim 10^{-6}$ to 10^{-5} m s^{-1} (Figure 1). The problem is that the model does not account for velocity-strengthening. The only parameter in the model that exhibits rate dependence is hardness and that dependence leads to a reduction in the coefficient of kinetic friction and not to an increase with increasing velocity. An increase of two orders of magnitude in sliding velocity, for instance, leads from Equation (4) to an increase of about a factor of two in hardness and to an expected reduction in the friction coefficient by about the same factor.

The thermodynamic model, incidentally, appears not to be compatible with one based upon the thermally activated deformation of asperities. This may be seen as follows: Consider a block of ice of length L , width W and height H . Allow the upper half to be displaced over the lower half by a distance b through the action of a shear force F_s . The shear force does work on the system, given by $F_s b$. The displacement creates two steps, each of area Wb , and thus new surface of energy $2\gamma Wb$ where γ is surface energy per unit area. If the shear force arises owing to a process that operates within the surface zone, as assumed in the shear strength model, then the shear force during sliding is the product of the shear strength τ_s of the interface and the area WL of the sliding surface, $F_s = \tau_s WL$.

Upon equating the work done on the system to the increase in surface energy ($\tau_s W L b = 2\gamma W b$) the shear strength is given by $\tau_s = 2\gamma / L$. This implies that the strength of the interface, and hence the coefficient of kinetic friction, decreases as the length of the block increases. There is no evidence that ice behaves in that manner.

2.2 Velocity-weakening

The discussion so far has focused on dry sliding. At some transition velocity, denoted V_t , sufficient frictional heat is generated and retained at certain points of contact to allow localized melting. It is at that point, we suggest, that velocity-weakening sets in. Beyond that point, progressively more asperities melt. The melt water lowers sliding resistance by serving as a lubricant. The coefficient of friction given by Equation (4) is then attenuated and, subject to the limitation noted below, may be described by the modified relationship:

$$\mu_k = \mu_{k,d}(1 - \eta) \quad (5)$$

where η denotes the fraction of the interface that is wet. (This description ignores the friction of wet patches for which the coefficient, based upon the value for a fully wet interface (Oksanen and Keinonen, 1982), is estimated to be around 0.01-0.02). In other words, with increasing velocity beyond V_t , inelastic deformation may play a decreasing role and localized melting may play an increasing role in sliding resistance.

What is the evidence of localized melting? Marmo et al. (2005), using low-temperature scanning electron microscopy, reported refrozen water on the surface of ice that had rubbed against steel at -3.4°C at 0.02 m s^{-1} . Under those conditions, the coefficients of kinetic friction of ice on steel and of ice on ice have similar values, $\mu_k = 0.1$ - 0.05 . Given that steel has greater thermal conductivity than ice and thus a greater propensity to conduct frictional heat away from a sliding interface, even more melt water is expected when ice slides upon itself. Of greater relevance to the ice-on-ice scenario is the observation (Fortt and Schulson, 2011) that upon rapidly sliding (at 10^{-3} to 10^{-2} m s^{-1}) a short distance following their formation at -10°C , shear faults possessed cohesion that probably developed as sliding-induced melt-water solidified. Of still greater relevance are the tiny, globular shaped features that appeared on the surface of Coulombic shear faults upon sliding at $8 \times 10^{-4}\text{ m s}^{-1}$ at -10°C (Fortt and Schulson 2007). We take those features to be frozen drops of water.

To estimate V_t , we begin by assuming that sliding on the scale of the asperities exhibits spatiotemporal character – analogous, perhaps, to the intermittent character of crystallographic slip within crystals of ice through the action of dislocation avalanches (Weiss and Grasso, 1997; Taupin et al. 2008). Accordingly, we imagine that certain asperities momentarily slip at a local velocity V_l that is greater by a factor f than the average applied velocity, leading to flash heating sufficient to melt the asperities. A thin layer of water of thickness δ is then produced. To avoid freezing, the asperities must slip past each other in time t_s less than the time needed t_c to conduct the heat of fusion into the surrounding ice. The slip time scales with asperity diameter and inversely with velocity and the conduction time scales inversely with the temperature difference ΔT between the surface and the sub-surface. Thus, the larger the asperity and the colder the ice, the faster must the ice slide to prevent the lubricating water layer from freezing.

