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ANALYSIS OF BOREHOLE JACK ICE STRENGTH DATA USING QUANTILE REGRESSION

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ABSTRACT

The magnitude of ice strength is a topic that impacts the design of offshore structures and the use of ice as a construction material (ISO, 2010). There is a large body of confined compressive strength values on old ice, first year and fresh water ice obtained using Borehole Jack (BHJ) measurements in the field. In some cases the corresponding values of ice temperature and ice salinity have been obtained. The ice strength measurements have often been analysed by plotting the BHJ ice strength as a function of the brine volume and performing least-squares analysis on the data. This approach has had limited success mainly due to the large scatter in the data. In least-squares analysis, the scatter is treated as an aspect that is to be minimised so that the underlying trend can be quantified. Using quantile regression in contrast, the scatter is treated as an essential part of the data. In this paper we use the quantile regression method to generate values of the BHJ strength at quantiles, from 0.05 to 0.95, using 236 strength measurements from multi-year, first-year and flooded ice. The analysis showed that the ice strength can be modelled using a linear function of root brine volume. The Normal, Log-normal, Gamma and Gumbel probability distributions were tested as suitable models for the generated quantiles. The Normal distribution was found to match the quantiles with the lowest RMS error. From the calibrated probability distribution, BHJ pressure at other quantiles can be obtained. For example at the 0.99 quantile (1% probability of exceedence) and at zero brine volume, the ice strength is 53.5 MPa. The calibrated probability distribution of the BHJ ice strength is compared with the recommendations contained in ISO 19906.

INTRODUCTION

Strength measurements have been conducted in laboratory settings on harvested or manufactured samples as well as conducted in the field on natural ice. Field measurements can often be both difficult to make and expensive to perform. One tool that has been used in the field over the last few decades is the Borehole Jack (BHJ). This device is described in various publications (Masterson, 1996; Masterson and Graham, 1992; Sinha, 1986). A nominally 150 to 160 mm diameter hole is made in an ice sheet or ice feature. The BHJ device is lowered into the hole and at the desired depth, the small diameter indenter in the BHJ is forced into the side wall of the hole. The oil pressure controlling the indenter along with the displacement of the indenter are both recorded. From the time series of displacement and pressure, the strength of the ice is determined. The BHJ is not a first-principles device but generates strength values that are considered to be index values (ISO, 2010). The strength values are however, indicative of the confined compressive strength of the ice feature. In some instances the access hole is made using a core barrel rather than an auger and the retrieved core can then be used to determine the temperature and salinity of the ice at the depth where the BHJ test was conducted. Usually a series of BHJ tests are conducted at different depths in the access hole. Measurements to a depth of 4 - 7 m have routinely been performed although measurements in excess of 11 m also have been made (Johnston, 2014). The depth to which measurements have been achieved emphasises the ability of the BHJ to sample ice strength at large depths in the ice feature.

A review of BHJ strength data collected on multi-year ice has recently been published (Johnston, 2014). The report contains strength data from many investigations along with information on the in-situ temperature and salinity. From the temperature and salinity measurements, estimates of brine volume can be made (Frankenstein and Garner, 1967). As described in ISO (2010) many ice strength characteristics on saline ice are analysed as a function of the brine volume or total porosity. In general, a larger brine volume results in a "weaker" ice. Reviewing the various plots from Johnston (2014) indicated that data at the larger brine volumes were sparse for the multi-year ice results presented. To supplement the data set for the larger brine volumes, BHJ measurements collected on constructed sea-ice roads were added to the multi-year ice data set (Masterson and Yockey, 2000; Spencer et al., 2001).

