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LOCAL ICE PRESSURE AND PLATE STRESS

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ABSTRACT

In ISO 19906 there are recommendations on deterministic Local ice pressure values that are expressed in terms of a pressure-area relationship Pressure = 7.4 Area$^{-0.7}$. These pressure values are used as input for determining the size and thickness of the exterior plating of Gravity Based Structures (GBS). ISO 19906 indicates *The size and placement of the local contact areas shall be selected to ensure that the most critical cases are addressed* but does not provide detailed guidance on how this Local pressure should be applied to the structure. One method that is commonly used is to calculate a constant Local pressure over the plate area defined by the spacing of the bulkheads and stiffeners (ISO A.8.2.5.5). Elastic analysis is then used, along with other design constraints, to determine an appropriate thickness for the exterior plating.

In this paper we use the fluctuating high pressure zones that generate the majority of the ice load and which produce plate pressures that are far from uniform. The elastic bending stress in the plating for both a uniform pressure distribution and various non-uniform distributions representing high pressure zones are calculated. Simply supported and clamped boundary conditions to the plate are addressed. It is shown that even with a uniform pressure distribution in combination with a range of pressure-area relationships, unexpected results can be produced, namely that as the spacing between bulkheads increases the plate bending stress can decrease. For non-uniform loading while the total applied load is less than for the uniform loading case, particular loading geometries produce elastic plate bending stresses that are much higher. Guidance is provided on selecting appropriately dimensioned load patches that may be used in conjunction with numerically intensive design tools such as non-linear finite element modelling.

INTRODUCTION

In the structural design of Gravity Based Structures (GBS) many factors and constraints have to be taken into account. When the GBS is deployed in ice covered waters, often the Global ice forces generate the major environmental loading which then governs the required global foundation resistance. In some geographical regions, wave forces or seismic forces can generate the largest environmental loads (Spencer and Bound, 2010).

When the ice feature includes thick first-year ice, glacial ice or multi-year ice then the Local ice pressures govern the strength requirements of the outer steel shell plating plus influence the sizing and separation of the various bulkheads and stiffeners. The Local ice pressures are generally larger than the Global ice pressures at the same contact area (ISO, 2010). For steel structures, the spatially varying Local ice pressure is resisted by the shell plating, the shell plating transfers the ice load to the various bulkheads and ultimately into the GBS foundation or seabed. For concrete or mixed construction, the detailed load paths may be different.

The Local ice pressure (MPa) for thick (>1.5 m) ice features is represented as a power law on contact area (m^2) between an area of nominally 0.1 m^2 and 10 m^2 (ISO, 2010).

$$\text{Pressure} = 7.4\ \text{Area}^{-0.7} \qquad (1)$$

From (1) it can be noted that the pressure decreases as the area under consideration increases. For steel structures, the strength requirement for the main bulkheads and weldments may be determined by the load that the bulkhead has to carry. The Local ice load (MN) is calculated from (1) and given as

$$\text{Load} = \text{Pressure}*\text{Area} = 7.4\ \text{Area}^{0.3} \qquad (2)$$

In ISO, (2010), there are recommendations that the (uniform) Local pressure is applied over the tributary area for the bulkhead or the area of the plating between horizontal and vertical bulkheads and horizontal stiffeners. From (2) the load increases with increasing area and thus the maximum Local load on the bulkhead corresponds to the uniform loading situation. For the steel shell plating on the other hand, the maximum plate bending stress depends on both the total ice load and the spatial distribution of the applied ice load. The structure designers are concerned with determining the minimum thickness of the shell plating that can safely carry the Local ice loads. This thickness can be determined using elastic, plastic or other calculation approaches with appropriate factors of safety.

To illustrate the process, thin plate theory and linear elasticity (Young and Budynas, 2002) will be used. Rather than determining the plate thickness, the plate bending stress will be calculated instead. The actual plate thickness can be determined from the calculated stress using the appropriate factors of safety etc, a topic outside of the scope of this paper. For the shell plating, the basic design area is a horizontal rectangle with the width a few times the height (pers. Comm. Alec Bound, 2014). In addition, the support and boundary for the basic plate design area will depend on structural details and can vary from simply supported to fixed conditions. The number of variables will be reduced in the calculations by having a constant plate height of 1.0 m and a constant plate thickness of 0.1 m. The width of the plate will be varied from 1.0 m to 5.0 m, ie a plate aspect ratio varying from 1.0 to 5.0. Typical dimensions for the basic design area are smaller than those selected but the analysis presented later shows that the general trends do not depend strongly on the actual plating area selected.

