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Combined translational and rotational sliding of gravity based structures loaded by ice

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ABSTRACT
Structures in ice-infested waters are subject to horizontal and vertical ice actions. These actions will be transferred to the base of the structure and the subsoil. For gravity based structures (GBS) various verifications need to be performed in a stability analysis. These verifications are similar to the stability of a shallow foundation, such as sliding, overturning and bearing capacity.

The methods presented in most handbooks are basically 2D and consider actions in the x-z (vertical) plane. This paper however focuses on actions in the x-y (horizontal) plane. The effect of eccentric loading in the x-y-plane, causing both rotational and translational effects, is investigated. The method is applied to circular structure and rectangular structures. Furthermore these structures are modelled in a 3D FEM model to verify the results.

INTRODUCTION
There is a growing interest in building near shore port structures and offshore oil and gas exploration and production platforms in Arctic and cold regions. Structures like these in ice-infested waters are subject to horizontal and vertical ice actions. The load scenarios that need to be taken into account for design depend on both the ice regime and the type of structure. Structures need to be checked for global and local loads which have a strong dependency on the loading width. In addition wide structures can be subject to eccentric loading due to ice actions at the edge of the structure. These ice actions will be transferred to the base of the structure and the subsoil. For gravity based structures (GBS) various verifications need to be performed in a stability analysis. These verifications are similar to the stability of a shallow foundation, such as sliding, overturning and bearing capacity. In fact the verification of the bearing capacity for inclined loading also includes the other two. Both the horizontal and vertical actions are considered, and the overturning moment at the base. Various publications have been written in this subject and reference is made to Randolph (2011).

The methods presented in most handbooks are basically 2D and consider actions in the x-z (vertical) plane. This paper however focuses on actions in the x-y (horizontal) plane. It will be further investigated what the effect is of eccentric loading in the x-y-plane, causing both rotational and translational effects. A new analytical method is developed based on the analogy of a beam that is designed for a combination of loads. The method is applied to a circular and rectangular structure. Furthermore these structures will be modelled in a 3D FEM model (PLAXIS). The yield function has been determined by incremental loading in the FEM model for a range of loading conditions. These results are compared with the analytical method; conclusions and recommendations are made.

ANALYTICAL METHOD FOR ECCENTRIC HORIZONTAL LOADING

Geometries

The analytical method has been developed for both circular and rectangular shapes, which are most commonly used. Three different geometries have been selected with the same foundation area equal to 2000 m2, these are:

- Circular GBS with 50.46 m diameter.
- Square GBS of 50 m by 40 m.
- Rectangular GBS of 100 m by 20 m.

The circular and square geometry are almost the same size and are therefore interesting to compare. The geometry is typically comparable to a foundation structure of a platform or jetty. The rectangular structure is twice as long as the square structure and can be compared too. The geometry is typically of an ice protection structure, see figure 1.

