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A STATISTICAL ANALYSIS OF SPATIAL STRENGTH HETEROGENEITY OF SEA ICE COVER

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ABSTRACT

Data published recently show that the ice strength in the same field varies significantly. Spatial strength heterogeneity of the ice cover was found in the field measurements of ice strength in different sites in the world: Spitsbergen, The Far East of Russia and the Gulf of Ob. In most cases the small scale heterogeneity has been extensively studied (sizes of an ice field were smaller or equal to 180 m by 180 m) and seems to be a suitable research object. The investigations of the ice strength spatial heterogeneity in first-year level ice in several regions have been done in this study. Statistical analysis of field data of the ice strength distributions was performed. The analysis consisted in calculation of the main probabilistic parameters of the ice strength distribution and determination of the theoretical distribution law fitting the experimental data. The goal of the work was to determine the strength frequency. The ratio of field area A_i corresponding to the ice strength R_{ci} to total area A_{total} of ice field was revealed to characterize the strength frequency and the probability of the strength R_{ci}. Several theoretical distributions (normal, lognormal, gamma, Weibull and Generalized beta) of the strength frequency for fields in different seas were chosen and compared.

INTRODUCTION

The formation of ice cover in full scale conditions gives varying thermo-mechanical properties, both through the thickness and in the horizontal plane. As the ice cover grows its thickness may also vary spatially due to accumulation of snow and non-constant oceanographic conditions. The variation of properties through the thickness is basically given by vertical temperature profile, but the spatial variation in the plane (hereafter simply called spatial variation) is more random. Therefore the ice strength may vary randomly in the plane and systematically through the thickness. The heterogeneity of the ice thickness is taken into account in different codes when design ice load is evaluated. But spatial heterogeneity of ice strength has not been taken into account yet.

Some recent field measurements of the spatial variation of ice strength show that an ice field contains areas of high or low strength. Truskov et al., (1996) performed field work in Far-East Russia, Kovalev et al., (2004) is the Gulf of Ob and Shafrova and Moslet (2006) in Spitsbergen. An example from the latter is shown in Figure 1, where one can observe significant ice strength variation in the field. It was decided to analyse data on spatial strength heterogeneity of the above mentioned papers. The data is represented as maps of the ice strength distribution over the area measurements in the polygon of investigations (see Figure 1).

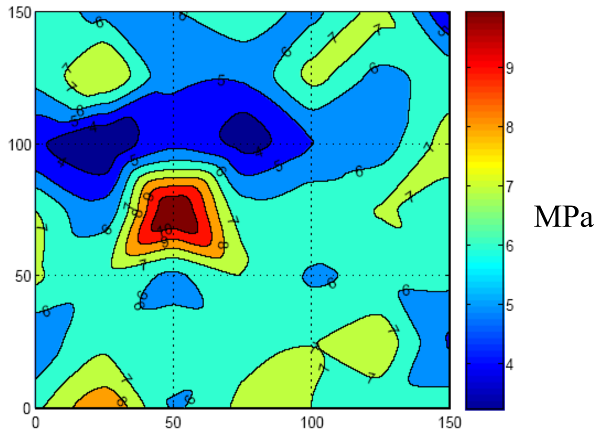


Figure 1. Distribution of the ice strength (MPa) over the area 22500 m² from Spitsbergen, 2004 (Shafrova and Moslet, 2006).

EXPERIMENTAL BASIS

Small scale heterogeneity has been extensively studied (sizes of an ice field were smaller or equal to 180 m by 180 m) and seems to be a suitable research object. Seliverstov et al (2001) proposed that the sizes of ice polygons should be not less than 100 m by 100 m because at such size the heterogeneity of ice cover is commensurable with the area of distribution of a stress-strain state zone arising at the ice –structure interaction.

All studies in present work will be done in the non-dimensional form. There it will be possible to use the method has been developed in this paper for the larger scale conditions provided that the ice strength heterogeneity in these conditions is known.