The combined BHJ strength data set are plotted as a function of root brine volume in Figure 1 where the three data sets are indicated with different symbols. The number of data points for the MY, 2000 FY and 1999 FY are 125, 71 and 40 respectively for a total of 236 data points. The MY data were digitised from Johnston (2014). The FY 2000 set contained a few measurements on very low salinity material, natural first-year ice and free-flood build-up ice (Spencer et al., 2001), the FY 1999 set contained natural first-year and build-up free flood ice (Masterson and Yockey, 2000). From Figure 1 it can be seen that in spite of different BHJ devices, different ice types being used to collect the data and different groups processing the data, reasonable agreement between the various sets is apparent. Also shown in Figure 1 is a least-squares fit to a linear function. The correlation coefficient for the fit is low (r^2 = 0.34) due to the large scatter in the data. To date, the low correlation coefficient has impeded detailed analysis or interpretation of the BHJ strength data.

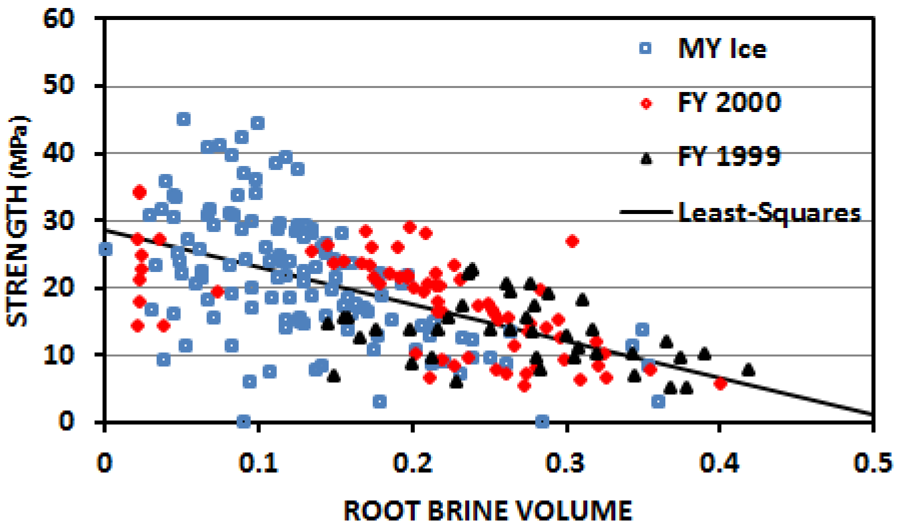

Figure 1. Borehole Jack Ice Strength Data

An alternative analysis technique called quantile regression (QR), is however available. This recognised statistical analysis technique, while used in many areas of science and technology (Koenker, 2005) has not been extensively used in the analysis of ice strength. Recent publications (Morrison and Spencer, 2014; Spencer and Morrison, 2014; Spencer, 2014) introduced the QR technique for interpreting local and global ice strength data. In least-squares analysis the aim is to reveal an underlying trend that has been obscured by scatter in the data. The scatter is considered to be just an impediment to the analysis and not to contain any useful data. Quantile regression on the other hand considers that the scatter contains useful information. For the BHJ data shown in Figure 1, the maximum strength values at certain brine volumes are of interest and the various "high" strength values are considered to be meaningful and not just the result of random noise.

QUANTILE REGRESSION OF BHJ DATA

Similarly to least-squares analysis, in QR a fitting function has to be selected. This may be determined from theoretical considerations or from reviewing the data itself. Presented here are two fitting functions, an exponential relationship between strength and root brine volume and a linear relationship. Both of these have been used and presented in ISO (2010) in the interpretation of ice strength data. The quantile analysis can determine the values of the various parameters in the fitting function to generate a curve that has a certain probability of exceedence. This is in fact a quantile and can take on values of, for example 0.50, also known as the median, or 0.90 etc. Additionally, QR can provide standard error estimates of the various fitted parameters. For this paper we have used the "R" language to conduct the quantile regressions. "R" is an open source statistical analysis package that is available on the web (R, 2014).