The maximum bending stress for a uniform pressure applied to the whole plate using (1) is presented in Figure 1 for three different traces. For the simply supported plate (SS), the maximum bending stress occurs in the geometrical centre of the plate. For the fixed boundary conditions (CC), there are two localised bending stress maxima, one also in the plate centre with another in the centre of the long edge support. From Figure 1 note that as the plate aspect ratio (or equivalently plate width) increases from 1.0 to 5.0 the three plate bending stress' increase reaching a maximum at an aspect ratio of around 2.0 for the simply supported boundary and around 1.5 for the fixed boundary condition. At the larger aspect ratios using the pressure area relationship defined in (1), the plate bending stress is seen to decrease. For a typical GBS framing arrangement, the plate aspect ratio is in the range of 3 to 5.

From the data presented in Figure 1, the plate bending stress and hence the plate thickness can be decreased by increasing the spacing between the vertical bulkheads when the aspect ratio is larger than about 2.0. This is a counter intuitive result where it would be expected that as the aspect ratio increases, the plate bending stress would increase asymptotically reaching a constant value independent from the aspect ratio. This relationship is shown in Figure 1 as the two traces where the pressure does not decrease with area but remains constant at 7.4 MPa. Thus the pressure area relationship given in (1) applied uniformly over the plate produces unexpected effects on plate bending stress.

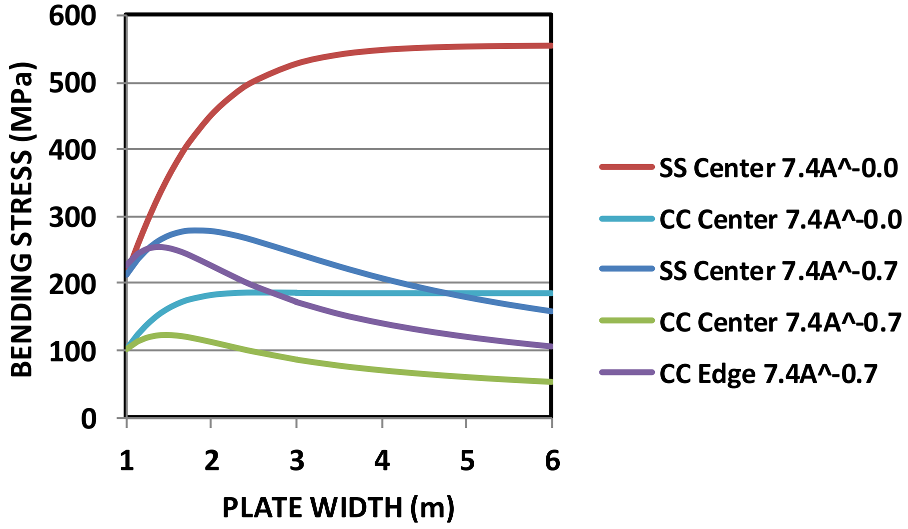

Figure 1. Bending Stress for uniform pressure over the whole plate

It would thus appear that applying the pressure as in (1) uniformly over the shell plate defined by the panel size is not a reliable approach for determining the plate thickness even though for the sizing of the main bulkheads it appears to be a reliable approach. ISO (2010) indicates *The size and placement of the local contact areas shall be selected to ensure that the most critical cases are addressed*. Also, *It is necessary to use judgment in determining the combination of pressures for use in the design. Some combinations are more critical than others, and the process of determining these is one of trial and error. Because of the considerable variability of hull configurations, no prescriptive set of combinations can be specified*. The solution to the problem of selecting the appropriate loading situation is the main focus of this paper. The solution is based on recognizing that the pressure applied to the plating is not uniform but has combinations of high and low pressure zones (ISO, 2010).

