



LOSSES FROM FAILURE: RAMS ANALYSIS IN EXTREME COLD OPERATING CONDITIONS

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ABSTRACT

Offshore field development, especially in the Arctic region, is a complex activity involving risks and uncertainties from a wide range of sources. The conventional *RAMS* (Reliability, Availability, Maintainability and Safety) analysis has been used, for a long time, as a performance measure of the system. Moreover, it has been applied to identify weaknesses and select the more reliable system. However, choosing the more reliable system does not always mean that less losses from its failure. The purpose of this paper is to modify and adapt a methodology for risk-based *RAMS* analysis of a production system, based on losses from failures, by considering the effect of the extreme cold operational conditions. The adapted methodology uses risk analysis as a key component for the *RAMS* analysis. The potential sources of uncertainties and risks involved in the *RAMS* analysis has been assessed and identified. Further, the losses from failures have been estimated, for case-specific design features, by considering the effect of operational conditions.

Keywords: Arctic, Cold region, Losses from Failure, Operational conditions, Risk, *RAMS* Analysis, Uncertainties

1. INTRODUCTION

Exploration of new areas such as the Arctic region for more petroleum production has driven by an increased demand of petroleum in the world. Barents Sea is one of the Arctic areas where the Norwegian oil and gas industry has been focusing on the exploration and development of oil and gas fields. However, there are several complex challenges when the offshore industry expands into High North compared to the well-established practices of exploration and production in the Norwegian Continental Shelf (*NCS*) (Homlong, 2010, Ayele et al., 2013). Due to Arctic operational environmental factors such as large variations in temperature during a short period of time, sudden wind increase and large changes in wind direction, icing, snow, and inadequate weather forecasting, it is expected that the uncertainty will be magnified and the risk involved will be much higher than North Sea (Barabadi et al., 2009, Ayele et al., 2013, Barabadi et al., 2012). Furthermore, petroleum production activities in the Arctic region may face unforeseen challenges which also increase the uncertainty and the risk involved (Markeset, 2008, Barabadi et al., 2012). It is then important to identify and assess all influence factors which can affect the production performance and safety of the

system. Further, it is relevant to understand how the uncertainties are involved in the *RAMS* (Reliability, Availability, Maintainability and Safety) assessment process such that they can be taken into consideration as a decision support.

For a long time, the conventional *RAMS* analysis has been oriented towards selecting the more reliable system and preoccupied with maximizing the reliability of systems (Todinov, 2007). However, selecting the more reliable system does not necessarily mean selecting the system with the smaller losses from failures (Todinov, 2007). Further, failure of a system always exists, and hence there might be losses associated with these failures. There is no guarantee that failure of the more reliable system contributes to less losses. Therefore, choosing the more reliable system does not always mean less losses from failures. It is then important to consider the losses given failures of the system and do the risk analysis based on these losses and combine with the uncertainty analysis to make better decisions. As a result, *RAMS* analysis should necessarily be risk-based, that is, it should be associated with the losses from failures. Minimizing the overall risk profile and increasing production performance of these complex projects is the main objective of risk-based *RAMS* analysis.

Risk-based approaches encourage a deeper understanding of the risks associated with the failures of purpose-built offshore facilities than is possible under a generic *RAMS* analysis. By analyzing case-specific design features and identifying and quantifying the unique risks involved, the industry can take appropriate measures to mitigate those risks. Furthermore, taking appropriate mitigation measures could improve system reliability and reduce losses from failure. The risk-based approach considering *RAMS* analysis plays an important role to optimize the production performance by dealing with losses from failures and uncertainties. That is, uncertainty and risk analysis must be integrated with *RAMS* analysis in order to reduce the losses from failure and to ensure the performance requirement.

The rest of the paper is organized as follows: Section 2 introduces an overview of *RAMS* and sources of uncertainty. Section 3 presents losses from failures. Section 4 discusses an illustrative case study. Section 5 provides the conclusion.

2. OVERVIEW OF BASIC RAMS PRINCIPLES AND UNCERTAINTY

In this section, an overview of basic *RAMS* principles and uncertainty is presented. It is relevant to consider the uncertainty analysis and combined with the risk, which can be related to the uncertainty regarding the outcome of the event, to be able to support the *RAMS* analysis to make better predictions.