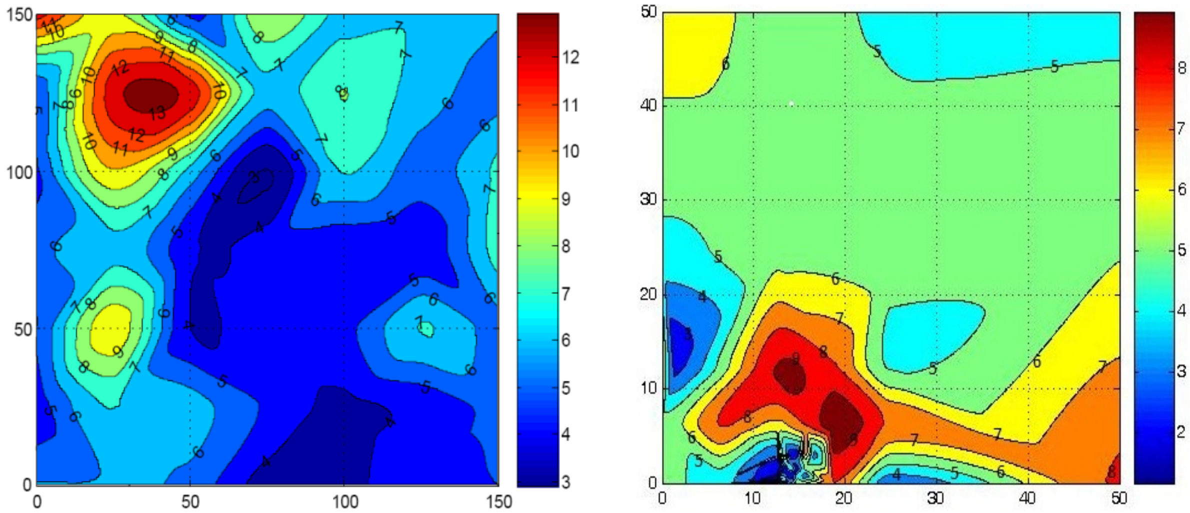
Six maps with strength distributions in ice fields in different seas have been collected. The following investigations were used for analysis:

1. Experiments in the Spitsbergen fjords (Shafrova and Moslet, 2006) given in Figs. 1-2. The ice strength was determined through the uniaxial compression tests. Vertical ice samples were taken from the land fast ice sheet and compressed. The ice was sampled from the same ice depth was assumed to have the same properties.
2. Experiments in the Far East of Russia (Truskov et al., 1996), the map is presented in Figure 3. The ice strength was determined through the uniaxial compression tests. The ice was sampled from one ice field horizon. Cylindrical ice cores were taken normal to the freezing plane and compressed.
3. The maps from experiments on The Gulf of Ob (Kovalev et al., 2004) are presented in Figure 4. The ice strength was determined through the borehole jack tests.

The general information about these ice fields is presented in Table 1. The maps with strength distributions in ice fields(4 and 6) smaller than 100 m by 100 m were taken into account for further comparison with ice fields of greater sizes.

Table 1. Location, area, strength variation and mean strength of the sites.

The ice field	Location	A_{total}, m^2	Strength variation, MPa	R_{mean}, MPa
1	The Far East of Russia	25600	1.3-3.7	2.04
2	Spitsbergen	22500	3-10	6.26
3	Spitsbergen	22500	3-13	6.17
4	Spitsbergen	2500	1-10	5.89
5	The Gulf of Ob	32400	9-25	18.22
6	The Gulf of Ob	6250	18-26	22.22



a). b).

Figure 2. The ice strength (MPa) distribution over the areas 22500 m² (a) and 2500 m² (b) from Spitsbergen, 2005 (Shafrova and Moslet, 2006).