NON-UNIFORM PLATE LOADING

The Local pressure is applied in an idealised rectangular high pressure zone (HPZ) with zero pressure outside of this zone. For simplicity, the axes of the loading rectangle are aligned with the plating axes. The ice pressure within the idealised HPZ is treated as uniform and given by (1). The size, aspect ratio and position of the rectangular HPZ were varied so that the largest plate bending stress was generated. The calculation of the bending stress for the non-uniformly loaded plate used finite difference (Dolićanin et. al., 2010) for the fixed boundary conditions. For the simply supported plate, the method used a double Fourier series (Ventsel and Krauthammer, 2001) or the finite difference method. The actual methods used in this paper were selected based on mathematical simplicity and the ability to readily determine the worst-case load patch dimensions, other methods such as finite element analysis could have been used.

At each plate width, the size and location of the load patch that generated the largest plate bending stress was found by a grid search method varying the width, height and location of the load patch. For the simply supported boundary, the load patch was located in the geometrical centre of the plate. For the fixed boundary conditions two different load patches were considered, one at the geometrical centre of the plate for maximizing the central bending stress and one adjacent to the centre of the long edge support. This second load patch maximized the bending stress at the plate edge.

The bending stress (σ_{yy}) contour plots for a fixed boundary plate are shown in Figures 2 to 4 for a representative 1 m by 3 m plate. The finite difference calculations used for Figures 2 to 4 had a grid spacing of 0.05 m. The worst case patch loading is shown in Figures 2 and 3 as the black rectangle and uniform loading shown in Figure 4. The loading and stress parameters for Figures 2 to 4 are given in Table 1.