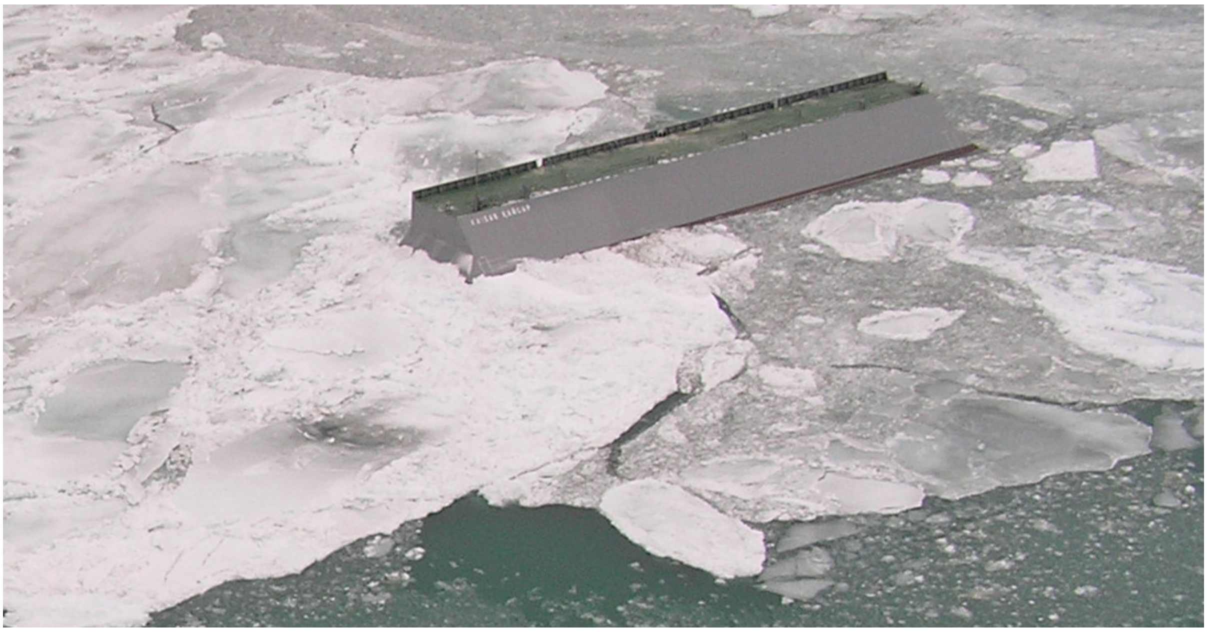

Figure 1. A steel ice protection structure in the North Caspian Sea.

For all geometries a range of eccentric loads has been applied. This is done by first applying the load over the full width and then in reducing the exposed width in steps of 10%. The following series of load cases have been used:

- In the first series the load per linear metre is constant, with decreasing exposed width, the total load decreases too.
- In the second series the total load on the GBS is constant, with decreasing exposed width, the total load remains constant.
- In the third series the load per linear meter is a function of the exposed width, with decreasing exposed width the load per linear metre increases but total load decreases. This last series is typical for ice loading.

Figure 2 presents an example of the first series with an exposed width of half the structure. These load series should be regarded as representative for structures loaded by ice as an eccentricity larger than the structure width is not realistic. It would be interesting to investigate the maximum eccentricity for any structure in Arctic conditions as normally the focus is on the maximum exposed width.

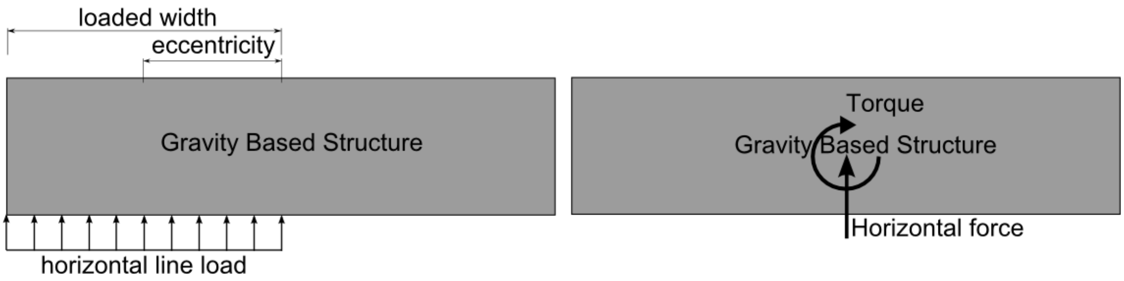

Figure 2. GBS with eccentric loading in top view (x-y plane).

Analytical solution
Analytical solutions exist for either translational or rotational sliding. These equations are used in structural engineering to determine the plastic capacity of a beam subjected to shear force and torsion. This can also be applied to the base resistance of the GBS. The soil is modelled as a Coulomb material, with strength parameter the undrained shear strength. Similar analysis can be performed for a frictional soil but will not be discussed in this paper as it will complicate the analysis.

With translational sliding the resulting horizontal force is at the centre of the base, so no eccentricity or accompanying torque. With rotational sliding there is only torque and no resulting horizontal load. The equations for a circular GBS are presented below:

$$
\begin{aligned}
R_{TS} &= \tau \cdot A \\
R_{RS} &= \tau \cdot \frac{I_{c;r}}{d_{c;r}} \\
I_c &= 0.5 \cdot \pi \cdot r^4 \\
I_r &= \frac{W \cdot L^3 + L \cdot W^3}{12}
\end{aligned}
\qquad (1)
$$

With:
R_{TS} is the translational sliding resistance in [kN].
R_{RS} is the rotational sliding resistance in [kNm].
τ = soil shear resistance (undrained shear strength Su) in [kN/m^2].
A = area of GBS in [m^2].
r = radius for circular GBS in [m].
I_c = rotational moment of inertia for circular GBS in [m^4].
d_c = equivalent plastic distance for circular GBS, taken as 2/3·r, in [m].
L = length of the rectangular GBS in [m].
W =width of rectangular GBS in [m].
I_r = rotational moment of inertia for rectangular GBS in [m^4].
d_r = equivalent plastic distance for rectangular GBS, taken as the diagonal/3.6, in [m].