The results of QR using an exponential or a linear fitting function to the data set presented in Figure 1, are given in Figures 2 and 3. In these figures the various solid lines represent 19 quantiles at equal 0.05 increments from 0.05 to 0.95 inclusive. The lowest solid line in Figures 2 and 3 corresponds to a quantile of 0.05 and the highest solid line to a quantile of 0.95. For the exponential function in Figure 2 there were two fitted parameters and for the linear function of Figure 3 also two fitted parameters.

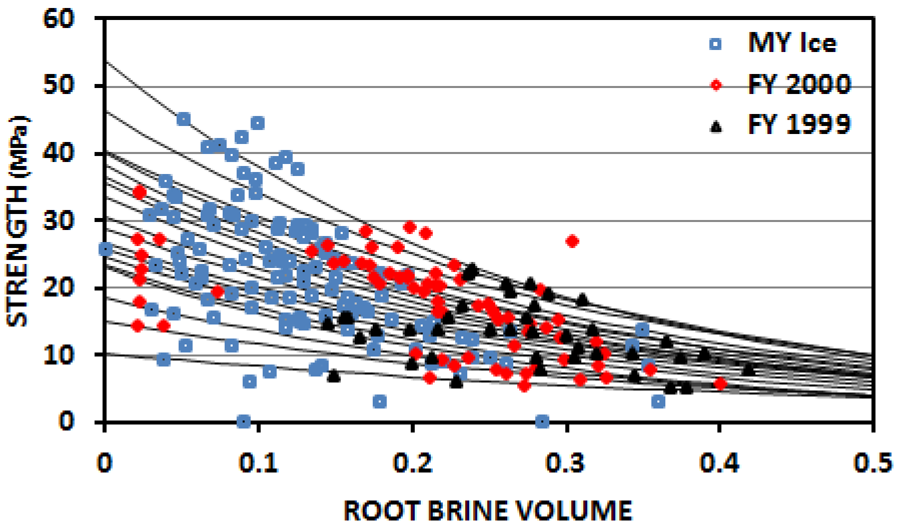

Figure 2. QR Using an Exponential Fitting Function

From Figures 2 and 3 either the exponential or linear fit appears to fit the data. The exponential fit does have a number of instances where the various quantile lines intersect. The exponential fit also has a difficulty in that at a root brine volume of 0.5 or more, the strength is still significant. Even at a root brine volume of 1.0 corresponding to an ice matrix that is all brine and no solid ice, the exponential fit generates strength values between 1.4 and 1.6 MPa.

For the linear fit shown in Figure 3, the various quantile lines intersect at a root brine volume of close to 0.5.

The value of the root brine volume that corresponds to zero strength is plotted as a function of quantile in Figure 4. As may be noted, apart from the 0.05 quantile, the zero strength root brine volume shows a consistent value with low scatter over a wide range of quantiles. We take this observation as evidence that the linear function is an appropriate fitting function. The mean of the root brine volume values, excluding the 0.05 quantile point of the data shown in Figure 4 is 0.5169. We then conducted another QR on the data but forcing the quantile lines to have zero strength at a root brine volume of 0.5169.