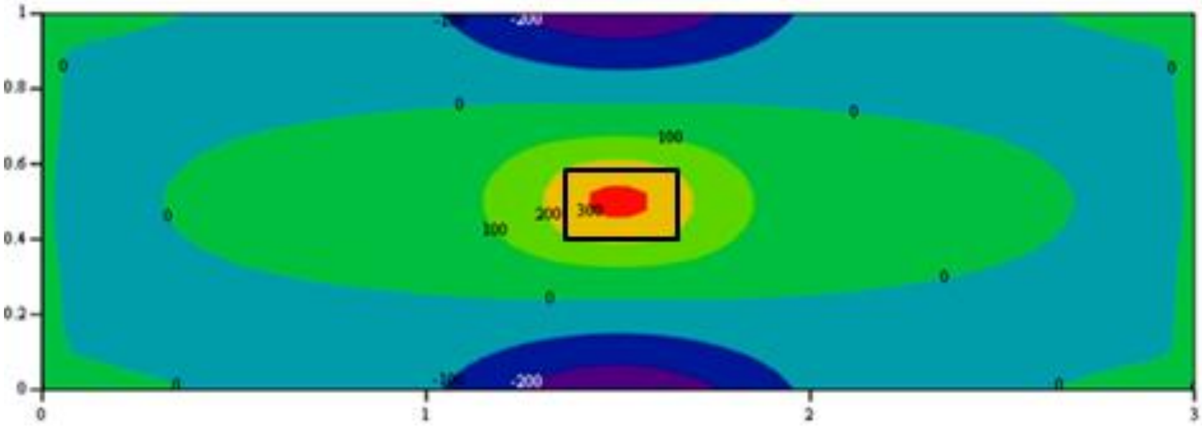

Figure 2. Plate Bending Stress (σ_{yy}) from Central Load Patch

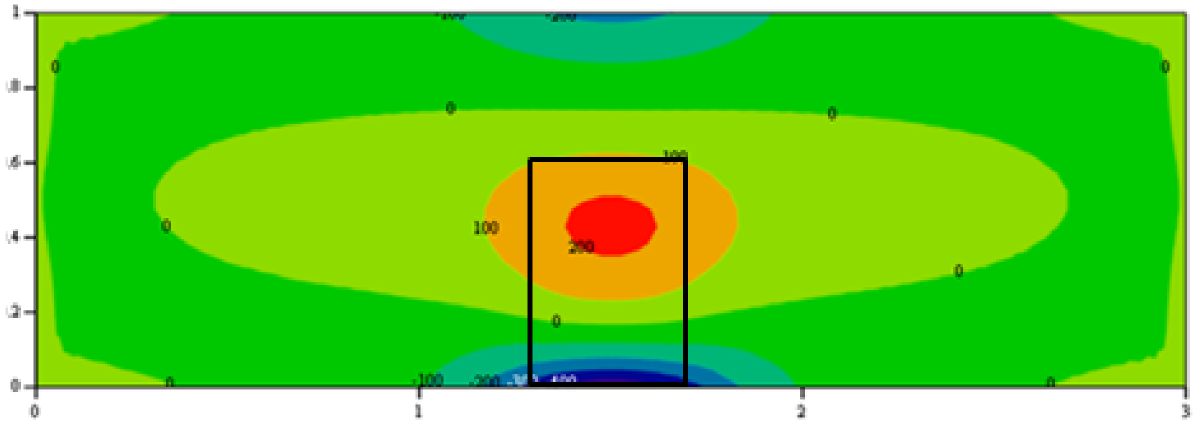

Figure 3. Plate Bending Stress (σ_{yy}) from Offset Load Patch

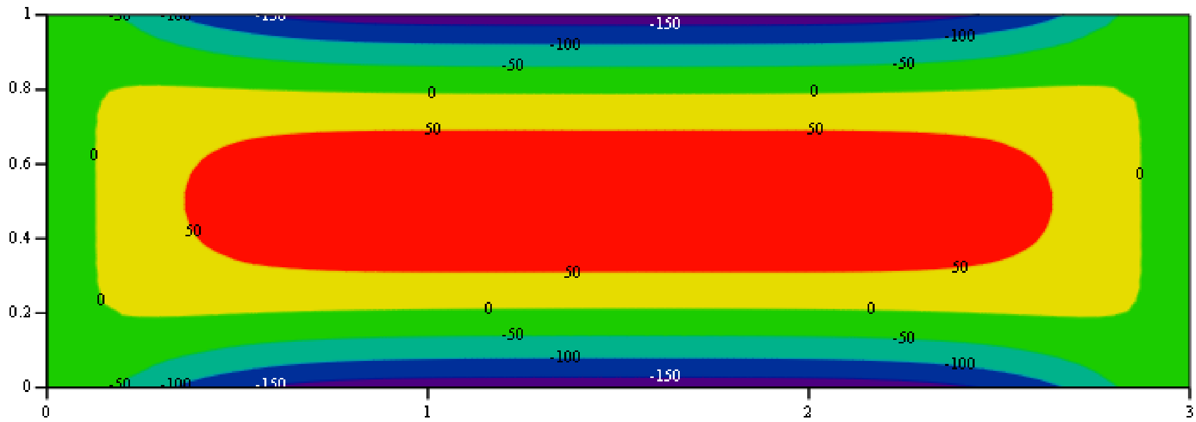

Figure 4. Plate Bending Stress (σ_{yy}) from Uniform Loading

Included in Table 1 are the bending stress values that would be generated by using (1) applied uniformly over the complete plate, illustrated in Figure 4. It can be seen from Table 1 that the bending stress values from the patch load are larger than what would be generated using a uniform pressure to the whole plate even though the total Local ice load is smaller. From Figures 2 and 3, note that the regions of high stress only cover a small portion of the plate surface and from Table 1 the load patches are small in area. Stress values for the uniform loadings with the simply supported and fixed boundary plate were compared with published values (Batista, 2010), the agreement was better than 0.2%.

Table 1. Parameters for Loadings Illustrated in Figures 2 to 4

	Central Patch (Figure 2)	Offset Patch (Figure 3)
Patch width (x)	0.295 (m)	0.395 (m)
Patch height (y)	0.190 (m)	0.595 (m)
Patch area	0.056 (m^2)	0.235 (m^2)
Patch Pressure	55.6 (MPa)	20.4 (MPa)
Patch Load	3.1 (MN)	4.8 (MN)
Max Bending Stress	317.7 (MPa) (centre)	-487.6 (MPa) (edge)
Max Bending Stress for uniform loading (Figure 4)	86.0 (MPa) (centre)	-172.3 (MPa) (edge)
Load for uniform pressure (Figure 4)	10.3 (MN)	10.3 (MN)

The worst-case bending stress calculations were repeated for plate widths from 1.0 to 5.0 m. The bending stress, patch area, patch aspect ratio and stress ratio plots are given in Figure 5.