With these equations translational and rotational sliding can be checked seperately. However a GBS can be subject to eccentric (horizontal) ice loading. This condition causes a combination of translation and rotation.

This similar to a beam subjected to shear force and torsion. There are basically two ways to deal with this:

- Method 1: the outer part of the section is used to accommodate the torsion, whereas the inner part is accommodated for the shear force.

- Method 2: for both torsion and shearing the utilization factors is determined and combined to one overall utilization factor.

Method 1 can be applied to a GBS. The GBS can be regarded as stable if the sum of the area required for both translational and rotational sliding is less than the total area. The method has some disadvantages as it requires quite some steps and provides no clear definition of the stability factor.

Method 2 can be applied to a GBS. For both conditions the utilization factor can be determined, which is defined as:

$$U_{TS} = \frac{F}{R_{TS}} \qquad U_{RS} = \frac{M}{R_{RS}} \tag{2}$$

With:
U_{TS} is the utilization factor for translational sliding [-].
U_{RS} is the utilization factor for rotational sliding [-].
F is the horizontal force action in [kN].
M is the horizontal torque action in [kNm].

The combined sliding utilization factor is then defined by the following approximation:

$$U_{CS} = U_{TS} + U_{RS} \tag{3}$$

The stability factor is the reciprocal of the utilization factor (SF=1/U) and stability factors higher than 1 are representing a stable situation. Method 2 is preferred above method 1 as it provides the utilization factor and will therefore be used in this paper.

Results

The results of the analytical model will be presented in some graphs. Figure 3 presents the yield function (stability factor=1) for a series of (horizontal) forces and torque due to load eccentricity. The three lines represent the 3 geometries (circular, square and rectangular).

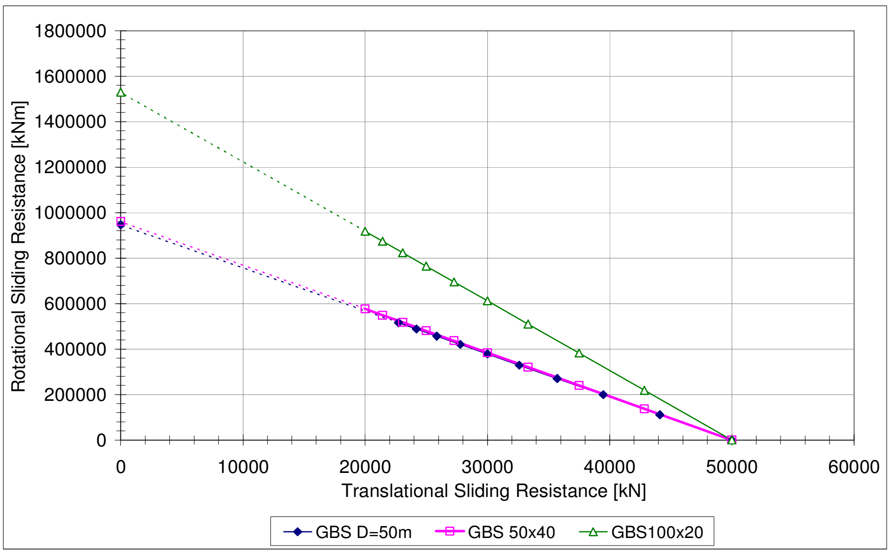

Figure 3. Yield function sliding resistance analytical model.

From the comparison between the circular and square geometry it can be concluded that the results are almost similar. From the comparison between the square and rectangular it can be concluded that the rectangular has the same translational sliding resistance but a significantly larger rotational sliding resistance.

Figure 4 presents again the yield function (stability factor=1) for the 3 geometries (circular, square and rectangular). Here the maximum (horizontal) force is presented against the relative eccentricity, defined as the absolute eccentricity divided by the diagonal (or diameter). For the maximum eccentricity (0.4 to 0.5) the allowable force decreases from 50000 to about 20000 kN (40%). The circular geometry has a slightly higher value.

