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ICE INDUCED FREQUENCY LOCK-IN VIBRATIONS - CONVERGING TOWARDS CONSENSUS

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ABSTRACT

A moving ice field or ice feature hitting a fixed offshore structure will cause different ice load actions as ice can be breaking in different ways depending on ice velocity. At low ice velocities loads are pseudo static or intermittent load fluctuations, and at high ice velocities exhibit random variations. In between is a velocity range where the most severe dynamic ice loading on structure may occur: the frequency lock-in range where ice failures repeat continuously close to a natural frequency in a resonant type of a loading. The capricious frequency lock-in vibrations were encountered already 50 years ago, but even now, the true understanding of the ice-structure interaction based on physical and mechanical properties of both the ice and the structure are not fully understood. Recent advances in scale model testing and numerical analysis of measurement results provide deeper insight on what is really happening in dynamic ice-structure interaction. This development provides a refined understanding on the phenomena during ice failure process against the structure, and how the structure is responding. The final proof is still ahead: acquiring dedicated full-scale data to verify the findings of scale-model tests and the predictions of numerical analysis.

INTRODUCTION

Moving ice fields initiated severe structural vibrations to the first Cook Inlet offshore structures in Alaska over 50 years ago. These events were investigated scientifically and results published, Peyton 1966. Later similar dynamic ice-structure interactions occurred at different locations around the world. However, only few full-scale measurement data sets - all with limited instrumentation - have been captured, e.g. Peyton 1965, Määttänen 1975, Xu et al. 1981. To understand the physics of dynamic ice-structure interaction many scale model tests have been carried through in ice tanks. However, most of these measurements have had only a single degree of freedom vibration model to present real structures with a plethora of degrees of freedom. As ice failure during dynamic ice structure interaction occurs very fast - including many different ice load frequency components - it is obvious that a SDOF model cannot imitate the true ice-structure interaction. Hence ice researchers have developed handicapped theoretical models that are based on these measured truncated data sets. The true coupling to the underlying physical and mechanical principles during dynamic ice-structure interaction needs more comprehensive test set-ups. Various - and even strongly contradicting - models and explanations have been presented, from simple mechanical models to nonlinear self-excited theoretical vibration models. Now, after various scale model tests, limited full-scale measurement data sets, and with improved data analysis methods, a cohesive understanding of the ice-induced vibration is emerging.

This paper is an extension to my earlier papers, Määttänen, 2014a and b, describing the history of dynamic ice structure interaction during 50 years. Here the perspective is on how the originally divergent explanations have started to converge into a coherent and scientifically justified common understanding.

REPORTED ICE-STRUCTURE INTERACTION EVENTS

The Cook Inlet oil/gas production structures in the Cook Inlet, Alaska, at 1960´s, were the first offshore structures that instead of constant ice crushing loads exhibited significant dynamic response while level ice was steadily moving and crushing against the platform legs. The measurement data indicated dynamic loads both close to the lowest natural frequency of the structure as well at other frequencies. The analysis of dynamic ice action events led to a explanation that ice has a tendency to break at certain frequency, Peyton 1966.

The transition from concrete caisson lighthouses to mono-pile steel structures in the Gulf of Bothnia, Finland, at the beginning of 1970's, brought with severe dynamic response modes in the structures whenever level ice was moving. Observations and measurements verified that the frequency of ice load failures was close to the first or second natural mode frequency of the structure. Such a dynamic ice action caused soon fatigue failures in the superstructures, Fig. 1a. The problems were solved after the design was based on structural fatigue, or having superstructures isolated from the foundation vibrations by a vibration isolation section, Fig. 1b, Määttänen 1987.

Fig. 1a Fatigue failures in Kemi-1, 1973 2b.Vibration isolation in Kemi-2, 1981

Severe ice induced vibrations have been recorded at the Norströmsgrund Lighthouse in the Swedish side of Gulf of Bothnia, Engelbrektson 1983. For comparison, a close by located almost similar lighthouse Kemi-1 Lighthouse in the Finnish side never had significant ice induced vibrations even though ice conditions are slightly more demanding. This proves the peculiar nature of ice-induced vibrations. The Nordströmsgrund data has been analyzed in many projects and numerical dynamic ice-structure interaction models have been tuned based on this databank, e.g. the model by Kärnä , 1992...1999.