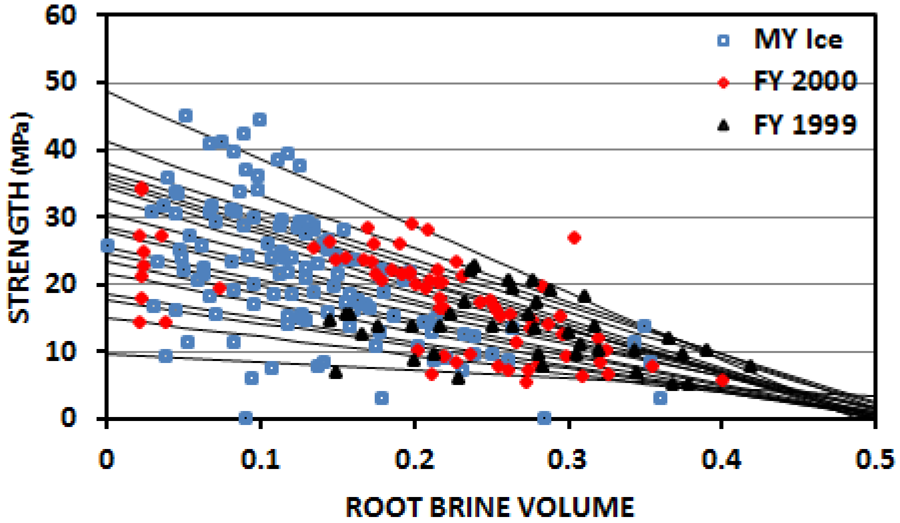

Figure 3. QR Using a Linear Fitting Function

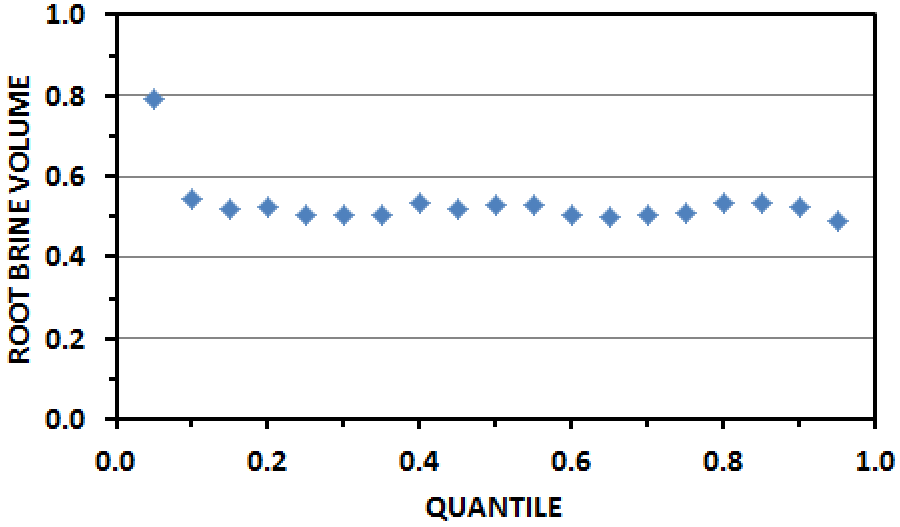

Figure 4. Root Brine Volume Corresponding to Zero Strength

The results of this additional QR are shown in Figure 5 as solid lines for quantiles between 0.05 and 0.95. The dashed lines labelled "Extrapolated" will be described in the next section. The values of the strength at zero root brine volume shown in Figure 5 are given in Table 1. The data in the fitted Normal column will be described in the next section as well.