To find a relationship between ΔT , asperity diameter $2a$ and velocity V_l we proceed as follows: The heat content of the water film is given by $\pi a^2 \delta L_v$ where L_v denotes the latent heat per unit volume. That heat is conducted in time t_c across an interface of area πa^2 and

down a thermal gradient $\Delta T / \Delta z$ on either side of the interface where $\Delta z \sim \sqrt{h_t t_c}$ and where h_t denotes thermal diffusivity given by $h_t = \kappa / \rho C_p$ where κ denotes thermal conductivity, ρ mass density and C_p specific heat. Upon equating the heat content to the heat transferred, it follows that:

$$t_c \sim \frac{\left(\frac{L_v \delta}{2}\right)^2}{\kappa \rho C_p \Delta T^2} \quad (6)$$

where the factor of 2 accounts for conduction into both sides of the interface. The slip time is given by:

$$t_s = \frac{2a}{V_t} = \frac{2a}{fV_t} \quad (7)$$

Upon equating Equations (6) and (7) and rearranging, we obtain the relationship:

$$V_t = \frac{8\kappa \rho C_p a \Delta T^2}{f L_v^2 \delta^2} \quad (8)$$

Thus, $V_t \propto a \Delta T^2$.

It is not easy to assess this model: neither the factor f by which the average velocity is enhanced nor layer thickness δ is known and only an average value of asperity size is available from the analysis of static strengthening by Schulson and Fortt (2013). However, in the interests of a rough assessment, we assume that $\delta \sim 1 \mu m$ and assume further that $f=100$, as might be the case for an avalanche-like de-pinning of asperities and allowing for slip in directions inclined to the direction of macroscopic sliding. (We recognize that such a large value implies a very large strain-rate gradient within the deformation zone.) With those assumptions and upon taking the experimentally-derived average asperity diameter $2a \sim 30 \mu m$ (Schulson and Fortt, 2013) and using the values $L_v = 320 \text{ MJ m}^{-3}$, $\kappa = 2.1 \text{ W m}^{-1} \text{ K}^{-1}$, $\rho = 917 \text{ kg m}^{-3}$ and $C_p = 1900 \text{ J kg}^{-1} \text{ K}^{-1}$, Equation (8) yields the estimate that $V_t \sim 4 \times 10^{-3} \text{ m s}^{-1}$ for ice initially at -10°C (i.e., $\Delta T = 10$). On a logarithmic scale this is about mid-way within the velocity-weakening regime observed in the experimental results shown in Figure 1. However, asperities almost certainly range in size: should that range be $0.1 \mu m \leq 2a \leq 1 \text{ mm}$, the transition velocity would be expected to vary over the range $\sim 10^{-5} \leq V_t \leq 10^{-1} \text{ m s}^{-1}$. In other words, the range of velocity over which velocity-weakening is observed at -10°C may be a measure of the size range of the interacting asperities.

The model implies that the velocity that defines the onset of velocity-weakening in warm sea ice is expected to be about an order of magnitude higher than in freshwater ice. This follows from the fact that the heat capacity of warm sea ice is about an order of magnitude greater than that of freshwater ice (Sakazume and Seki, 1978, Hoyland 2009), owing to the presence of brine (i.e., $C_p \sim 30 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ at -10°C for sea ice of ~ 5 ppt salinity; the values of density, thermal conductivity and latent heat of fusion are similar in both materials). This implication agrees with the experimental results: velocity-weakening upon sliding across Coulombic shear faults at -10°C appears to begin at $\sim 10^{-4} \text{ m s}^{-1}$ in sea ice but at $\sim 10^{-5} \text{ m s}^{-1}$ in freshwater ice (Fortt and Schulson, 2011).

Concerning the role of temperature, if flash heating continues to generate a thin layer of water on certain asperities as temperature decreases, then the model dictates, as already noted, that V_f will increase. A drop from from -10 °C to -50 °C, for instance, is expected to lead to an increase of about a factor of $(50/10)^2 = 25$. The experimental results shown in Figure 1 for sliding across a relatively smooth interface indicate that over this same range of temperature, the velocity at which the coefficient of friction reaches a maximum does indeed increase, but by the smaller factor of ~ 10 . Of greater concern, perhaps, is the apparent absence of an effect of temperature on V_f for sliding across Coulombic shear faults, where the data (Fortt and Schulson, 2011) indicate that over the range from -10 °C to -40 °C temperature exerts little detectable effect. These discrepancies need to be addressed, although they may be more a reflection of uncertainty in the exact value of V_f , owing to the fact that the experiments were performed at relatively coarse increments of sliding velocity.