The Bohai Sea in China has sub-arctic ice conditions with ice driven at diurnally varying tidal current velocities. This provides favourable combinations for ice thickness and velocity dependent ice-induced vibrations. First ice induced vibration events were reported in 1971 and 1973, Xu et.al. Due to shallow sea and marginal ice fields these production platforms are lightweight mono-piles or multi-legged structures. The first of these have persisted already over 30 years. Mostly the dynamic response has been only un-convenience to the operating personnel. Only few fatigue failures have occurred in secondary structures.

In addition to resonant type crushing loads against vertical piles, continuous and sometimes also resonant type ice bending failure related loads were encountered with structures that had small cones at waterline to avoid ice induced vibrations. This repeating cone ice failure load is basically a pure forced vibration excitation. The ice failure rate is not normally coupled to elastic movements of the structure. It is almost totally the result of ice velocity and cone angle, as the ice failure occurs after the cone has raised ice edge high enough to cause ice sheet bending failure. If the ice velocity is such that the ice bending failures occur close to a structures natural frequency, a resonance state is achieved and response amplitudes grow. This is a forced vibration case, not a self-excited type ice failure against a conical structure, as the latter would require an impractically flexible cone support, e.g. the elastic cone movement under ice action should be of the same magnitude as the ice sheet bending failure breaking radius, typically over 4-times the ice thickness.

The previous experience suggested that the ice induced frequency lock-in vibration is only a problem for slender offshore structures. Completely different type of construction is Molikpaq, a low-rise wide annular steel structure with a sand core as stabilizing ballast. When a thick multi-year ice field ice hit in 1986 and started to crush against wall of the Molikpaq, Fig. 2a. It produced repeating severe ice load fluctuations that hit close to a natural frequency, April, 1986, Jeffries et.al. 1988. The instrumentation in the Molikpaq proved that the normally uncorrelated local area ice crushing events started to synchronize, the whole 80 m long contact area exhibited in phase ice pressure variations and deflections, Fig. 2b. The frequency varied but sometimes persisted close to that of a low natural mode. The structural damping was high as the structure itself is low and wide, it lies on sand berm, and the inner part of annular structure is also filled with sand. Continuously repeating in-phase ice failure cycles persisted only relatively short times. Dynamic response can be explained more likely by ice failure spatial synchronization than by a self-excited structural response.

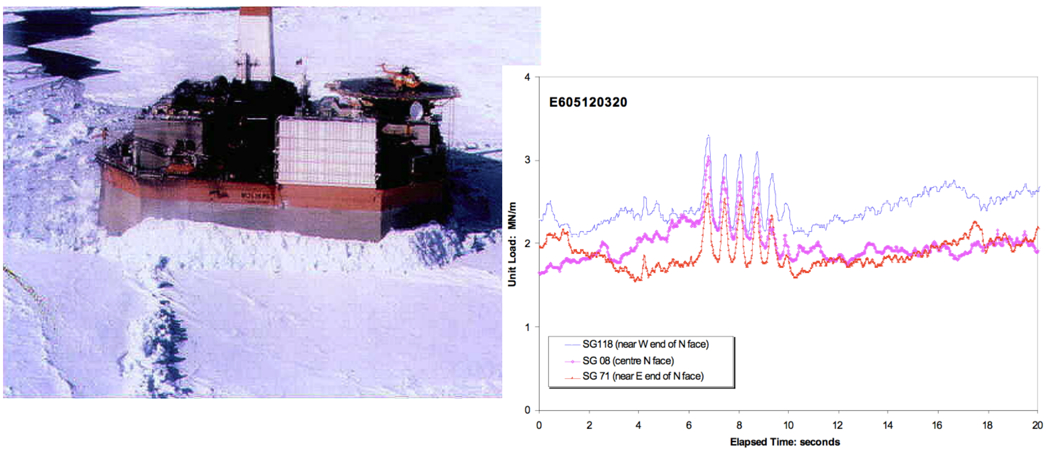

Fig. 2a. Molikpaq and multiyear ice action 2b. Ice load synchronization along the side wall.

