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MODELLING SHIP PERFORMANCE IN ICE USING BAYESIAN NETWORKS

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ABSTRACT
Navigation in ice has received substantial attention over recent decades. This increased attention has led to the development of numerical and semi-empirical models that characterize ship performance in ice. These models consider numerous parameters such as; level ice thickness, ridged ice thickness and ice concentration in additive fashion. However, they fail to account for the joint effect of the above ice features on ship speed. Moreover, the effect of ice compression on ship performance is usually omitted. This paper introduces probabilistic models, based on field observations, that predict a ship's speed and the situations where a ship is probable to get stuck in ice based on the joint effect of selected ice features, such as the thickness and concentration of level ice, ice ridges, rafted ice, and ice compression. To develop the models a Bayesian Belief Network is used. The case study presented in this paper considers a single and unassisted trip of an ice-strengthened bulk carrier between two Finnish ports in the presence of challenging ice conditions and the obtained results show very good prediction power of the models.

INTRODUCTION
Ship performance in ice has been given a lot of attention in the recent years. This increased attention has led to the development of methods that estimate ship resistance in ice, see for example (Lindqvist, 1989; Kaj Riska et al., 1997) and tools that simulate ship transit in ice, see (LaPrairie et al., 1995; Mulherin et al., 1996; Su et al., 2010; Lubbad and Løset, 2011). These simulation tools are used in detailed evaluation of required power for a ship at the design stage or for predicting her performance in ice at the operational stage; however the ice conditions which are modeled are often limited to the level ice and ice channel. In select cases, the effect of ice ridges was also taken into account, see for example (Kaj Riska et al., 1997). The effect of ice compression on ship speed has not been researched in-depth, thus it is usually expressed in a qualitative manner, see for example (Mulherin et al., 1996). However, a recent study has allowed the quantification of compression on ship performance using a concept of added resistance, simply produced by an ice sheet in contact with a ship's hull, see (Kaups, 2012) .

Most of these models evaluating ship performance are deterministic, and they require detailed input data in terms of ice features and ship characteristics. If such models are to be used for operational purposes, where the estimation of ice conditions is burdened with significant uncertainty and the parameters of ships are not readily available, the reliability of an outcome of such modeling process can be questioned. Although the above mentioned methods have been utilized for optimizing shipping routes in ice-covered waters, see (Kotovirta et al., 2009), there are still numerous issues which need further studies, for instance: the effect of ice compression on ship performance, or the quantification of the joint effect of ice conditions (level ice, ridges, compression, the relative angle at which ice reacts on a ship) that can bring a ship to a halt. Moreover, suggestions have been made to move towards probabilistic models, see for example (Kotovirta et al., 2009).

The most common modeling practice in the field is to adopt quantity-oriented models, which describe the relation among a set of input variables and the factor of interest. The most challenging task here is a proper reflection of the joint effect of ice conditions on a ship speed and to provide sufficiently detailed input variables. Therefore another modeling technique can be adopted leading to an event-oriented model, where the conditions under which an event of interest occurs are reflected. This type of modeling does not provide an insight into the physics of the process of ice breaking, but simply quantifies the joint effect of various ice features on ship speed and determines the most probable speed of a ship along a given route. The identification of areas and/or sea states that should be avoided, due to a high calculated probability for a ship to experience significant speed reduction could also be generated.
There are event-oriented models estimating ship speed in given ice conditions which are based on the full scale measurements, but their focus is either on a single ice feature affecting ship speed, as the ice thickness see, (Haas et al., 1999); or the model relies on a subjective interpretation of existing ice conditions, based on visual observation carried out from a ship, see (ENFOTEC Technical Services Inc. et al., 1996). Therefore the joint effect of the relevant ice features on ship performance is not addressed.

This paper introduces a probabilistic framework for predicting performance of a ship navigating in ice, meaning the probability for a ship to attain certain speed, considering the following parameters: thickness and concentration of various types of ice such as, level ice, ridged ice, and rafted ice. Ice compression and its relative direction with respect to a ship is taken into account as well. Bayesian learning techniques have been adopted and detailed records on ship movements obtained from AIS system are utilized together with the information on ice conditions which are obtained from the ice forecast model, called HELMI, which has been developed at the Finnish Meteorological Institute, see (Haapala et al., 2005). The structure of the model, which is developed here, and the probabilistic relations among the input parameters are defined by applying two learning algorithms, called PC and Naïve Bayes. Finally, the obtained framework has been cross-validated and the final results show very good convergence with the real world conditions. The obtained models and their results are valid for a particular ship type, which is an ice going bulk carrier with ice class of IA Super - see Table 1 - which is navigating under certain set of hydro-meteorological conditions.