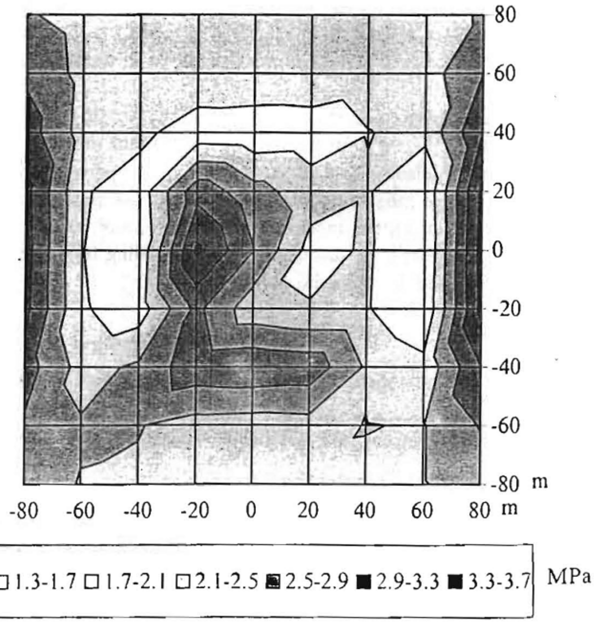
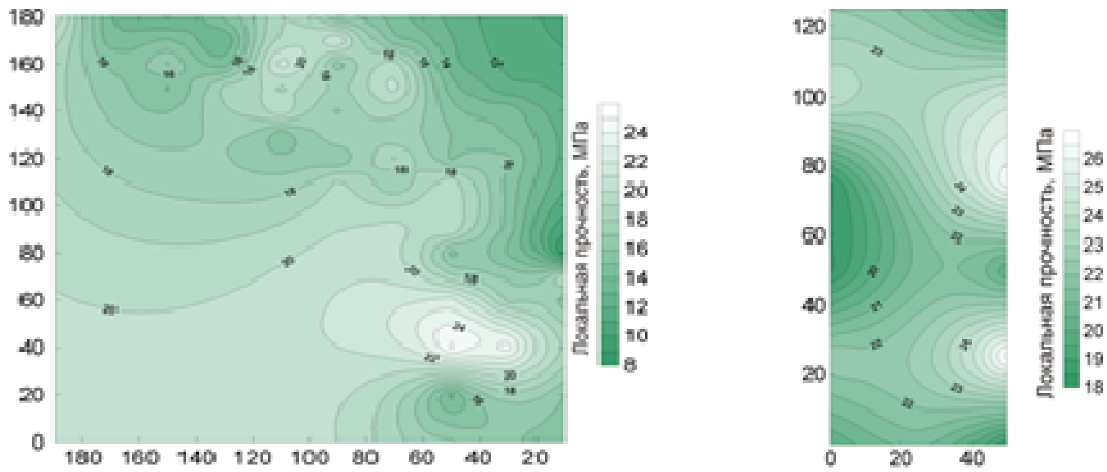


Figure 3. The ice strength (MPa) distribution over the area 25600 m² from the Northern Sakhalin shelf, 1993 (Truskov et al., 1996).



a). b).

Figure 4. The ice strength (MPa) distribution over the areas 32400 m² (a) and 6250 m² (b) from the Gulf of Ob (Kovalev et al., 2004).

ANALYSIS OF ICE STRENGH DISITIBUTIONS FROM DIFFERENT FIELDS

Calculation of the main probabilistic parameters of ice strength distributions

As can be seen (Figs. 1-4) the field data are represented as zones of different strength (in some ranges). But it is not sufficient to know the strength variation in different parts of the ice field. The part of the total polygon area corresponding to a particular strength is an important parameter that determines the practical "importance" of this strength. Therefore the task for the following investigation is to determine the strength/corresponding area link. One can meet different areas with the same strength in the same polygon. If all fields corresponding to a particular strength will be united the heterogeneity influence would be emphasized.

It may be difficult to compare the data from the different experiments because of difference of environmental conditions and corresponding ice properties. The strength level may be different and we will take this into account by introducing a ratio of the spatial strength to the mean one will not depend on the local environmental conditions.

The investigated ice fields were divided to zones with the strength in the specified intervals and the mean strength of each interval was determined. Then the mean strength of the ice field R_{mean} was calculated and the mean strength of each zone was referred to the mean strength of the ice field for the transition to dimensionless (relative) values:

$$R_{mean} = \frac{\sum R_{ci} A_i}{A_{total}} \qquad (1)$$

where R_{ci} is the mean value on the strength interval, A_i is the area of zone of the particular strength interval, A_{total} is the total area of the ice field. The areas A_i were defined using the AutoCAD program. The maps of the investigated ice fields were downloaded to the AutoCAD and the areas A_i were measured. The mean relative strength of the ice field was defined as

$$R_{\substack{mean\\relative}} = \frac{\sum R_{\substack{ci\\relative}} A_i}{A_{total}} \quad (2)$$

where $R_{ci\ relative}$ is R_{ci} divided by the mean strength of the ice field R_{mean}.
The main probabilistic parameters that characterize a random strength distribution are the mean, the dispersion (variance) D, the standard deviation σ, the coefficient of variation v, the skewness r_3, and the kurtosis r_4.

The coefficient of variation is defined as the standard deviation divided by the mean and it may be used to compare the variability of different distribution. Therefore the coefficient of variation can be assumed as the measure of the ice strength heterogeneity. The correlation between the coefficient of variation and the total area of the ice polygons is presented in Figure 5. The skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable. It is positive if the right tail of the distribution is longer than left tail, and is negative in the opposite case. The kurtosis is a measure of the "peakedness" of the probability distribution of a real-valued random variable in probability theory and statistics. If $r_4 = 3$, then the distribution is close to normal. The main probabilistic parameters of the ice strength distributions from different experiments are presented in Table 2.