2.1. Introduction to Basic RAMS Principles

RAMS is a central element in many different application fields, which are ultimately linked to the study of the failure, maintenance and availability of systems. The aim of *RAMS* is to generate input data in a life cycle of a system so that based on these data one can assess capability of a system. That is, it provides data on failure rates of the system, possible failure modes, mean down time (*MDT*), maintenance operations, hazards and their consequences, etc (Simões, 2008). In the Arctic operational conditions, even though there are several factors that can significantly affect the *RAMS* of a system, there is a lack of both data and experience related to operation and design of petroleum production facilities. Hence, it is difficult to get exact available information about operational conditions in the Arctic, and hence leads to

increase uncertainty and risks related to health, safety and environment (*HSE*) (Barabadi et al., 2011).

Therefore, it is relevant to consider the uncertainty analysis to investigate the uncertainty of variables that are used in decision-making problems. In other words, uncertainty analysis aims to make a technical contribution to decision-making through the quantification of uncertainties in the relevant variables. Hence, it has a positive contribution to the risk, which can be related to the uncertainty regarding to the outcome of the event.

There are different methods that have been using to estimate the parameter uncertainty. Each of the methods has then their own advantage and limitation, and thus, it is important to choose the best suitable method based on the given conditions.

2.2. Sources of Uncertainty and Methodology

Uncertainty can be classified as model, parameter (data), and incompleteness (Drouin, 2009). Model uncertainty is rely on the validity of model assumptions. It happens because the models and their assumptions are not always valid or perfect due to the limitations in including the natural variability in the real life system.

- **Model uncertainty** reflects the inability of a model and or design technique to represent precisely the system's true physical behaviour, and therefore, it will, up to a certain degree, always exist.
- **Parameter uncertainty** can be occurred due to inability to quantify the inputs and parameters of the model. In the Arctic region, data availability is limited. Thus, the lack of data, and hence *RAMS* models use assumptions to overcome this, leads statistical uncertainties in the estimated parameters, and will be reflected in the final results. Generic databases are established to provide data for *RAMS* analysis, but they also introduce uncertainty due to lack of relevance.
- **Incompleteness uncertainty** is another type of uncertainty, which is either known or not known during in the course of *RAMS* assessment. However, both of them are not included in the assessment, off course, the unknown is unknown. The known uncertainties may be due to omission of factors, like failure modes, assumed to be negligible for the assessment's results or outside the scope, whereas, the unknown uncertainties may be due to lack of knowledge (in particular, in the Arctic region), like the exclusion of unknown failure modes, or from interaction between foreseeable events.

Thus, the potential sources of uncertainties so as to predict *RAMS* of the system (equipment) in the Arctic region based on the available data such as *OREDA* can be summarized as (Barabadi et al., 2011):

- "Limited field data and information about the surrounding environment and failure data in the Arctic region (non-representative of historical data).
- Random error in measuring the time to failure(*TTF*) or time between failures (*TBF*) (measurement errors).
- Inconsistency and non-homogeneity of *TBF* or *TTF* data.
- Systematic bias due to miscalibration of device.

- Misclassification or handling and transcription error in the filed data.
- Lack of human failure data during operation and maintenance process.
- Estimating the uncertainty for unobserved systems in the Arctic region.’’

As mention above, it is important to select the appropriate method to do the uncertainty analysis based on the given condition. Uncertainty analysis can be analytical and simulation. Some of the powerful methods that have been using to describe the uncertainty parameter and to quantify the uncertainties are Monte Carlo (probabilistic) (Doubilet et al., 1984, Doucet et al., 2001) and *FAC* (Fuzzy Alpha-Cut) (non-probabilistic) (Yang et al., 2008, Wong et al., 2000). Both methods are different either interms of characterizing the input parameter uncertainty or ways of propagating from parameter level to model output level. That is, fuzzy logic and probability are different methods that have been used to express uncertainty (its propagation). However, Monte Carlo method is time consuming technique, it provides better output results compared to *FAC* for more than one uncertain input variable (Abebe et al., 2000). Whereas, for a single or limited input variables, *FAC* provides better option than Monte Carlo, and *FAC* is fast. As a result, the uncertainty analysis may provide an important result for decision-makers, and hence for the risk analysts.