METHODS AND MODELS

The approach taken towards development of the probabilistic models presented in this paper utilizes techniques of Bayesian learning from data. For this purpose, two learning algorithms were used to find correlation and causation in the data to develop probabilistic models capable of forecasting ship performance in ice. For the description of the algorithms and discussion about they usability the reader is referred to the following publications (Friedman et al., 1997; Cheng & Greiner, 1999; Acid et al., 2004; Darwiche, 2009). These models first determine and quantify the relations between all the analyzed variables and second they quantify the joint effects of ice features on ship speed, allowing probabilistic analysis of ship performance in ice, for any given ice forecast. From the engineering perspective, quantification of the joint effect of various ice features on ship performance is a novel approach.

To develop the models, two data source were used. First the reanalyzed ice forecast for the sea area was taken under consideration. This dataset provides information about ice features for the interval of 1 hour and spatial resolution of 1 by 1 NM. Second, the database containing ship positions was analysed. This analysis included ship course and speed obtained from AIS, recorded with an interval varying between 10 sec to 3 min, depending on the speed of the vessel. Then, these two data-sources were matched in tempo-spatial fashion, and Bayesian machine learning techniques were applied to determine models' structure and contents. The obtained models and their results are valid for a specific ship type, which is an ice going bulker with very high ice class (IA Super) - see Table 1 for ship data.

Date and the area of interest

The date and area of interest have been selected specifically to capture challenging ice conditions, meaning high concentration of level ice, the presence of ridged ice and ice compression that changes in time. For this reason, the day of 6^{th} of March 2011 was chosen and the sea area between two Finnish harbors, in the Bay of Bothnia on the Baltic Sea, called Vaasa and Kokkola was selected, see Figure 1. The case study presented here is based on records of a single trip of the bulk carrier between two positions after boarding of a pilot, meaning that the stage of high-sea navigation is considered, where the ship is supposed to proceed with full engine power. During her trip, the ship covered distance of 94 NM in 12 hours, and the ice conditions hampered her significantly, making her ram the ice several times, and forced her to idle in ice for three hours. This means that the data set can be considered appropriate for the model development; as it contains significant variability in the recorded data and strong effect of environmental conditions on ship's speed, see Figure 2.

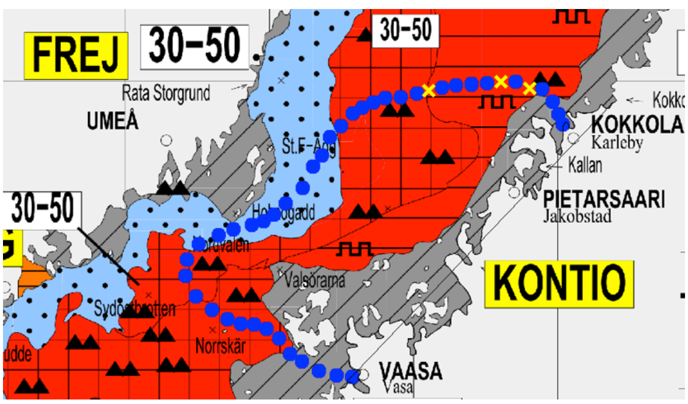

Figure 1. Ice chart representing ice conditions for the analyzed date and location.

Ship data

For the purposes of this study a ship navigating alone was selected, meaning she was not assisted by an ice-breaker nor following a convoy. The chosen ship is a general cargo ship having the ice class of IA Super according to Finnish-Swedish Ice Class rules. This means that she has such structure, engine output and other properties which make her capable of navigating in difficult ice conditions without the assistance of icebreakers. The design requirement for this ice class is a minimum speed of 5 knots in 1 m thick brash ice channels with a 0.1 m thick consolidated layer of ice on top, see (Transport Safety Agency, 2010). The ship is equipped with an "ice knife" at the bow, which additionally eases the process of ice breaking. In order to quantify the joint effect of ice conditions on ship speed, the following parameters of ship motion were retrieved from the AIS records: time, ship position, speed over ground, course over ground, and true heading. Then for each time step and position, the relevant ice characteristics were obtained from the ice forecast model. Once the ship parameters had been aligned with the ice forecast, one additional parameter was calculated allowing for the quantification of an effect of direction of compression with respect to a ship at a certain speed. This is called "relative direction of compression", which is an angle between ship's centre line and the resultant direction of the ice compression. This parameter is expressed on a scale from 0 deg (ice pressing from the bow) to 180 deg (ice pressing from the stern), where the value of 90 deg means that the ice compression acts perpendicularly to a ship.