Table 2. The main probabilistic parameters of ice strength distributions.

The ice field	$R_{mean\ relative}$, (-)	σ,(-)	v,(-)	r_3,(-)	r_4,(-)
1	1	0.185	0.185	0.715	3.85
2	1	0.172	0.172	0.036	4.208
3	1	0.314	0.314	1.046	3.822
4	1	0.183	0.183	0.670	3.987
5	1	0.142	0.142	-0.666	3.230
6	1	0.067	0.067	-0.027	2.960

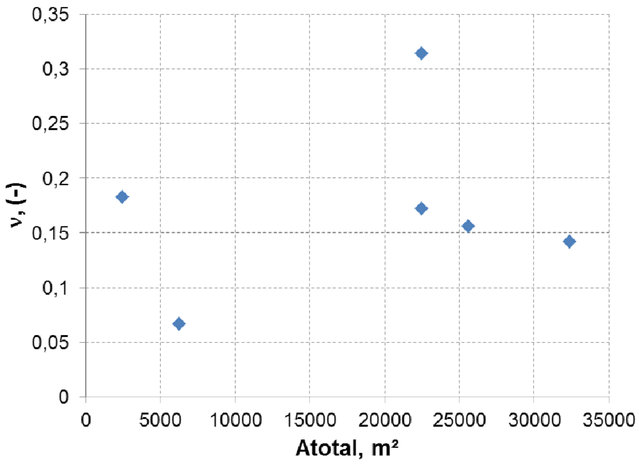


Figure 5. The dependence between the coefficient of variation and the area of the ice fields.

From Figure 5 we can see that there is no clear relationship between the coefficient of variation and the area of the ice polygon when for areas from 2500 m^2 to 32,400 m^2.

Determination of theoretical distribution law fitting the experimental data

The goal of this part of the paper is determination of the theoretical distribution law which fits the experimental data. First step is plotting the histogram of the ice strength frequencies (dependences of the correlation of the spatial strength in some interval and relative area corresponding to this interval) and experimental probability density function of the ice strength distribution over the area. The second step is comparing experimental probability density function of the ice strength frequencies with existent theoretical distributions.

Several theoretical distributions of the strength frequency for fields in different seas were chosen in order to determine the best fit distribution law: normal, lognormal, gamma, Weibull and Generalized beta distribution. All these distribution are characterized by unknown parameters, which needed to be determined.

The normal distribution is characterized by mean value μ and standard deviation σ. The lognormal distribution is characterized by parameters μ and σ, where μ is shape parameter and σ is scale parameter. The Gamma and Weibull distributions are both characterized by shape parameter α and scale parameter β. The Generalized beta distribution also is characterized by two parameters α and β, but they are both shape parameters.

The unknown parameters of the theoretical distributions were estimated using the methods of the moments (MOM), the least squares method (LSM) and the maximum likelihood method (MLM). Then the coefficient of determination R^2 was calculated for every distribution:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{\sum_{i=1}^{n}(x_i - x_{it})^2} \quad (3)$$

where x_i are observed values, $\bar{x}$ is the mean of the observed data, x_{it} *are* modelled values. The coefficient of determination R^2 has values between 0 and 1. When R^2 is closer to 1, it means that models data are closer to experimental data.

The results are summarized in Table 3 and Figs. 6-9. Weibull and Generalized beta distributions are found to be the most appropriate theoretical distribution laws fitting the experimental data. As noted above, the unknown parameters of the theoretical distributions were estimated using MOM, LSM and MLM. These methods can give different values of the parameters of the theoretical distributions. The best values of the unknown parameters (when R^2 is closer to 1 and the experimental probability density function is close to theoretical probability density function for the ice fields) were conducted using MOM and LSM methods. Comparison of the theoretical distributions for the different ice fields is demonstrated in Figure 10.

Table 3. Parameters of the theoretical distributions.