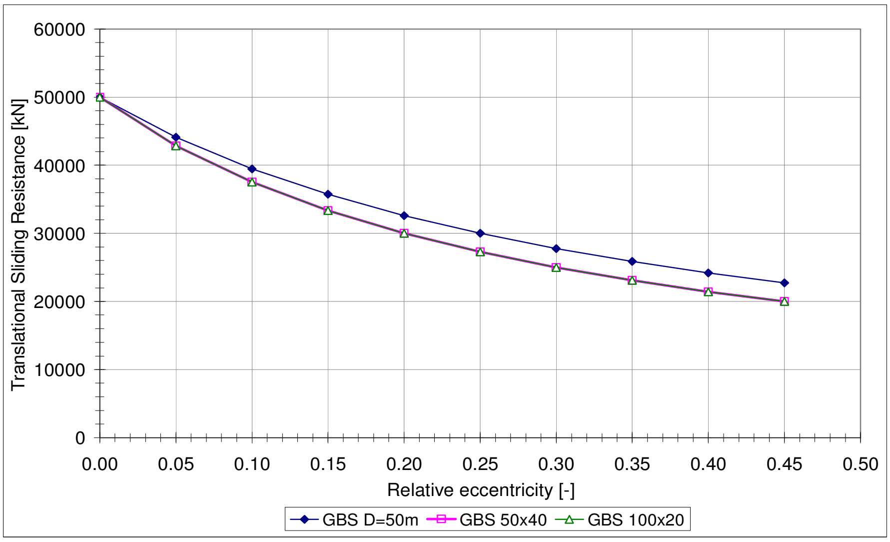

Figure 4. Maximum translational sliding resistance for eccentric loading, linear analytical method.

Figure 5 presents the utilization factor for the three series of ice loads on a circular GBS. The x-axis is the relative loaded width with 1.0 representing 100% loaded width and 0.1 being 10% of the loaded width. The upper line has a constant load per linear metre, so the total force is proportional with the loaded width. The lower one has a constant total force, regardless of the eccentricity. The middle one has the most realistic load function, with q is proportional to ($L^{-0.5}$) and F is proportional to ($L^{0.5}$). Such ice load functions can typically be found in the ISO19906 (2010). In all cases the load has been chosen such that for pure translational sliding the stability factor is 1.

It can be concluded that for a realistic ice load function ($F \sim L^{0.5}$) the most unfavourable case is loading over about half the exposed width and with an eccentric of $L_{max}/4$. This is where the utilization factor is the highest (in this example about 1.2, which is higher than 1 and thus instable). For conditions with constant load per linear metre the decisive case is for loading of about 90% of the width. For the condition with constant total force the stability factor is lowest at the maximum eccentricity. This might be the case for structure loaded by long ridges approaching a GBS at an angle or from one side.

Figure 6 presents the utilization factors for a realistic ice load function ($F \sim L^{0.5}$) as can typically be found in the ISO19906 (2010). The lower line represents the utilization of the rotational sliding resistance, the middle line the utilization of the translational sliding resistance, the upper line the utilization of the combined sliding resistance which is the sum of the other two. As already concluded from figure 5 it can be seen that the most unfavourable load case is ice loading over one half side of the structure (at 60% in this example). The combined ultilization factor is 1.2, the translation utilization is 0.8 and the rotational utilization is 0.4. The ratio between utilization of translational versus rotation is about 2:1.