The resolution of data describing ship motion is much finer (10 sec) than the data obtained from the ice forecast (1 hour time interval and 1x1 NM in space), therefore a code in Matlab was created to extract the ice features which correspond to ship location in the given time. The difference in resolutions produces the variability in the value of parameters describing ship performance even if the modeled ice conditions remain unchanged, see Figure 2. This effect is removed at the stage of model development, where all variables are discretized and divided into classes, as required by Bayesian learning algorithm which has been applied. Thereby, the speed of ship has been mathematically binned into three classes as follows: *below 5 kn*, *between 5 and 10 kn* and *above 10 kn*. The variability of speed given constant ice conditions occurs mostly within the classes, see Figure 2. However, one important parameter, which describes ship's inertia and her ability to break the ice, meaning mass of the ship (displacement) cannot be determined accurately. We know that the ship was steaming from Riga with the load of coal, and her first arrival harbor was Vaasa, where she discharged some amount of coal, and then she continued to Kokkola to discharge the remaining cargo. Unfortunately, specific load quantities of the ship are unknown, thus we made an assumption about half-load conditions on her arrival to Kokkola.

Table 1. Ship's data

Type	General cargo
Ice class	IAS
DWT	21353 t
Length	149.3 m
Breadth	24.6 m
Draught	9.4 m
Power	9720 kW

Ice data

All the ice data used to develop the probabilistic models presented in this paper was obtained from the reanalyses performed with the use of the HELMI multicategory sea-ice model, see (Haapala et al., 2005). The HELMI model resolves ice thickness and concentration for five level ice categories. These categories include rafted ice and ridged ice together with the thermodynamics of sea-ice, horizontal components of ice velocity and internal stress of the ice pack; the latter refers to ice compression.

Ice motion is determined by the momentum balance equation, which takes into account the Coriolis force, wind and water stresses, sea surface tilt term and an internal stress. The magnitude of internal friction is used as the principal model variable to describe compression. It is to be noted that the viscous-plastic rheology does not describe elastic stresses and the internal stress arises from the interactions of moving ice. Forces arising in a static ice are included by assuming a negligibly slow viscous creep. Roughly, the internal friction term can be interpreted to describe the forces arising when ice floes are pushed and sheared against each other, or broken and heaped into ridges. Thus it is a good descriptor for the interaction between dynamical ice cover and an ice-going ship. This manifests as ice forces against the ship hull and as closing of channels, or other phenomena that navigators associate to compressive ice conditions. The internal friction magnitude has typical values ranging from 0 to 10 N/m2. The magnitude, possibly scaled to semi-empirical compression numeral 0-4, acts a proxy for ice compression. However, to estimate the actual local forces additional scaling arguments must be taken into account such as floe size and other ice cover geometry.

The HELMI model was discretized in a c-crid and in curvi-linear co-ordinates. The grid has 415 nodes from west to east and 556 nodes from south to north. The SW lower corner coordinates are 16.72 E 56.74 N, NE corner coordinates 30.48 E 65.99 N and the increment is 1/30 degrees eastwards and 1/60 degrees northwards. This is approximately 1 NM in both directions at 60N latitude. The model was forced by the HIRLAM regional weather model forecasts or reanalyses of winds and air temperature. The length of forecast is 54 hours and interval 3 hours, while reanalyses are stored at 1 hour intervals. The present set-up of the ice prediction system doesn't include any dynamical ocean component, thus ocean currents are neglected. Sea surface temperature is prescribed and updated once a day and thus an ice edge is very much controlled by this procedure. Ice forecasts have been validated against the observed ice situations.