The risk analysts should use risk assessment methods to point out the most accurate result. Risk estimation involves the use of identified failure or hazardous data to estimate possible consequence and overall risk level using combination of qualitative and quantitative methods. Risk estimation process begins with the estimation of consequences of each failure event using qualitative methods if the identified event may not be readily quantifiable. However, if the level of uncertainty is very high, subjective safety analysis methods such as fuzzy reasoning approach which has the ability to deal with uncertainty may prove to be more appropriate in executing this task (Homlong, 2010). Some of the risk assessment methods are Preliminary Hazard Analysis (PHA), Hazard and Operability (HAZOP) Analysis, Failure Modes and Effects Analysis (FMEA), Fault Tree Analysis (FTA), Event Tree Analysis (ETA), etc. Risk matrix can also be used for qualitative analysis of risk.

3. LOSSES GIVEN FAILURES AND PROBABILISTIC RISK ASSESSMENT

3.1. Losses given Failures

In the Arctic, failure is always a threat for petroleum production activities. Every engineered system (component or equipment) will fail sooner or later, even with the best design, construction, maintenance, and operation (Blischke and Murthy, 2011). Hence, the frequency of failure can significantly increase in the Arctic operating condition due to several factors such as low temperature, ice, snow, etc. This may also increase losses associated with failures depending on severity of damage. The losses from failures are remarkably high for many production systems, in particular, in harsh operating condition, due to the complex challenge from the Arctic environment. For example, major components of the losses from failures for oil and gas production systems are the amount of lost production which is directly related to the amount of lost production time, the cost of mobilisation of resources and intervention, and the cost of repair or replacement (Todinov, 2007). A critical failure in a deep-water oil and gas production system, in particular, is required long downtimes and extremely high costs of lost production and intervention for repair. Furthermore, due to the Arctic sensitive environment to disruption, on one hand, but harsh and unforgiving on the other the environmental impacts can take longer to heal and cost more to remediate. In addition, such

failures can have disastrous effects on the company profile. They can be represented in number of fatalities, lost production time, volume of lost production, mass of released harmful chemicals into the environment, lost customers, warranty payments, costs of mobilisation of emergency resources, insurance costs, etc (Todinov, 2007).

Losses from engineering failures can be classified as (Todinov, 2007):

- “Loss of life or damage to health
- Losses associated with damage to the environment and the community infrastructure
- Financial losses including loss of production, loss of capital assets, loss of sales, cost of intervention and repair, compensation payments, penalty payments, legal costs, reduction in benefits, losses due to change of laws, product liability, cost overruns, inflation, capital costs changes, exchange rate changes, etc.
- Loss of reputation including loss of market share, loss of customers, loss of contracts, impact on share value, loss of confidence in the business, etc.”

3.2. Probabilistic Risk Assessment

Generally, the risk of failure (expected loss or the classical risk), R_{fr} , is expressed as:

$$R_{fr} = P_{fr} \times C_{fr}, \quad (1)$$

where, P_{fr} is failure probability and C_{fr} is cost given failure.

For instance, the cost given failure to an operator of production equipment C_{fr} , based on Todinov (2007), may include: cost of lost production, cost of cleaning up polluted environment, medical costs, insurance costs, legal costs, costs of mobilisation of emergency resources, cost of loss of business due to loss of reputation and low customer confidence, etc, and the cost of failure to the manufacturer of production equipment may include: warranty payment if the equipment fails before the agreed warranty time, loss of sales, penalty payments, compensation and legal costs.

From (1), the probability of failure at time t is given by:

$$P_{fr} = \frac{R_{fr}}{C_{fr}} \quad (2)$$

To determine the probability of failure at time t for risk-based Arctic RAMS analysis, we have adapted the Todinov's (2007) methodologies. Let R_{fmx} be the maximum acceptable risk of failure and P_{fmx} be the corresponding maximum acceptable probability of failure at time t . Then, equation (2) can also be presented as:

$$P_{fmx} = \frac{R_{fmx}}{C_{fr}}. \quad (3)$$

Therefore, for a system consist of only one component, requirements of system reliability can be expressed using (3). That is, the minimum reliability, R_{mn} , of the component required to keep the risk of failure at time t at least equal to the maximum tolerable risk R_{fmx} can be determined as:

$$R_{mn} = 1 - P_{fmx}$$

$$= 1 - \frac{R_{fmx}}{C_{f_r}} \quad (4)$$

Equation (4) should then be used for the Arctic *RAMS* analysis. It does basically show that so as to maintain the risk of failure below the maximum tolerable level, R_{fmx} , a component whose failure is associated with large losses should be more reliable compared to a component whose failure is associated with smaller losses (Todinov, 2007). This is, in particular, the root of the risk-based design for Arctic *RAMS* analysis of the system which even consists of identical components in its hierarchy. It is obvious that more production units can be affected due to failure of the critical or higher component of the system in the hierarchy. Therefore, it is relevant to consider the required minimum reliability level of this component to be large.