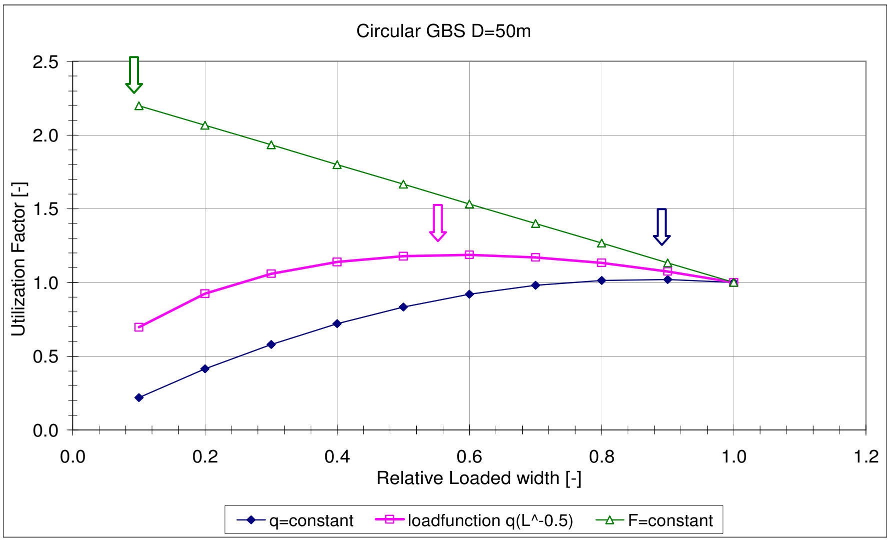

Figure 5. Example of utilization factors for various load functions and widths, linear analytical method.

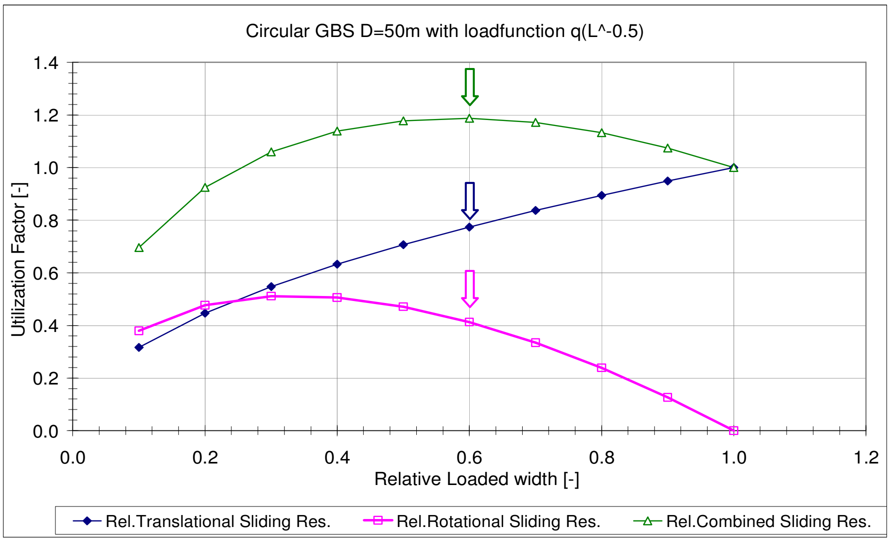

Figure 6. Example of each component in the utilization factors, linear analytical method.

DNV method

The DNV-OS-J101 (2010) provides an alternative method. In this method the verifications remain purely in translational sliding. The horizontal load is increased to account for the eccentricity or torque. The DNV (2010) equation is presented below:

$$F_{eq} = \frac{2 \cdot M}{L_{eff}} + \sqrt{\left(F^2 + \left(\frac{2 \cdot M}{L_{eff}}\right)^2\right)} \qquad (4)$$

With:

F_{eq} is the equivalent horizontal load corrected for eccentric loading in [kN].
M is the torque (horizontal load times eccentricity) in [kNm].
L_{eff} is the effective foundation width in [m].

The DNV (2010) method has the advantage of one simple verification and unity check. Also the equivalent horizontal load can be used in other shallow foundation calculations where also vertical eccentricity is taken into account. Disadvantage is however that the resulting horizontal load is artificially increased and has no physical meaning. Application of the DNV (2010) for loadcases where torque is dominant should be carefully examined. These conditions will however not occur under normal ice loading as then the eccentricity (generally) is not larger then half the structure width. Figure 7 presents again the stability factors for the most realistic load function as can typically be found in the ISO19906 (2010) for both the DNV (2010) approach as the new analytical approach. It is concluded that both methods fit very well in the normal range ice loading. This is promising as both methods have their advantages in application.