Bayesian learning from the data

In this section we describe the learning process from the data resulting in the development of the probabilistic, event-oriented models estimating ship performance in ice. An event-oriented model reflects the ice features under which ship's speed changes or a ship gets stuck in ice. These types of models are able to quantify the joint effect of various ice features on ship's speed, which in turn make it possible to determine the sea areas which should be avoided as a ship may experience significant speed reduction there. The analyzed datasets, representing speed of ship navigating in changing ice conditions are illustrated in Figure 2, whereas the correlation matrix for the variables describing ship performance and ice conditions is depicted in Figure 3.

To extract knowledge form the available datasets, we applied a Bayesian Belief Network (BBN) which suits the purposes, being a renowned technique for transferring data into knowledge in the presence of uncertainty or limited information and for drawing inferences based on that knowledge. Moreover the BBN and its learning techniques make it possible to combine the knowledge extracted from the data with the existing background knowledge and understanding of the analyzed phenomena and relations between certain factors. This is realized at the stage of definition of the model's variables, where the relations between variables can be specified, meaning forced or forbidden. Moreover the temporal order of variables can be defined at this stage, meaning that the BBN, to some extent, is able to capture also dynamics of a system that has been analyzed. Two ways of learning Bayesian networks are in use: Constraint Search-Based Learning (CSBL) and Bayesian Learning (BL). In the first case, an algorithm searches the data for independence relations to determine the causal relations, and in the second case the space range of models is searched over and each model found is scored using the posterior probability of the model given the data, and the model which gets the highest score is adopted, see for example (Cooper & Herskovits, 1992; Spirtes et al., 2000). In this paper two learning algorithms were utilized and their results compared. The first algorithm called "PC" belongs to the CSBL group and the second called Naïve Bayes (NB) represents the BL group. The models are developed with the use of software package called GeNie, which is a versatile and user-friendly development environment for graphical decision-theoretic models, see (Druzdzel, 1999). Moreover it offers a wide range of learning algorithms, depending on available data and model requirements.