Furthermore, a new measure of the loss from failure in addition to the classical risk equation has adapted from Todinov (2007) for the Arctic *RAMS* analysis. The method considers the unexpected loss instead of the expected loss from failure. Losses from failure can further be categorised as potential loss and conditional loss (Todinov, 2007). The concepts of potential loss and conditional loss are introduced because the classical risk equation only estimates the average value of the potential loss from failure. Thus, a new measure of the loss from failure which avoids the limitations of the classical risk measure is incorporated and it is the cumulative distribution of the potential loss. Potential loss is a loss related to a single and only premature failure, which can happen before the a specified time t . That is, it is unconditional quantity. Potential losses can be associated with multiple failures in the time interval $(0, t)$. However, conditional loss is a loss given that failure has occurred. Hence, it is conditional quantity. Both potential loss and conditional loss can be used for non-repairable and repairable systems whilst the concept potential losses can only be for repairable systems. Furthermore, both the conditional loss and the potential loss can be random variables. The distribution of the conditional loss can only be determined using historical data which is related to the losses from failures, whereas the distribution of the potential losses requires an estimate of the probability of failure in addition to the historical data.

According to Todinov (2007), the distribution function $F(x) = P(X \leq x)$ of the potential loss can be expressed as the probability that the potential loss X will not be greater than a specified value x . The probability $F(x)$ that the potential loss X is not greater than x is then expressed as a sum of the probabilities of two mutually exclusive events, that is:

$$F(x) = P(X \leq x) = (1 - p_{f_r})H(x) + p_{f_r}F(x | f_r), \quad (5)$$

where

- f_r is the given failure.
- p_{f_r} is the probability of failure.
- $H(x)$ is the Heaviside unit step function representing the conditional probability that the loss is not greater than x given that no failure.
- $F(x | f_r)$ is the conditional probability that the loss is not greater than x given failure.

The conditional distribution of the loss given failure (the conditional loss) is then given by:

$$F(X \leq x | f_r) = \sum_{k=1}^N p_{k | f_r} F_k(x | f_r), \quad (6)$$

where

- $F_k(x | f_r)$ is the conditional distribution of the loss given failure characterised by the k^{th} failure mode.

- $p_{k|f_r}$ is the conditional probability that given failure, the k^{th} failure mode has initiated the failure first ($\sum_{k=1}^N p_{k|f_r} = 1$).
- N is number of system components arranged in series with mutually exclusive failures, and characterized by N mutually exclusive failure modes.

The distribution of the potential loss associated with mutually exclusive failure modes is then:

$$F(x) = P(X \leq x) = (1 - p_{f_r})H(x) + \sum_{k=1}^N p_k F_k(x | f_r), \quad (7)$$

where p_k is the probability that the k^{th} failure mode will initiate failure in the time interval $(0, t)$, and it is given by :

$$p_k = \int_0^t f_k(s)[1 - F_1(s)] \dots [1 - F_{k-1}(s)][1 - F_{k+1}(t)] \dots [1 - F_N(t)] ds \quad (8)$$

where $F_k(t)$ and $f_k(t)$ are the distribution and density functions respectively for the times to failure characterising N statistically independent failure modes.

Furthermore, the probability that the potential loss will exceed a specified critical quantity x can be determined as:

$$P(X > x) = 1 - F(x) = \sum_{k=1}^N p_k [1 - F_k(x | f_r)] \quad (x > 0) \quad (9)$$

$$= p_{f_r} \sum_{k=1}^N p_{k|f_r} [1 - F_k(x | f_r)] \quad (10)$$

In addition, the probability that given failure, the loss will be larger than a specified limit is given by

$$P(X > x | p_{f_r}) = \sum_{k=1}^N p_{k|f_r} [1 - F_k(x | f_r)]. \quad (11)$$

Therefore, from (10) and (11), the probability that the conditional loss will exceed a specified quantity is always greater than the probability that the potential loss will exceed the specified quantity, that is, $P(X > x | p_{f_r}) > P(X > x)$. For example, this specified quantity can be the optimal value which the company will be able to pay.