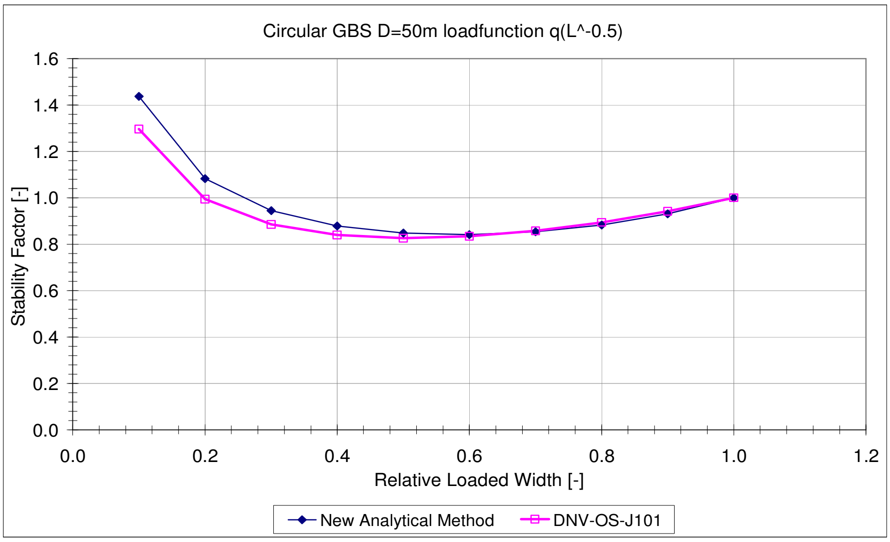

Figure 7. Comparison Stability Factors for analytical and DNV method.

3D FEM VERIFICATION

Load cases

3D FEM calculations have been performed in order to verify the linear analytical method. The calculations have been performed with PLAXIS 3D v2012. The soil has been modelled with a linear elastic-plastic constitutive model (Mohr-Coulomb). The undrained shear strength is 25 kPa and the undrained elasticity modulus is 10000 kPa. In addition another soil profile with drained frictional properties has been verified too, but it will not be presented here.

The GBS has been modelled as a 1 m thick plate with elasticity modules of 10 GPa. The FEM model size is typically 20 m wider and deeper than the GBS dimensions and has about 15000 elements. Figure 8 presents the circular GBS model and figure 9 presents the deformed mesh (scaled) of the rectangular GBS model after load application. The load cases in the FEM approach performed for the whole range, from pure translational sliding (force) to pure rotational sliding (torque) and all combinations in between. This has been done for investigation purposes to be able to derive standardized yield functions. The unity checks in the FEM calculations can be derived in two ways, either by load increase or shear resistance reduction. In these models the method of incremental loading is applied where translational force and torque increase simultaneously.

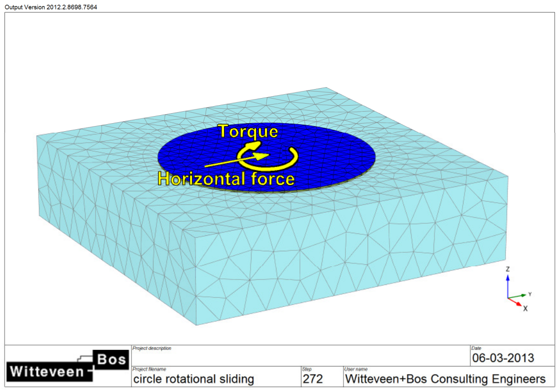

Figure 8. PLAXIS FEM model for circular GBS.

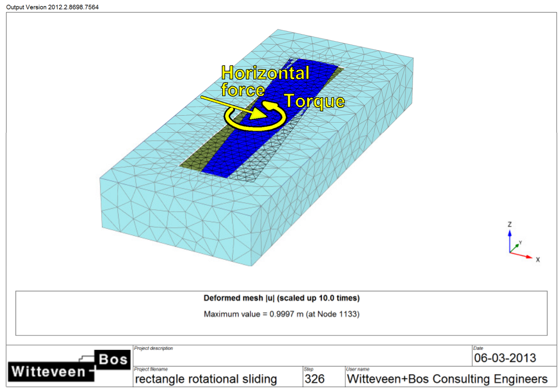

Figure 9. PLAXIS FEM model with deformed mesh of rectangular GBS.

Results

The full range from pure (horizontal) load to pure torque and all combinations in between has been calculated. The ultimate sliding resistances are presented in Table 1 together with the results of the analytical method. The circular and square show very similar results. The rectangular has a higher rotational resistance as could be expected. The total resistance for pure sliding is the same as the area is the same for all geometries. It is also the same as in the analytical method. The total resistance for pure rotational sliding is slightly less than predicted with the analytical method.

Table 1. Maximum sliding resistances.