The ice field	Location	Normal distribution	Lognormal distribution	Weibull distribution	Beta distribution	Gamma distribution
1	The Far East of Russia	μ=1.000 σ=0.185 R^2=0.942	μ=-0.016 σ=0.180 R^2=0.933	α=6.200 β=1.075 R^2=0.917	α=1.366 β=3.530 R^2=0.934	**α=29.127** **β=0.034** **R^2=0.944**

2	Spitsbergen	μ=1.001 σ=0.169 R^2=0.875	μ=-0.004 σ=0.167 R^2=0.465	**α=6.880** **β=1.064** **R^2=0.954**	α=3.722 β=4.303 R^2=0.919	α=35.625 β=0.028 R^2=0.715
3	Spitsbergen	μ=0.968 σ=0.297 R^2=0.879	μ=-0.045 σ=0.296 R^2=0.462	α=3.602 β=1.073 R^2=0.915	**α=1.227** **β=2.542** **R^2=0.960**	α=10.408 β=0.095 R^2=0.738
4	Spitsbergen	μ=1.000 σ=0.183 R^2=0.942	μ=-0.016 σ=0.183 R^2=0.754	**α=6.117** **β=1.064** **R^2=0.973**	α=7.203 β=6.050 R^2=0.947	α=30.002 β=0.033 R^2=0.855
5	The Gulf of Ob	μ=1.000 σ=0.141 R^2=0.894	μ=0.006 σ=0.134 R^2=0.699	**α=8.500** **β=1.058** **R^2=0.967**	α=4.890 β=3.437 R^2=0.953	α=54.658 β=0.018 R^2=0.783
6	The Gulf of Ob	μ=1.000 σ=0.066 R^2=0.929	μ=-0.001 σ=0.066 R^2=0.868	α=18.000 β=1.030 R^2=0.891	**α=3.602** **β=3.632** **R^2=0.980**	α=222.878 β=0.004 R^2=0.893

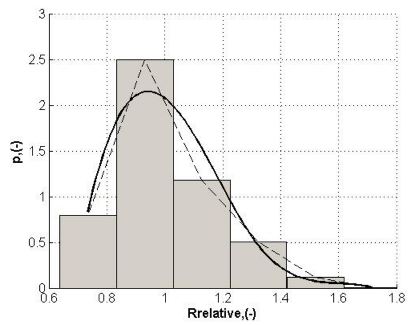
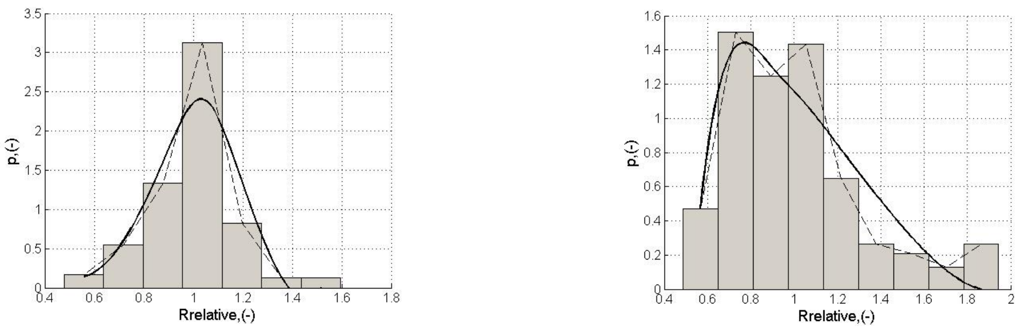


Figure 6. Density histogram for ice field 1 (The Far East of Russia).
Dashed line – experimental probability density function,
solid line – theoretical probability density function (Gamma distribution).



a). b).

Figure 7. Density histogram for ice fields 2 (a) and 3 (b) (Spitsbergen).
Dashed line – experimental probability density function, solid line – theoretical probability density function (Weibull distribution (a), Generalized Beta distribution (b)).

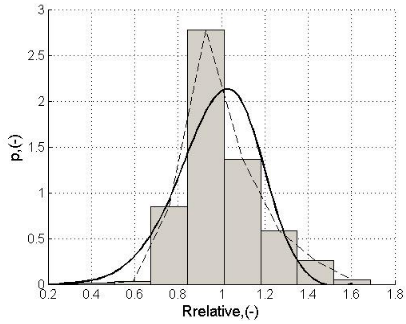
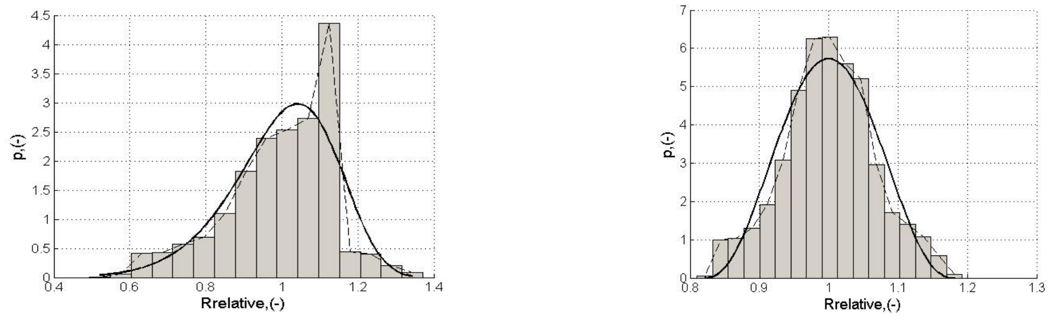