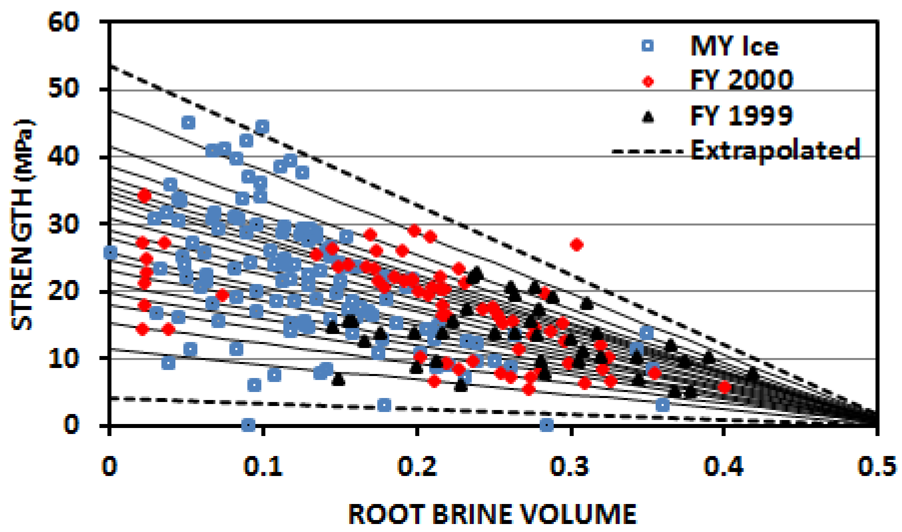

Figure 5. QR using Linear Fit Forced to Horizontal Intercept = 0.5169

Table 1. QR Results at Zero Brine Volume

Quantile	Strength at Zero Brine Volume (MPa)	Standard Error (MPa)	Fitted Normal (MPa)
0.01	n/a	n/a	4.10
0.05	11.36	1.40	11.33
0.10	15.28	1.17	15.18
0.15	17.91	1.07	17.78
0.20	19.68	1.15	19.85
0.25	21.18	1.14	21.62
0.30	23.15	0.94	23.22
0.35	24.46	0.83	24.69
0.40	25.66	1.04	26.09
0.45	27.72	1.13	27.46
0.50	29.04	1.11	28.78
0.55	30.03	1.11	30.11
0.60	32.49	0.86	31.47
0.65	33.75	0.77	32.87
0.70	34.57	0.61	34.34
0.75	35.61	0.74	35.94
0.80	36.86	0.66	37.71
0.85	38.53	1.14	39.78
0.90	41.43	1.72	42.38
0.95	46.93	1.77	46.23
0.99	n/a	n/a	53.46

PROBABILITY DISTRIBUTION FUNCTION

From Table 1 it can be seen that at both the low and high quantiles the uncertainty in the calculated value increases. For engineering use the values at various extreme quantiles would be needed, for example the strength at the 0.99 quantile. Because of the limited number of data points available for the QR, there is, in general, a limit to the largest and smallest quantile that can be determined. The method that we shall use to estimate these extreme quantiles is to fit a continuous probability distribution to the various quantile values given in Table 1. As was done in Spencer and Morrison (2014), the Gumbel, Log-normal, Gamma and Normal distributions were evaluated. These four distributions can each be defined by two parameters, a mean and a standard deviation (Wikipedia, 2014).

The fitting was performed by a grid search method where the mean and standard deviation of the distribution are varied and the combination that produced the lowest mean-square deviation of the quantile pressures was used as the best fit values. The procedure was repeated for each of the four probability distributions. The fitting procedure produced slightly different best-fit mean and standard deviation values for each of the four distributions. The goodness of fit parameter was the lowest for the Normal distribution as shown in Table 2.

The quantiles from the QR analysis and the best-fit Normal, Log-normal, Gamma and Gumbel distributions are shown as QQ plots in Figure 6. The pressure error estimates from the QR are also shown in Figure 6 as the red bars. For a perfect fit all of the data points would lie along the solid line at 45 deg. As can be seen, all of the distributions fit the quantiles in the middle of the range but the Normal distribution fits the largest and smallest quantiles as well.

The calculated strength values for the Normal distribution with mean and standard deviation specified in Table 2 are given in Table 1 along with the values at the strength values calculated at the 0.01 and 0.99 quantiles. These two extrapolated quantile lines are also shown in Figure 5 as the dashed lines. For the 236 data points used in the QR, it is expected that 2.36 data points would lie above the 0.99 quantile line and 2.36 data points below the 0.01 quantile line. Obviously the number of points would have to be an integer, but the data shown in Figure 5 indicates that these extrapolated quantile lines are reasonable.