Multi-legged Sakhalin platforms offshore Hokkaido exhibited unexplained ice induced vibrations, not close to any of the lowest natural modes, but at a higher natural frequency over 4 Hz. This caused shaking in the superstructure even though it is supported on earthquake isolation modules that should filter out also vibration due to ice action. The vibrations have been more like a nuisance instead of a threat to the structure itself. Counteractions are still being developed.

One of the major pitfalls in understanding the physics of ice-induced vibrations is the quality and amount of the available full-scale data. The direct measurement of true ice load has not been possible and the needed instrumentation to distinguish the origin and physical factors from the indirect data have been inadequate. Also in most measurement campaigns the actual ice movement during the measurement periods has been much more rare than anticipated. The amount of well-documented data is still very rare.

Sodhi, 2001, defined three different ice crushing behaviour ranges – ductile, intermittent, and brittle - in dynamic ice structure interaction, Fig.3. This has been very useful in distinguishing the physically different ice failure modes, both in the design of structures and in interpreting measurement results. The ductile and brittle ice failure modes allow structural design with simple engineering methods. At ductile range the structure is practically stationary - passive - in reference to ice creep failure. At brittle range the random brittle ice failures induce random dynamic response to the structure. The design for intermittent range has been a more difficult "grey area" as the dynamic response of the structure starts to interfere into the ice failure process. These three different ice failure modes have been already adopted in the ISO 19906 (2010) ice load code. Different approaches have been proposed but the generally accepted method is only slowly emerging. E.g. the self-excited vibration model can reproduce frequency lock-in range ice-induced vibrations.

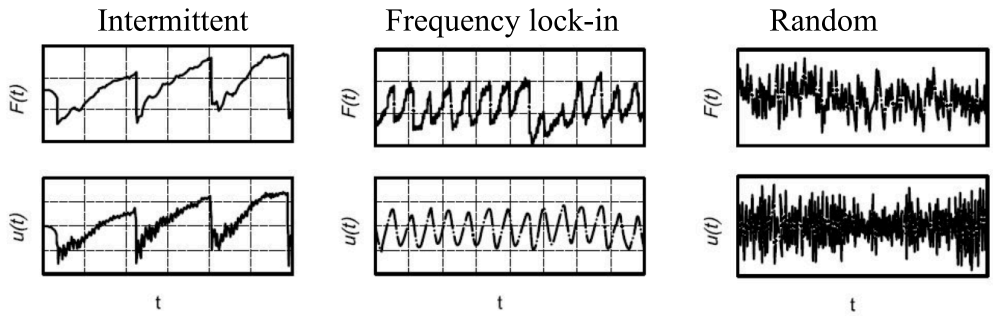

Figure 3. The different ice failure modes, Sodhi 2001.

DIFFERENT MODELS

The first proposal based on Cook Inlet observations was that ice has a tendency to fail repetitively at a constant frequency, about 1 Hz, Peyton 1964. Later this model was proven wrong as with the similar ice but with a different ice velocity or different structure a different frequency was recorded. Peyton`s another proposal in 1968 was that ice is having a constant failure length, e.g. that a constant distance of ice is destroyed during each crushing cycle. This was supported by Neil 1976, Michel 1978, Sodhi and Morris 1986, and Sodhi 2001. Peyton was the first to measure dynamic ice load histories while the natural sea ice was crushing against his test pile that was fixed in front of a leading main leg of a Cook Inlet offshore structure. At some instances resonant type vibrations were encountered.

Some experts such as Peyton (1968) and Neil (1976) had the opinion that steady state vibration caused by ice is a resonant vibration that relates to the characteristic failure length of ice. Michel (1978), Sodhi (2001), Sodhi and Morris (1986) concluded similarly based on their in field and lab tests. They reported that that the failure frequency of ice is directly proportional to ice velocity and inversely proportional to ice thickness.