Figure 8. Density histogram for ice field 4, (Spitsbergen).
Dashed line – experimental probability density function, solid line – theoretical probability density function (Weibull distribution).



a). b).

Figure 9.Density histogram for ice fields 5 (a) and 6 (b) (Gulf of Ob).
Dashed line – experimental probability density function, solid line – theoretical probability density function (Weibull distribution (a), Generalized Beta distribution (b)).

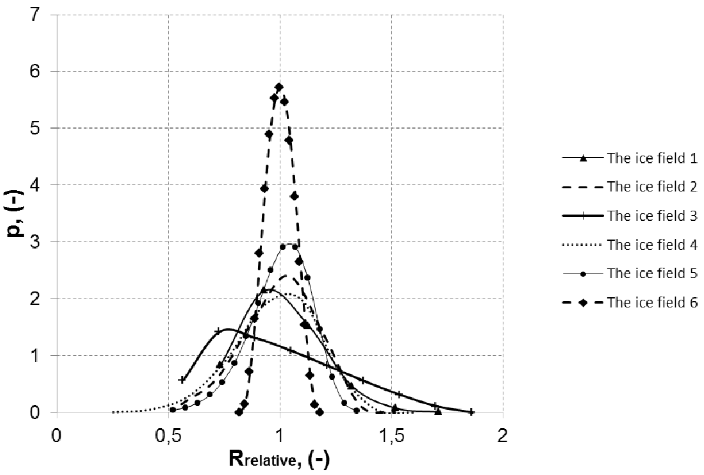


Figure 10. Theoretical distributions of the strength frequency for the different ice fields.

To find the better fitted theoretical distribution a Kolmogorov-Smirnov goodness-of-fit test was performed. The estimated parameters λ for all distributions and tests criteria λ_{crit} (with confidence probability 0.99) are given in Table 4 for distributions shown in Table 3.The tests criteria λ_{crit} are taken from statistic tables (Horev, 2002). If $\lambda<\lambda_{crit}$, the experimental distribution is close to the theoretical distribution. Parameter λ is defined as (Vadzinskiy R., 2008)

$$\lambda = D\sqrt{n}(1 + 0{,}12/\sqrt{n} + 0.11/n) \quad (3)$$

where $D = \max|p_i - H_i|$ is maximal difference between values of theoretical and experimental cumulative strength distributions,n-number of the considered above areas.

Table 4.Results of the Kolmogorov-Smirnov goodness-of-fit test for the ice strength distributions.

The ice field	λ_{crit}	λ	goodness
1	0.617	0.575 Gamma distribution	+
2	0.513	0.264 Weibull distribution	+
3	0.381	0.273 Beta distribution	+
4	0.404	0.321 Weibull distribution	+
5	0.392	0.267 Weibull distribution	+
6	0.381	0.065 Beta distribution	+

Results of the Kolmogorov-Smirnov goodness-of-fit test for the ice strength distributions showed that all distributions are close to the considered theoretical distributions.

CONCLUSIONS

A statistical analysis of field data of the ice strength distributions from different regions (Spitsbergen, Far-East of Russia and the Gulf of Ob) was performed. The analysis consisted in calculation of the main probabilistic parameters of ice strength distribution and determination of the theoretical distribution law fitting the experimental data. Themain results are:

- There is noclear relationship between the coefficient of variation and the area of the ice polygon for areas between 2500 $m^2$and 32,400 m^2.
- Several theoretical distributions (normal, lognormal, gamma, Weibull and Generalized beta) of the strength frequency for fields in different seas were chosen and compared.
- Weibull and Generalized beta distributions were found to be the most appropriate theoretical distribution laws.

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