Number	Maximum Load [kN] with no torque	Maximum Torque [kNm] with no resulting force
	Analytical / FEM	Analytical / FEM
Circular GBS D=50m	50,000 / 50,000	946,000 / 839,000
Square GBS 50x40m	50,000 / 50,000	960,000 / 864,000
Rectangular GBS 100x20m	50,000 / 50,000	1530,000 / 1300,000

Figure 10 presents the normalized yield function. The main difference with the analytical method is the shape of the yield function. It is clear that the yield function is not linear as in the analytical method. This yield function can be approached with a power (m). In figure 10 a fit with power (m) of 1.5 is presented. The equation of the yield function is presented below:

$$(U_{TS})^m + (U_{RS})^m = 1 \qquad (5)$$

From the curved shape of the FEM model it can be concluded that the combined sliding resistance is larger than derived by analytical method. The linear analytical method is thus a more conservative approach.

Figure 11 presents the yield function (stability factor=1) for the 3 geometries (circular, square and rectangular). Here the maximum force (ice load) is presented against the relative eccentricity, defined as the absolute eccentricity divided by the diagonal (or diameter). For the maximum eccentricity (0.4 to 0.5) the allowable force decreases from 50000 to about 20000 kN (40%). The circular geometry has a slightly higher value. The results are very similar to the analytical method, with a more conservative result based on the analytical method.

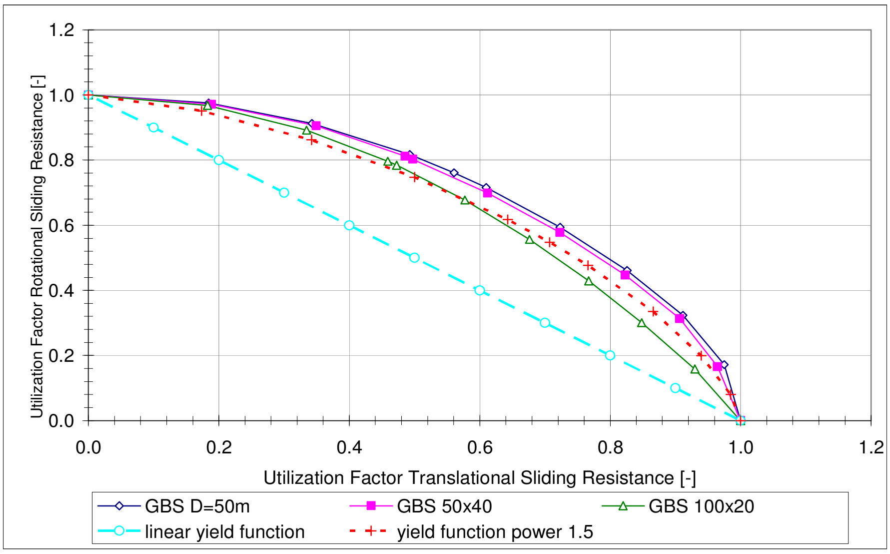

Figure 10. Normalized yield function sliding resistance, FEM model.

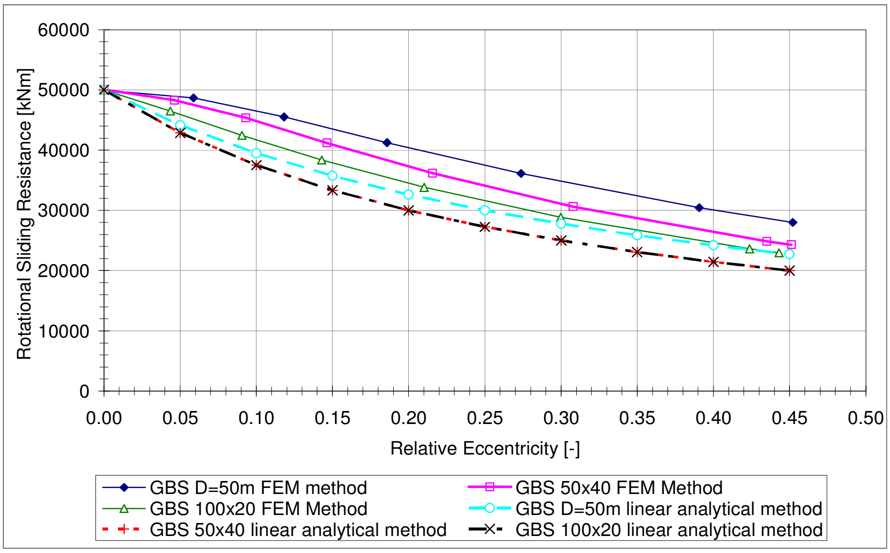

Figure 11. Maximum translational sliding resistance for eccentric loading, FEM model.