Returning to the coefficient of friction and to Equation (5), we do not have a measure of the parameter η . We know neither the asperity size distribution nor the number and area of the wet patches versus sliding velocity. As a result, we do not know by what fraction the interacting asperities is reduced. Further development thus requires more work.

Oksanen and Keinonen (1982) present a different interpretation of velocity weakening, albeit of weakening over the higher velocities of 0.5 to 3.0 m s⁻¹. Over that range, the coefficient of kinetic friction scales as $\mu_k \propto V_s^{-1/2}$ and has the value, for instance, of ~ 0.02 at -15 °C at 1 m s⁻¹. This behavior is explained not in terms of the deformation of asperities, but in terms of heat conduction away from the sliding interface where a thin layer of water is assumed to exist and to be self-balanced—self-balanced in the sense that “increasing frictional heat would melt more water and if the water thickness increases the reduction in frictional heat would cause a temperature drop at the contact below the melting point of water (sic) and the heat produced by friction is equal to the heat conducted into the two solids”. While Oksanen and Keinonen’s model accounts well quantitatively for their observations and for those of Evans et al. (1976) that were obtained under similar conditions, this explanation cannot account for the velocity-weakening apparent in Figure 1, or under the lower-velocity conditions under which the data in that figure were obtained, $V_s^{-1/2}$ functionality would imply a friction coefficient of ~ 3 at a velocity of 10⁻⁴ m s⁻¹ at -15 °C compared with the measured value of ~ 0.5 (albeit at -10 °C). (The difference of 5 degrees could not account for this discrepancy, given the relatively small effect of temperature implied by the data in Figure 1.) In other words, our sense is that there may be two regimes of velocity-weakening of warm ice, one that operates over lower velocities and results from a combination of inelastic deformation and localized melting and another that operates over higher velocities and results from thermal processes alone.

Absent from the above discussion is the liquid-like layer (for review, see Dash et al., 2006). Although that feature may play a role in the development of cohesion when, within the spatiotemporal context of sliding, asperities are momentarily static, it cannot account for either velocity-strengthening or velocity-weakening.

3. Friction and brittle compressive fracture

It is interesting to note that, just as the coefficient of friction falls from a maximum to a minimum value over about four orders of magnitude of applied velocity, the brittle compressive strength of ice falls from a maximum to a minimum over about four orders of magnitude of applied strain rate (for review of strain-rate softening of ice, see Schulson and Duval, 2009, Chapter 11). We think this correlation is not fortuitous. As noted in that review, frictional sliding underlies brittle failure and in the mechanistic-based relationships described therein the unconfined compressive strength σ_c scales as $\sigma_c \propto 1/(1 - \mu_k)$ for $\mu_k < 1$. This

means that a factor of ten drop in the coefficient, from a maximum of $\mu_k = 0.5$ to a minimum of $\mu_k = 0.05$ at -10°C , is expected to lead to a reduction in strength by a factor of two. This expectation is in good agreement with experiment (Carter, 1971; Schulson, 1990) which shows that the compressive strength of freshwater granular ice of ~ 1 mm grain size, although scattered, falls by about the same factor within the strain-rate weakening regime ($\dot{\epsilon} \sim 10^{-4} \text{ s}^{-1}$ to $\sim 1 \text{ s}^{-1}$ at -10°C).

The other correlation between friction and fracture pertains to very high-rate sliding and deformation. As noted in Section 1, at sliding velocities above about 0.1 m s^{-1} at -10°C the coefficient of kinetic friction increases with velocity, from $\mu_k = 0.020$ at 1 m s^{-1} to 0.025 at 3 m s^{-1} , scaling roughly with $\mu_k \propto V_s^{1/2}$; correspondingly, at strain rates above $\sim 1 \text{ s}^{-1}$ the brittle compressive strength of warm ice increases with strain rate (Jones, 1997; Shazly et al., 2006), through the relationship (Schulson and Duval, 2009) $\sigma_c = 9.8\dot{\epsilon}^{0.14} \text{ MPa}$ (for strain rate in units of s^{-1}). These are small effects and the scatter in the dynamic compressive strength is too large to allow meaningful quantitative comparison with $1/(1 - \mu_k)$ functionality. Nevertheless, the qualitative similarity hints at a fundamental link. Should frictional sliding be truly a significant factor in the compressive strength of ice under conditions of dynamic loading, fresh fracture surfaces would be expected to be wet.

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