Table 2. Best Fit Probability Distributions

Distribution	Mean (MPa)	Standard deviation (MPa)	RMS error (MPa)
Normal	27.78	10.61	1.91e-2
Gamma	29.66	11.00	7.81e-2
Gumbel	30.12	12.02	8.51e-2
Log-normal	30.18	11.83	8.77e-2

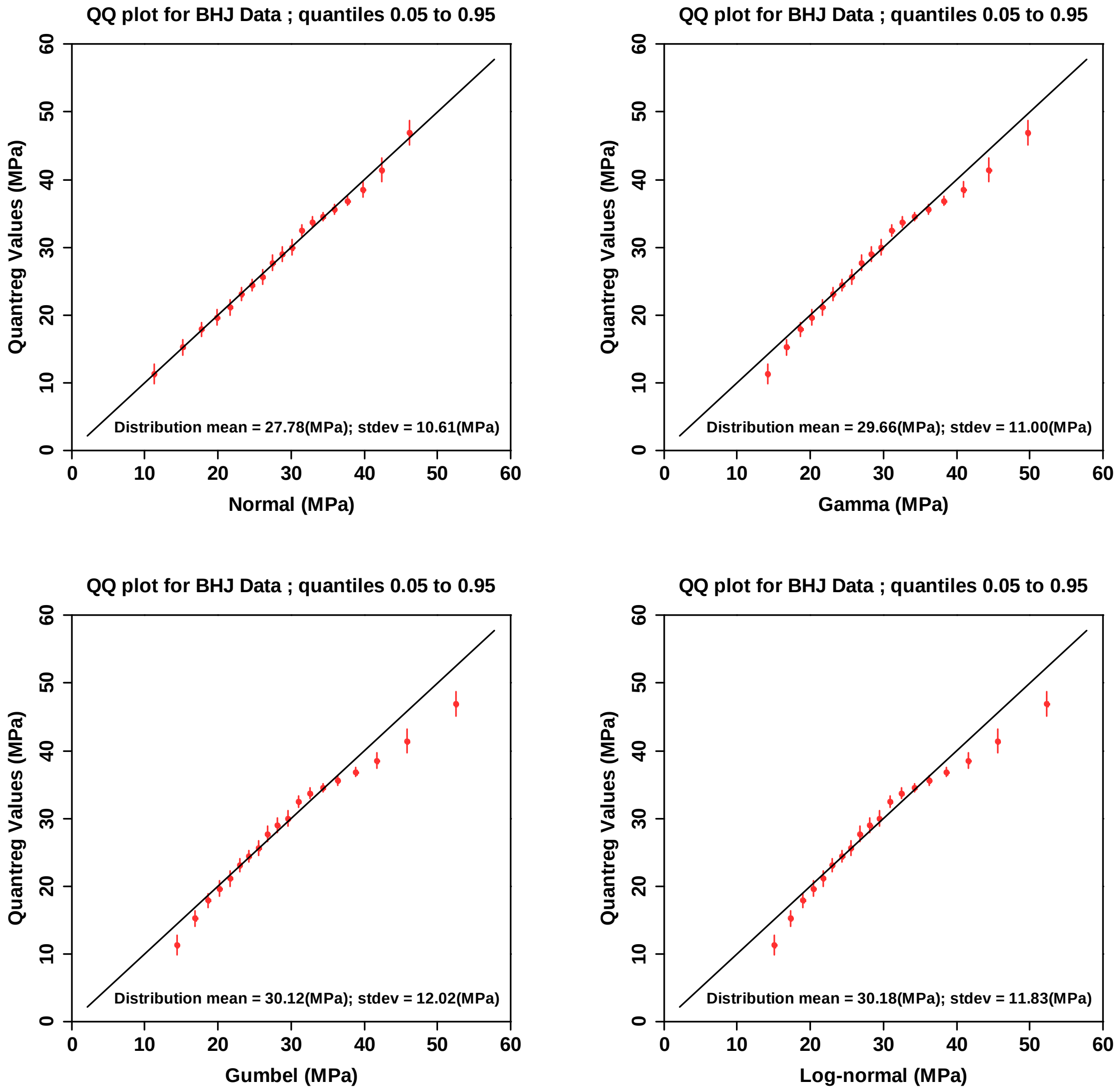

Figure 6. QQ Plots for Fitted Distributions

DISCUSSION AND POTENTIAL APPLICATIONS

From the data presented in Figures 5 and 6 in particular, it can be seen that much more information can be obtained from the set of 236 data points using QR than can be obtained using least-squares regression on the same data shown in Figure 1. The best-fit Normal distribution parameters given in Table 2 are for a zero root brine volume. For an ice with a non-zero brine content, the strength will be linearly scaled by root brine volume. This relationship is given in (1) where $N(\mu,\sigma)$ is the Normal distribution function and ν_b is the Brine volume of the ice. Note that the BHJ pressure is zero at a brine volume of 0.267 ($\nu_b^{0.5}=0.5169$).

$$\text{BHJ pressure (MPa)} = N(\mu,\sigma)\,[1 - (\nu_b/0.267)^{0.5}] \qquad (1)$$

In Sections A.8.2.8.2.2 and A.8.2.8.2.4 of ISO (2010) are discussions on the pressure observed using the BHJ. The BHJ pressures are given as the uniaxial compressive strength of the ice times a multiplier. Two multiplier ranges of 2 to 4 or 3.5 to 5.0 were quoted. Furthermore the uniaxial compressive strength of sea ice σ_c was defined, given in (2), as a function of total porosity ν_t. The maximum compressive strength from (2) is 10.78 MPa.

$$\sigma_c \text{ (MPa)} = 49*0.22\,[1 - (\nu_t/0.280)^{0.5}] \qquad (2)$$

Note that the total porosity includes contributions from both air voids and brine volume. For the data presented in this paper, the air voids have not been included in the analysis. Thus it would appear that in (1) a brine volume of 0.267 is probably consistent with a total porosity of 0.280. The current analysis on BHJ data is then consistent with the recommendations from ISO regarding the relationship between strength and brine volume.

Combining (2) with the pressure multipliers given above, provides the BHJ pressures given in Table 3. Table 3 also contains the results from the QR analysis.