The linear analytical method can be improved with the findings of the FEM verification. This can be done by adjusting the linear function of equation (5) to a power function as equation (7). In figure 12 both the FEM results as the adjusted analytical results (with power function) are presented. For the adjusted analytical method a power of m=1.5 (circular GBS) and m=1.25 (rectangular GBS) has been applied. In particular in the range of normal engineering with eccentricities less than half the structure the fit is very close. For conditions with extreme torque the analytical method is slightly unconservative. This is probably to the fact that in rotational sliding the shear stress is not fully mobilized under the whole foundation as deformations at the rotation point are always less than at the perimeter. This is correctly simulated in a FEM model whereas a analytical method assumes full plasticity.

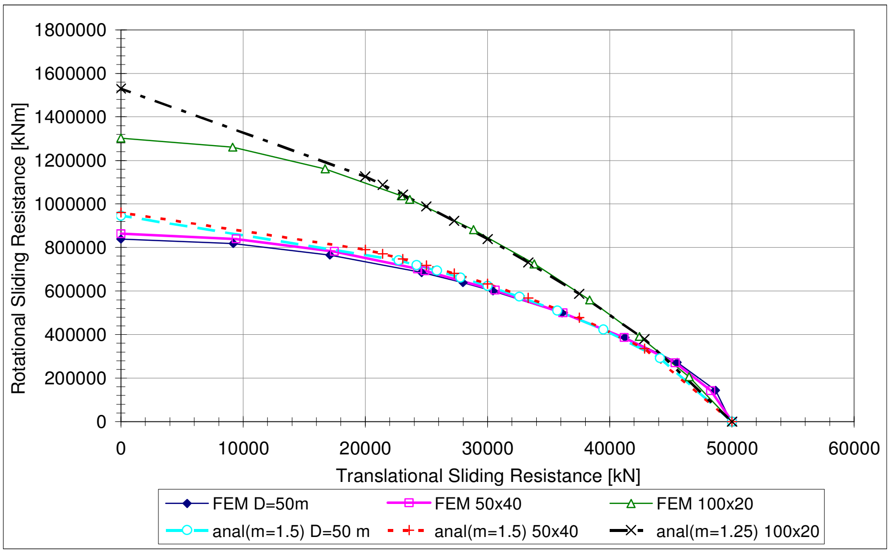

Figure 12. Yield function sliding resistance FEM model and analytical model with power function.

CONCLUSIONS

The linear analytical method as presented in this paper provides new and useful insight in the sliding resistance of GBS loaded by ice. The method has been compared to the DNV (2010) and with 3D FEM calculations.

Based on the comparison between the 3 methods it is concluded that the translational and rotational sliding resistance can be predicted reasonably well, although the FEM results for pure rotational sliding are slightly less. For normal eccentric loading conditions the linear analytical method and DNV (2010) are very close. The linear analytical method confirms in fact the existing DNV (2010) as an appropriate design standard for eccentric loading. Based on the linear analytical method is concluded that for realistic ice load conditions eccentric loading over one half side of the structure is decisive above loading over the full width. The difference in stability is about 1.2. This is also dependent on the assumption that the load per unit width increases with smaller widths, this should be checked for the relevant ice failure mode. There may be additional ice loading scenarios which should be checked for eccentric loading; such as a high force at the edge of a GBS. This might be the case for a consolidated ridge with high limit stress and limit force conditions.

The FEM analysis provides even more insight, it confirms that the analytical method can be used as lower bound method for calculation of the combined sliding resistance. Based on the FEM analysis a more accurate standardized yield function that can be determined which can also be applied to the analytical method. The adjusted analytical method has a power function with m=1.5 for circular GBS and m=1.25 for a rectangular GBS. The adjustment leads to a better fit but not necessarily to a lower bound value. It is advised to perform more FEM calculations to confirm the yield function for other geometries and soil conditions. In a next step the eccentric loading can be combined with inclined loading due to other ice load conditions.

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