In each plot of Figure 5 three lines are given, corresponding to the central stress for simply supported plate (SS Centre), central stress for the fixed plate (CC Centre) and edge stress for the fixed plate (CC Edge). For a plate width of larger than about 2.5 m, (plate aspect ratio >2.5) there is little variation in stress, patch area or patch aspect ratio. The patch aspect ratio is horizontal divided by vertical lengths. The stress ratio plot provides the ratio of stress generated with the worst-case patch to the stress generated for the uniform plate loading using (1). In all three lines the stress ratio is larger than 1.0 indicating that the uniform loading is not the worst case design loading and that the patch loading is a more severe loading.

In generating Figure 5 it has been assumed that the pressure-area relationship given by (1) applies to arbitrary small area patches. In ISO (2010) data for the Local pressure are only shown for area of greater than about 0.1 m^2 and ISO (2010) does not indicate a specific lower limit for the validity of equation (1). While a limit of 0.1 m^2 would appear to be adequate for the uniform loading of a plate, the data from Figure 5 shows worst-case load patches having an area of less than 0.1 m^2 even for a 1.0 m high basic design plate. For a 0.05 m^2 area load patch shown in Figure 5, the applied ice pressure would be from (1) 60 MPa. For smaller panel heights the load patch dimensions will be proportionally smaller and ice pressures higher.

There is thus a need to define the pressure-area relationship at an area of less than 0.1 m^2. From field measurements, see Spencer (2014) for a discussion, small scale pressures up to a maximum of only approximately 70 MPa was observed. To investigate the effect of potential deviations from the power-law pressure-area relationship at small area, the pressure is limited at an area of less than 0.1 m^2 to the pressure at 0.1 m^2 (37.1 MPa). Calculations indicate that the plate bending stress was between 95 and 98% of the unclamped case for the fixed boundary and central pressure. Calculations were also performed when the height of the plate was reduced from 1.0 m to 0.5m and the width ranged from 0.5 to 2.5 m. The smaller plate size had a smaller patch size and the effect of limiting the pressure was that the bending stress was now between 65 and 80% of the unclamped value. These calculations indicate that the details of the pressure–area relationship at area of less than 0.1 m^2 are important.