The Matlock mechanical model, Fig.4, was the first to couple both the ice and structural properties together into a dynamic vibration system, Matlock 1971. He modelled the ice by a train of moving cantilevers hitting a vibrating elastic structure. The "ice" fails after a certain combined relative movement of the ice (train) and structure deflection. The ice load increases with train movement and includes also the structure deflection variations. Hence this includes an augmentation to the characteristic failure length of ice. The failure load depends on the cantilever beam dimensions and the strength (= ice strength). However, the cantilever spacing and train velocity determine therefore directly the ice failure frequency. This kind of model is a predetermined forced vibration model and cannot reproduce the three different dynamic ice-structure interaction zones as defined by Sodhi 2001.

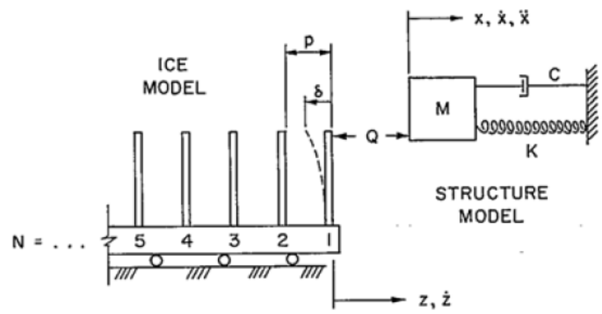

Fig.4. The Matlock model, 1969

Blenkarn, 1970, was the first to propose a physical explanation on the rise of ice-induced vibrations in the structure. As long as ice velocity is low the ice fails in ductile manner smoothly without any significant vibrations, or intermittently with relaxation vibrations after each ice failure without memory of previous ice failure effects. When ice velocity exceeds the range where stress or strain rate dependent ice strength starts to decrease after the ductile range but before the random high velocity ice crushing failure, the decreasing ice strength makes the structure to spring back against the ice with further increasing relative velocity and reducing ice resistance. The process is augmented by the "memory" of the decaying dynamic response of the previous ice failures. At this phase the static stability is lost and the process has become self-excited. The structure continues its stroke into the ice until the average constant strength brittle ice failure phase has dissipated both the excessive potential and kinetic energies. Then the structural deflection spring back starts – possibly including a gap formation between the ice edge and the structure before a new crushing stroke can start.

In the self-excited model the structure is modelled with its real properties and the ice action by the ice strain rate dependent interaction load. This allows to cover all the three different interaction ice velocity ranges: ice creep loads with no dynamics at low ice velocities, random ice load variations with random structural response at high ice velocities, and also the dynamic ice-structure interaction state at in-between velocities.

Määttänen, 1978, extended Blenkarn's proposal by utilizing FE-modelling to any structure. The chosen approach allowed to model realistically all the three ice velocity dependent ice failure modes and formed the basis for designing structures to withstand dynamic ice-structure interaction. As the nonlinear ice strength dependence on strain-rate was included, the dynamic response history has to be solved by numerical integration by observing the interaction rate dependent ice load. The model is capable to predict the most dangerous resonant type ice loading scenario: the frequency lock-in vibrations, ice load build-up at different ice action locations, and ice crushing cycles repeating continuously close to a natural mode frequency of the structure. Also, the model correctly predicts at high ice velocity the average brittle ice load, as well as random variations by including random brittle ice strength variations. Also, when ice velocity is changed, the model can predict the ice-structure interaction shift to another mode.

Hendrikse et.al. 2013, have extended Matlock's model into a grid of Matlock-elements to simulate two or even three-dimensional ice-structure contact loads, including local area ice failures synchronization. This approach can observe also the true strain rate dependent ice strength at ice contact points and couples the structural response effect into ice strength. This way tuned the Matlock model approaches true dynamic ice structure interaction.

PROBLEMS IN SCALE MODEL TESTING

Peyton, 1966, was also evidently the first to start scale model testing for dynamic ice-structure interaction. He pushed different thickness ice plates floating in water against a small vertical cantilever structure, with diameters at waterline from 6 to 51 mm, and recorded different dynamic response both at low and high ice velocities. He also measured dynamic ice loads on a small pile attached in front of the main leg of an offshores structure at Cook Inlet.