Table 3 BHJ Pressure at Zero Porosity

Uniaxial Compressive Strength (MPa)	Pressure Multiplier	BHJ Pressure (MPa)
10.78	2	21.6
10.78	4	43.1
10.78	3.5	37.7
10.78	5.0	53.9
QR Results at 0.99 quantile		53.5
QR Results at 0.95 quantile		46.2
QR Results at 0.90 quantile		42.4
QR Results at 0.50 quantile		28.8

From Table 3 it can be seen that the ISO multiplier of 5.0 is approximately consistent with a quantile of 0.99 or a 1% probability of exceedence, whereas the 4.0 multiplier is approximately consistent with a 10% probability of exceedence.

The analysis presented in this paper generally supports the recommendations made in ISO regarding BHJ strength and furthermore provides a means to associate a probability of exceedence with the ISO recommendations. Note that the analysis presented in this paper analyses the BHJ data directly and does not rely on a uniaxial compressive strength and a range of pressure multipliers. Thus the analysis is more robust than presented in ISO (2010). Furthermore our analysis used data from a variety of ice types supporting the idea that the strength is scaled mainly by brine volume with only potentially minor ice type variation.

As an example of the use of the analysis of this paper we consider the BHJ strength requirements for floating ice roads. From Spencer et al. (2001) the BHJ strength for safe operation of an ice road must be larger than 7.6 MPa averaged over the thickness of the ice sheet. Equation (1) is used to determine the brine volume that is equivalent to an ice strength of 7.6 MPa at a defined probability level. The brine volume is a function of both temperature and ice salinity (Frankenstein and Garner, 1967). This relationship is shown in Figure 7 where two lines are presented. One is for the average (p = 0.50) condition and one is for a higher probability (p = 0.90). The current design method uses the p = 0.50 condition and from Figure 7 we can see that for a thickness averaged ice temperature of -5°C, the thickness averaged ice salinity would need to be less than 1.79 ppt. On the other hand, if a higher probability were to be required (p = 0.90), then the thickness averaged salinity of the ice sheet would have to be less than 0.83 ppt.