The expected value of the potential loss from failures C is then:

$$C = \int x f(x) dx = \sum_{k=1}^N p_k C_k(x | f_r), \quad (12)$$

where $C_k(x | f_r) = \int x f_k(x | f_r) dx$ are the expected values of the loss given that failure has occurred, characterising the individual failure modes.

For example, based on failure modes characterised by constant hazard rates λ_k , and using (12) and regarding the expected loss given failure, the risk (the expected value of the potential loss) is:

$$\begin{aligned} R_{f_r} &= [1 - \exp(-\sum_{k=1}^N \lambda_k t)] \sum_{k=1}^N p_{k|f_r} C_k(x | f_r) \\ &= [1 - \exp(-\sum_{k=1}^N \lambda_k t)] \sum_{k=1}^N \left(\frac{\lambda_k}{\sum_{k=1}^N \lambda_k} C_k(x | f_r) \right), \end{aligned}$$

where the sum

$$\sum_{k=1}^N \left(\frac{\lambda_k}{\sum_{k=1}^N \lambda_k} C_k(x | f_r) \right)$$

is representing the expected conditional loss (given that failure has occurred before time t). Further more, for a non-repairable system with the hazard rate $\lambda(t)$ depending on time, and the probability of failure:

$$1 - \exp\left(-\int_0^t \lambda(s)ds\right)$$

before time t , the risk is then given by:

$$R_{f_r} = C = \left(1 - \exp\left(-\int_0^t \lambda(s)ds\right)\right) \sum_{k=1}^N p_{k|f_r} C_k(x|f_r). \quad (13)$$

4. AN ILLUSTRATIVE CASE STUDY

The case study is based on two simple systems and both were operating in the Arctic environment. Each of them is consisting of three components which are connected logically in series. Figure 1 shows the two systems with three components each, and all components are connected in series and characterized by a constant failure rate. The assumed data for both systems are presented in Table 1. The components of the first system are C_{11} , C_{12} and C_{13} , and for the second system are C_{21} , C_{22} and C_{23} as shown in Figure 1. Component C_{11} has failed on average twice a year and the losses associated with its failure are 3000\$. Component C_{12} has failed on average 9 times a year and the losses associated with its failure are 200\$, and component C_{13} has failed on average 2 times a year and the losses associated with it are 2000\$.