In scale model testing both the properties of ice and structure have to be scaled down. Most tests have been conducted in model tanks that were dedicated for ship model testing in ice. Ship model testing is based on the Froude scaling for correct wave actions. In dynamic ice structure interaction the structural response is more important while the wave loads are usually negligible. Hence Cauchy scaling shall be used. With saline model ice another problem comes in scaling down the ice strength by pre-warming before the test. The warm ice can weaken unrealistically soft and ductile and cannot reproduce natural brittle ice crushing at high strain rates. Ductile ice failure provides too much internal damping that may completely prevent the essential brittle ice failure as crushing starts after the load build. Hence many ice tank scale model tests have missed continuous frequency lock-in resonant type vibrations - the most dangerous loading scenario observed in full scale.

The true interaction ice load measurement in scale model tests is almost as challenging as in full scale. Instrumentation for the direct true ice loads measurement is not yet complete. Recently tactile sensors have provided promising results for direct load measurement but they have their own pitfalls: missing non-normal loads, endurance, calibration and frequency response. If the ice action area is supported on load cells, the mass effects bring with dynamic effects that are not a part of the true ice load. Indirect methods, with no transducers at ice action zone, are based on convolution with Fourier-transforms, require dynamic calibration for each different structural configuration, and "heavy" numerical analysis, but allow the solving of the true dynamic ice load. Good signal conditioning is essential, recent studies have indicated that the noise in measurement can reduce significantly the indirect measurement accuracy, Petersen, 2014.

The most common scale model test configurations have been single degree of freedom models (sdof). Real structures have always multiple degrees of motion freedom (mdof). Recent analysis, Nord 2014, has proven that during ice crushing the energy input and dissipation in the structure is a combined result from the contribution of many natural modes. Hence it is impossible to simulate the real structure dynamic ice structure interaction process with a single degree of freedom model, e.g. all the sdof-model tests findings apply only to "non-existing" sdof-full scale structures.

RUDIMENTARY FULL-SCALE MEASUREMENT DATA

The full-scale measurement data on ice-induced structural vibrations is both rare and incomplete. The offshore structures that have experienced severe ice-induced vibrations have neither been originally furnished with adequate instrumentation during manufacturing phase, nor can realize a comprehensive instrumentation installation afterwards. In most cases the gathered data has included only acceleration, strain, tilt or pressure panel data at few locations. The ever present noise in data makes it more challenging or often impossible to solve the real ice loads and the dynamic response of the structure. The redundancy of different measurement signals helps to mitigate this problem. Another problem is that with varying contact with ice edge the whole vibration system is changing. This effect can be taken into a account by utilizing Kalman filtering to the measurement data, Nord et.al, 2014.

ISO19906 REQUIREMENTS

The first edition of ISO 19906, Petroleum and Gas Industries – Arctic Offshore Structures, 2010, provides a simplified design method for dynamic ice loads, including lock-in vibrations. The main point is to check if any of the natural modes of the structure is prone for resonant type excitation, e.g. the modal positive damping is smaller than ice strength vs. strain rate dependent "negative damping", Blenkarn 1971. Then if this is the case, instead of self-excited ice-structure nonlinear response analysis, a simplified but more stringent deterministic maximum resonating saw-tooth like resonating ice load function is applied at ice action points until structural response velocity at ice action zone exceeds the ice velocity. Thereafter a gap forms between the ice edge and the structure, stops the energy inflow into the structure and limits further response amplitude growth. The analysis is repeated for all such natural modes that are prone to ice-induced resonant vibrations according to the Blenkarn's criterion. This way determined, dynamic state exceeds the true maximum response because the true more random ice/structure interaction ice load cannot feed as much energy into the structure due to random load and failure time variations during each crushing cycle. This approach was adapted about 40 years ago for the new Finnish steel lighthouses and is now an infield proven simple design approach.

Unfortunately the first ISO19906 edition included one additional - unintended - definition that predicts to any structure the same maximum vibration state regardless of the dimensions or stiffness of the structure. The ice velocity is the limiting factor - in self-excited state the maximum response velocity cannot become larger than the actual driving ice velocity Kärnä et.al. 2007, Määttänen 2008.The correction is coming to the second edition of ISO 19906.