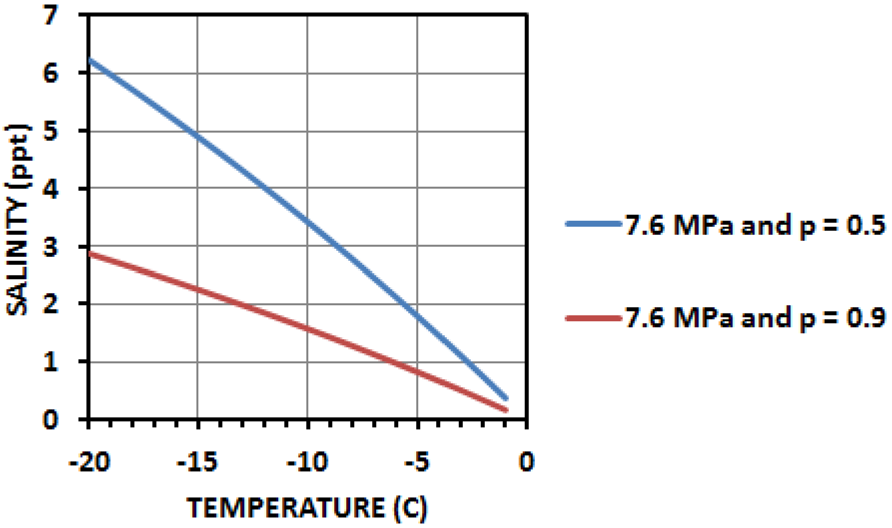

Figure 7. Temperature-Salinity Relationship for Safe Ice Road Operation

Another potential application of the analysis is in gravity based structure (GBS) design. In ISO (2010) there are design data on local ice pressures for thick ice features. These local pressures are used in the sizing of the structure bulkheads and the skin plating thickness. The local pressure data in ISO (2010) is defined in terms of a pressure-area relationship at a contact area of more than approximately 0.1 m^2. As shown in Spencer (2015) the details of the pressure–area relationship at areas of less than 0.1 m^2 are also required for assessing bending stress in the skin plating of steel structures. The area of the indenter that is used in the various BHJ devices is nominally 0.01 m^2. The statistical analysis of strength data from these BHJ measurements can be used to assist in defining the local pressure trends at contact area of less than 0.1 m^2.

Additional BHJ data and the corresponding brine volume obtained in multi-year and other ice types should be available in the near future (pers. comm. M. Johnston, 2014). These data could then be included into the data set analysed here in order to improve the reliability of the analysis. This improvement would come from having more data points to use plus the inclusion of second-year ice into the data set.

CONCLUSIONS

A data set of 236 BHJ strength values from multi-year, first-year and flooded ice with corresponding brine volumes has been analysed using quantile regression. From the analysis we have determined:

- The BHJ strength can be represented as a linear function of root brine volume ($\nu_b^{0.5}$) for quantiles evaluated between 0.05 and 0.95

- The BHJ strength is zero at a brine volume (ν_b) of 0.267
- Normal, Log-normal, Gamma and Gumbel probability distributions were evaluated for matching the quantiles
- The Normal distribution has the lowest RMS error and at zero brine volume is defined by mean = 27.78 MPa; standard deviation = 10.61 MPa
- At a 1% probability of exceedence and at zero brine volume, the BHJ strength is predicted to be 53.5 MPa
- The results generally support the recommendations contained in ISO 19906 regarding BHJ strength. However the QR analysis is more robust and quantifiable than the recommendations in ISO 19906, for example the ISO pressure multiplier of 5.0 approximately corresponds to a probability of exceedance of 1%
- There is evidence that brine volume is a major component in estimating BHJ ice strength and ice type is of secondary importance
- Additional BHJ strength data with corresponding brine volumes will improve the reliability and scope of the analysis

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