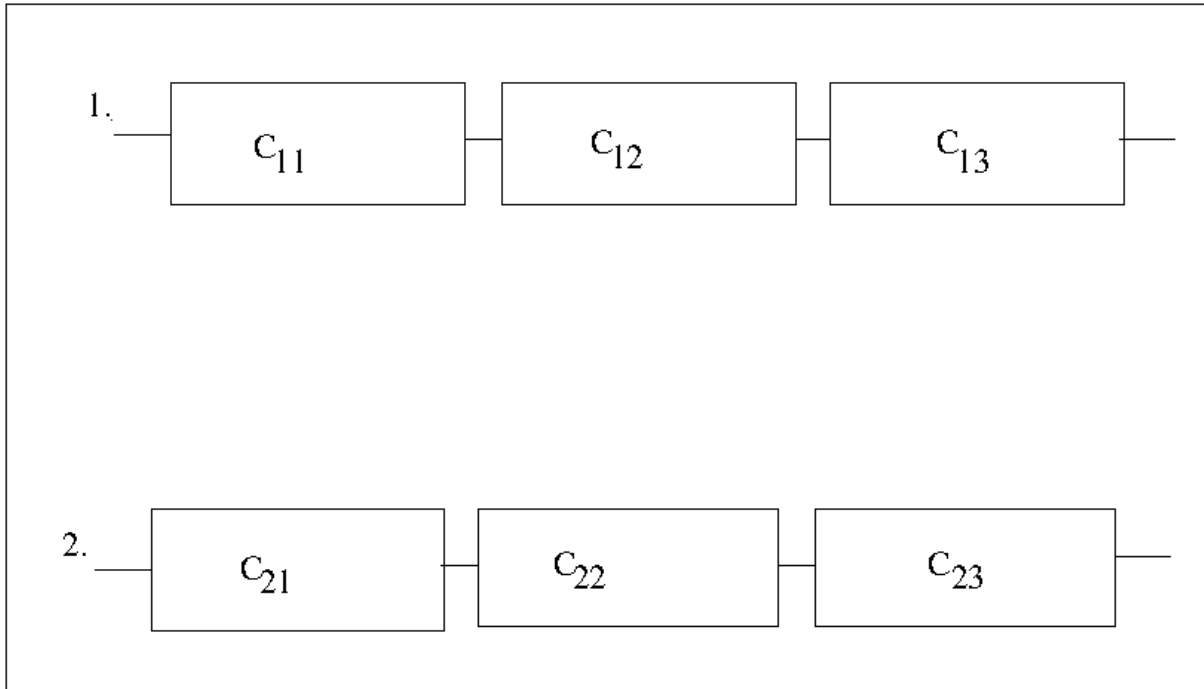


Figure 1: Two systems with two components each

The components of the second system are the same type with that of the first one, and have the same losses associated with failure of the corresponding components as the assumed data have shown. However, the probabilities of failure of both components correspondingly are different. That is, component C_{21} has failed on average 3 times a year, component C_{22} has failed on average 4 times a year and C_{23} has failed on average 3 times a year. It is clear that the first system will fail whenever either component C_{11} or component C_{12} or C_{13} fails because they are connected in series, and the same is true for the second system.

Table 1: Assuemed $MTTF_{ij}$ per year and Losses from failures (C_j)

	$MTTF_{ij}$ per year	Losses from failures (C_j)
C_{11}	2	3000
C_{12}	9	200
C_{13}	2	2000
C_{21}	3	3000
C_{22}	4	200
C_{23}	3	2000

Based on the conventional *RAM* analysis, the second system is more reliable than the first system because it has failed 10 times on average per year as compared to the first system, which has failed 14 times on average per year. However, the result is different based on the risk analysis model. That is, the total expected loss (EL) from failure of system one is:

$$\begin{aligned}
 EL_1 &= MTTF_{11} \times C_1 + MTTF_{12} \times C_2 + MTTF_{13} \times C_3 \\
 &= 2 \times 3000 + 9 \times 200 + 2 \times 2000 = 11800\$.
 \end{aligned} \tag{14}$$

And the total expected loss from failure of second system is:

$$\begin{aligned}
 EL_2 &= MTTF_{21} \times C_1 + MTTF_{22} \times C_2 + MTTF_{23} \times C_3 \\
 &= 3 \times 3000 + 4 \times 200 + 3 \times 2000 = 15800\$.
 \end{aligned} \tag{15}$$

From (14) and (15), based on the risk model, it is demonstrated that the expected loss associated with failure of the more reliable system, second system, is larger than the first system.

This illustrative case study verifies that it can be misleading if a system is chosen only based on its reliability. Thus, a system with larger reliability does not always mean a system with smaller losses from failures if system failures are characterized by different costs. However, a system with larger reliability does mean a system with smaller losses from failures if system failures are characterized by same costs. Hence, components associated with large losses from failures should be designed to a higher reliability level.

5. CONCLUSION

The conventional *RAMS* analysis has been using for a long time as a performance measure of the system. However, choosing the more reliable system does not always mean that less losses from its failure. In this paper, risk-based *RAMS* analysis methodology has been adapted, to improve the reliability performance analysis of a production system in the Arctic operating condition. Further, the basic *RAMS* principles and the main uncertainties involved in the *RAMS* analysis has been discussed. A simplistic case study has been presented, to estimate the total expected loss (EL) from failure of system deployed in the Arctic region. The result of the illustrative case study has demonstrated that the system with larger reliability *does not always mean* the system with smaller losses from failures if system failures are characterized by *different* costs. However, the system with larger reliability *does mean* the system with smaller losses from failures if system failures are characterized by *same* costs. That is, the system with large reliability is not always described by smaller losses from failures. Hence, a risk-based approach considering *RAMS* analysis play an important role to optimize the production performance by dealing with losses from failures and uncertainties. Therefore, *RAMS*

analyses related to production systems, in particular in the Arctic region, is essential in strategic decision.

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