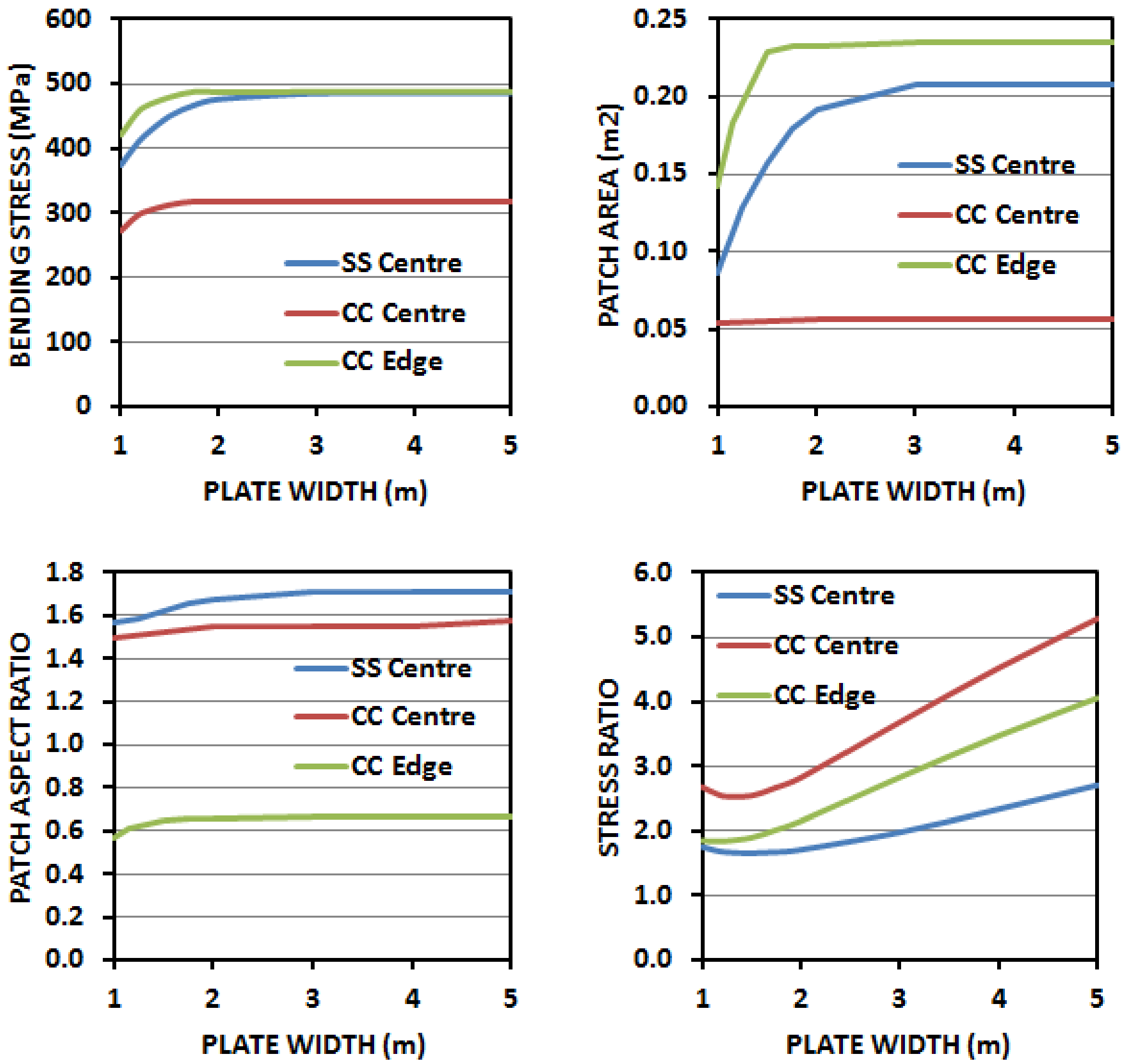

Figure 5. Worst-Case Patch Loading

LOCAL PRESSURE-AREA FORMULATIONS

There have been a number of pressure-area relationships for Local ice pressures for GBS design. Some of these are given in Table 2 along with the parameters defining them (Spencer, 2014; ISO, (2010); CSA, (1994); see Masterson et. al., (2007) for Molikpaq design).

Table 2. Local Pressure-Area Formulations

	Spencer	ISO	CSA	Molikpaq
Offset (MPa)	0.766	0	0	0
Multiplier (MPa)	7.916	7.400	8.100	5.119
Exponent	-0.716	-0.700	-0.500	-0.400

As can be seen, the exponent on the area varies from -0.400 for the Molikpaq design to -0.716 for the relationship given by Spencer (2014). The Local pressure as a function of area is given in Figure 6 for these four relationships for areas between 0.1 m^2 and 10.0 m^2.

From Figure 6 it can be seen that there are considerable differences in pressures between the various formulations. The tributary area for bulkheads in many GBS designs is a few square meters. From Figure 6 the difference in pressure is relatively small at these areas compared with the differences at an area of 0.1 m^2. Thus the effect of the pressure area relationship on bulkhead design may not be too severe.

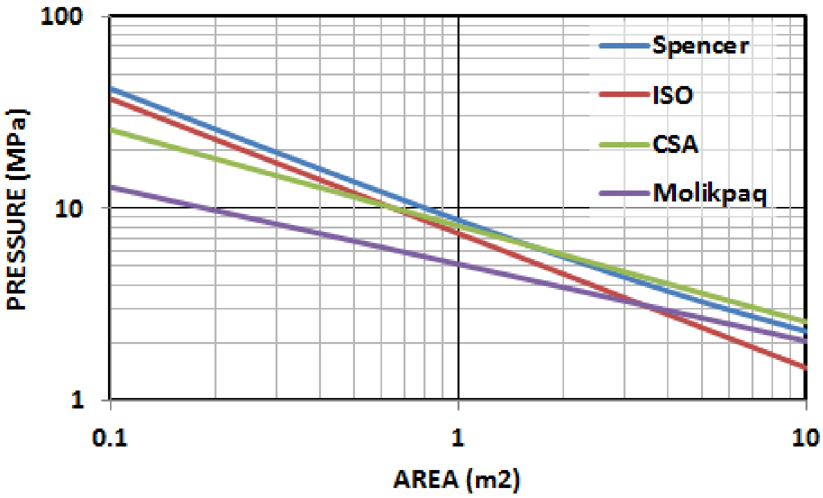

Figure 6. Local Pressure-Area Relationships

Calculations using patch loading were done for these four pressure-area relationships to illustrate the effect of the various exponents and multipliers on the plate bending stress. The results for the central stress in a simply supported plate are presented in Figure 7. There are similar trends for the central bending stress for the fixed plate boundary conditions.