PRESENT STATE OF THE ART – CONCENSUS

Most of the early conflicting opinions and conclusions have gradually merged towards harmonized general acceptance. One reason for original disagreement was that civil engineers most often designed the offshore structures with the same static and linear design criteria that were common for any stationary building. The normal construction materials are normally far below their homologous temperature - the ratio of actual temperature to the melting point temperature in Kelvin scale - where material nonlinearities are practically non-existent. This is not the case with ice that is normally very close to its homologous temperature, e.g. at -10 C the homologous ice temperature is 0,96 while with steel it is only 0,15. Especially the ice strength dependence on stress or strain rate has a strong effect on the rise of frequency lock-in vibrations. As ice is normally close to its melting point and behaves differently, creep is pronounced and strong strain rate effect exists. The concept of ice induced self-excited vibrations was initially hard to be adopted but gradually the observations on ice strain rate dependent behaviour has been accepted as the origin for ice-induced self-excited vibrations. Another factor was increasing amount of measurement data. By varying test parameters it was proven that ice has neither a specific failure length nor a failure frequency. So what was left to initiate ice-induced vibrations is the nonlinear ice strength dependence on loading rate.

Another factor is coupling. The severe vibrations in suspension bridges have been due to coupling of bridge deck bending and torsion vibrations to the aerodynamic lift in wind. The co-operation of two modes can produce a resonant type of lift loading. This can be compared to coupled ice crushing and structure movement during a vibration cycle. If the structure natural mode vibration changes the strain rate in harmony with a natural mode, the resulting ice load can be in phase with the structural movement, allowing energy input into the structure. Then the state of self-excited vibrations can develop into limit cycles (maximum steady amplitudes). However, the vibration response is limited: with increasing amplitude the strain rate increases and ends up to constant ice strength brittle range. Then the energy input levels but the damping dissipation still increases. Hence the maximum amplitude state of limit cycles is achieved. The ever-present ice properties randomness further reduces the most efficient resonant loading. In addition a gap can form between the ice edge and the structure stopping completely all energy inflow from ice into the structure. As the energy input from ice into the structure is limited, the maximum response amplitudes will be also limited and one can design structures that can withstand also ice induced self-excited vibrations.

The earliest dynamic ice load models – ice has a tendency to break at certain frequency, or ice has a characteristic failure frequency - were easily proved wrong by a simple test set up in which only one relevant parameter was changed, e.g. ice velocity, structural mass or stiffness. The physical explanation for frequency lock-in vibrations was more complicated and needed to observe the energy balance by comparison to other similar media/structure interaction processes in nature, like friction-induced vibrations, or aircraft components flutter. Common factor is energy input into the structure through a coupled media/structure interaction and response process. Hence understanding the energy flow into the structure at one phase of vibration and dissipation at the other explains the origin and the growth limit of ice induced frequency lock-in vibrations. As the nature always follows the easiest way, also the shift of vibration response from one natural mode to another with the ice velocity can change, and can be explained by the least energy dissipation principle.

CONCLUSION: TOWARDS COHERENCE

The first Cook Inlet dynamic ice load observations initiated a research to find out what is the origin of such severe vibrations, and how to design structures in such a way that the threat is avoided. The first explanations on ice having a certain crushing frequency or crushing distance were soon rejected based on the increasing data of ice crushing in different ice conditions and structure configurations. Quite long the original proposal by Blenkarn in 1970 - ice crushing providing negative damping into the structure - was denied. Gradually more and more momentum was accumulating to support the self-excited vibrations originating from the negative damping, Määttänen 1977, 1978,1983 and 1987, Toyama et-al, 1983, Xu et- al 2001, Vershinin et-al 2001. Based on scale model tests, Huang 2007 verified also the initiation and decay velocity ranges for the frequency lock-in ice induced vibrations. Dynamic instability can initiate also from negative mass. Indeed, negative mass during dynamic ice-structure interaction has already been measured in scale model tests, Hendrikse et. al. 2012.

All the continuously accumulating research together verifies the existence of the original Blenkarn's proposal. These findings confirm the rise of ice-induced vibrations due to dynamic instability, and the resulting continuous steady state frequency lock-in vibrations to follow the theory of self-excited vibration due to the presence of negative damping or negative mass. However, the good quality dedicated full-scale measurement data is still missing.

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