From Figure 7 the patch loading produced a range of central bending stress with the largest stress occurring for the pressure-area exponent = -0.716 (labelled Spencer) and the smallest with pressure-area exponent = -0.400 (labelled Molikpaq). The area of the worst case load patch also shows considerable variation with the largest area being generated from the smallest pressure-area exponent (= -0.400).

The applied ice pressure in the worst case patch also shows a similar trend with the type of pressure-area relationship. At a large aspect ratio, Figure 7 shows a patch pressure varying from a low of approximately 5 MPa to a high of approximately 25 MPa.

The stress ratio data indicates that the effect of the patch loading on the worst case stress is more pronounced for the steeper pressure-area exponent. For the case of the Molikpaq where the exponent =-0.400 the stress ratio is less than 2.0 whereas for the ISO line the stress ratio is as large as a factor of 5.0.

The results shown in Figure 7 illustrate that the value of the pressure-area exponent has a major effect on the worst-case patch loading geometry. With the more extreme exponent, the worst case patch area becomes smaller and the effect of the patch loading compared with the uniform loading becomes larger. The plating on ice going vessels appears to be designed using a different pressure-area relationship of -0.1 (pers. Comm. Andrew Kendrick, 2014). For such an exponent and using data shown in Figure 7, uniform plate loading would likely be appropriate for design.

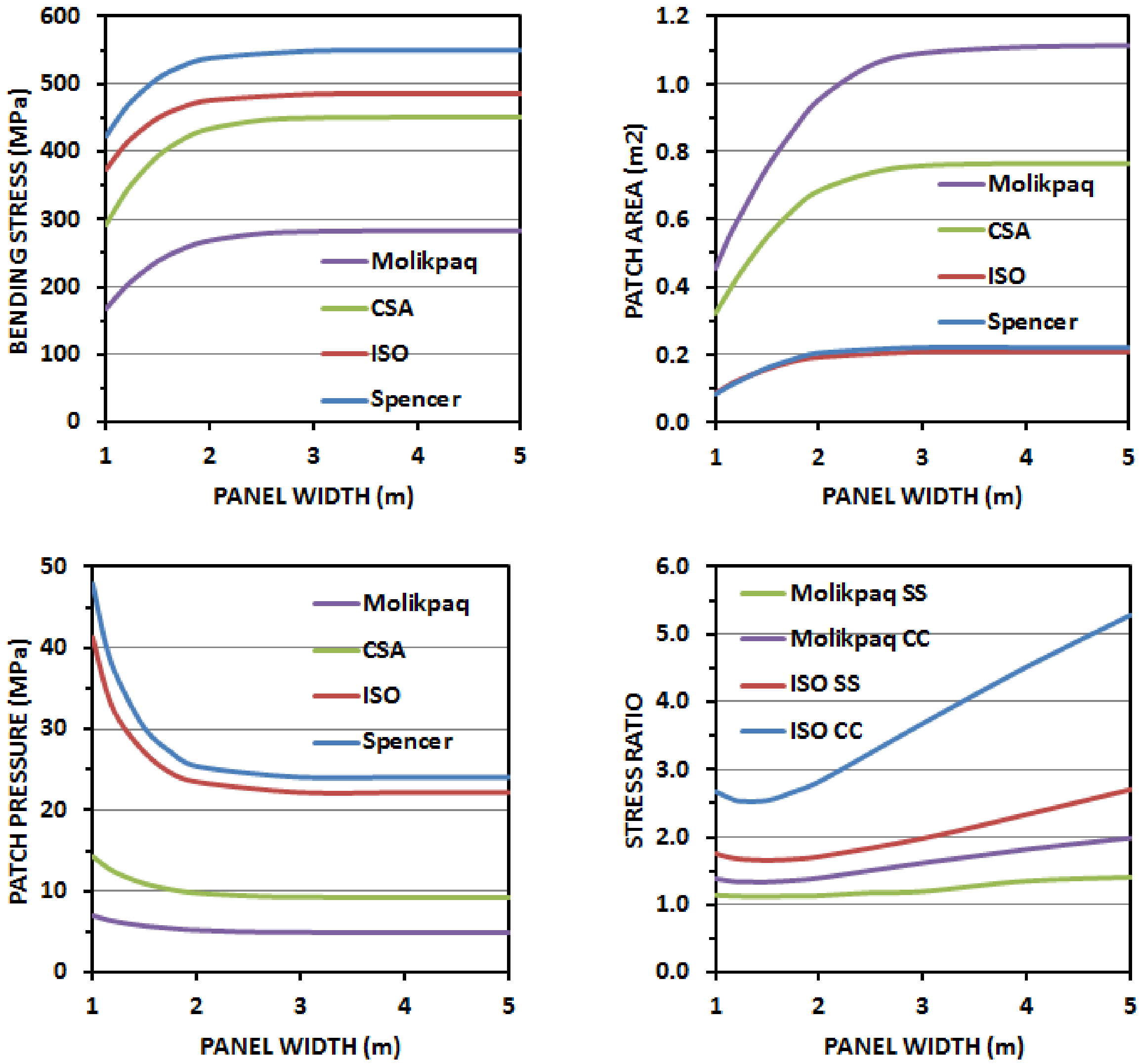

Figure 7. Simply Supported Plate Bending Stress and Pressure-Area Relationships

DISCUSSION

For initial structural design, established design codes are often used to estimate the shell plating thickness (pers. Comm. Alec Bound, 2014), as an example DNV (2011). Section F 301 of this code is a procedure for determining the minimum plating thickness. A review of this formula indicated that it is applicable to a uniformly loaded plate and not the non-uniform patch loading discussed in the current paper. It also contains the inherent difficulties associated with increasing plate width illustrated earlier in Figure 1. We note that section 5 paragraph F 301 of DNV (2011) appears to address the bending at the centre of the plate and not the (larger) stress at the midpoint of the long edge for clamped plates, see Figure 3 for a bending stress contour plot. As shown in Figure 3, the high stress at the plate edge is much more localised compared with the high stress at the centre of the plate.

The boundary conditions for the real plates in a GBS are closer to the fixed conditions than to the simply supported conditions. As was shown in Figure 5, the patch area for the worst case central stress for the CC condition is smaller than for the SS condition. Thus if this CC plate is being considered, then the patch area is likely to be smaller than the lower limit of validity of the ISO pressure area trend. Thus the central stress is likely to be overestimated for the small dimension load patch. Spencer and Morrison (2015) indicate that the use of ice strengths obtained using a Borehole Jack (BHJ) may assist in the defining of a pressure area relationship for contact areas smaller than the current 0.1 m^2.

The GBS designer may select to use elastic, plastic, membrane action or other approaches in determining the plate thickness. In all of these design methods a loading scenario has to be chosen. The small patch area with corresponding high pressure can generate a large ice load, as given in Table 1. The shear stress generated in the plate should also be checked, regardless of the bending stress considerations. The purpose of this paper is not to design the GBS steel plating when subjected to ice loading, but to provide the designers with some guidance on what ice loading geometries to use. From the data presented in Figure 5 for panels with an aspect ratio of greater than approximately 2.0, a fixed area patch and fixed aspect patch can be used. The actual worst case patch area and aspect ratio depends on if the panel has simply supported or fixed support conditions.

CONCLUSIONS

The bending stress in steel plates was evaluated using linear elasticity, thin plate theory with simply supported or fixed boundary conditions. The analysis presented here has shown:

- Uniform loading of the steel shell plating of a GBS is not, in general, a worst-case loading situation for evaluating plate bending stress from local ice pressure
- The exception is where the pressure-area relationship has an area exponent of close to zero.
- For the ISO 19906 local pressure-area relationship (Pressure = 7.4 $\text{Area}^{-0.7}$), a small dimensioned load patch can produce a bending stress in the plate up to 5 times larger than from the uniform loading of the plate.
- There is a need to determine and define a local pressure-area relationship for contact areas of less 0.1 m^2 to extend the formulation given in ISO 19906.
- Caution is needed in using standard design codes in dimensioning steel plating for GBS. These codes may not explicitly include consideration of the effect of the pressure-area relationship on worst-case plate bending stress.

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