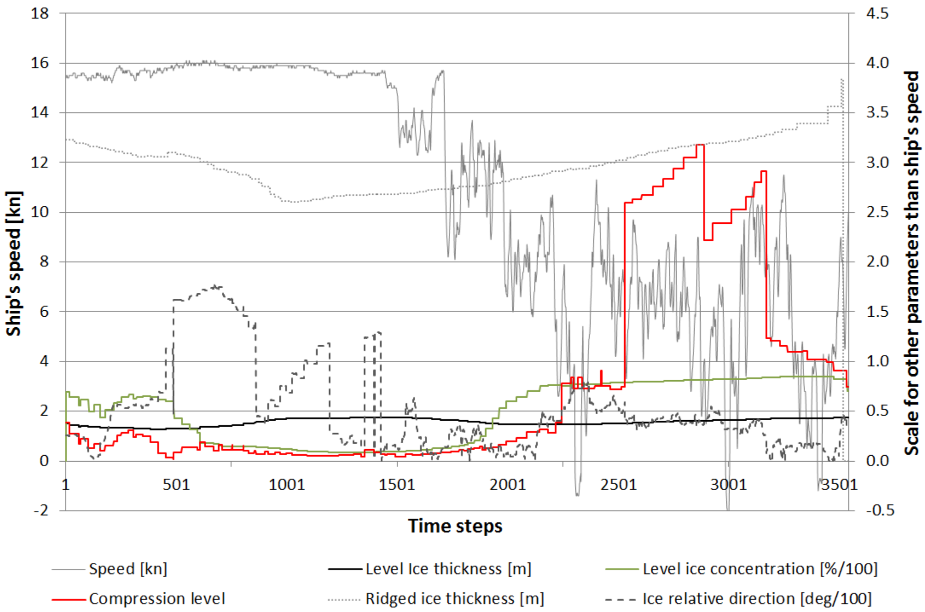

Figure 2 The time series of analyzed parameters

	Speed	Level Ice conc	Level Ice thickn	Ridged Ice conc	Ridged Ice thickn	Rafted Ice conc	Rafted Ice thick	Rel dir of compress	Rel dir of wind	Wind speed	Compressi on	Ship gets stuck
Speed	-											
Level Ice concentration	-0,61	-										
Level Ice thickness	-0,12	-0,01	-									
Ridged Ice thickness	-0,31	0,57	-0,55	-								
Ridged Ice concentration	-0,31	0,81	0,06	0,62	-							
Rafted Ice concentration	0,04	-0,03	-0,84	0,48	-0,25	-						
Rafted Ice thickness	-0,50	0,80	-0,28	0,68	0,79	0,01	-					
Rel dir of compression	0,38	-0,32	-0,32	0,00	-0,13	0,15	0,09	-				
Rel dir of wind	0,53	-0,62	0,01	-0,35	-0,44	0,01	-0,54	0,06	-			
Wind speed	0,51	-0,55	-0,38	-0,39	-0,70	0,49	-0,57	0,23	0,34	-		
Compression	-0,62	0,64	0,20	0,38	0,48	-0,15	0,56	-0,22	-0,49	-0,43	-	
Ship gets stuck	-0,40	0,15	0,04	0,09	0,11	-0,04	0,15	-0,05	-0,10	-0,24	0,10	-

Figure 3. The correlation matrix for all analyzed variables.

MODEL DEVELOPMENT

Model development is an iterative process, where at the first step a version of a model is obtained by adopting a learning algorithm. At the second step, the obtained model is cross-validated and if it passes the test it is stored, otherwise the process returns to the step one.

The analyses performed in this study have failed to deliver a single unified model, which simultaneously predicts the speed of the ship and situations where a ship gets stuck in ice. For all the cases analyzed, models which were obtained provided good prediction for either of these events but never for both of them. Therefore, two separate models been developed and are presented in the following section, one predicting each event. The models are obtained by applying two different types of learning algorithms (PC and NB), which were found the best for a given purpose; this means that the structure of the models is different. The first model predicting the ship speed, called model A – see Figure 4 - considers the causality discovered in the data; whereas the second model, called model B – see Figure 5 - disregards the causality, and searches for the best fit to determine the relations between the situations where ship is stuck in ice and the surrounding ice conditions.

Since the model A is rather complex – see Figure 4 - it encompasses 12 variables, which are connected with 29 arcs, having in total 29 states, thus producing large number of parameters (4415), it is impossible to illustrate all of them in the paper. Consequently, the description and presentation of the models is limited to its qualitative part – model structure – only, see figures 4 and 5.

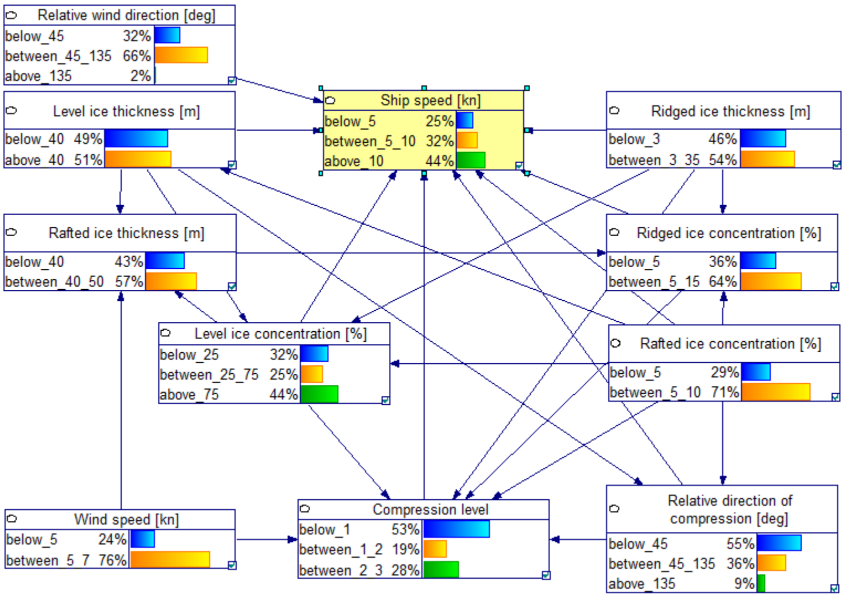

Figure 4 A probabilistic model predicting ship speed in ice, developed with the use of *PC* learning algorithm and incorporated background knowledge (model A).

MODEL VALIDATION

Bayesian Networks offer a wide variety of tests and analyses to validate the model, see for example (Pitchforth & Mengersen, 2013). One of the tests is the cross-validation the part of the data is used to learn the network, and a part to validate it. It tends to answer the question about the predictive power of a model with respect to a selected hypothesis, which is tantamount to evaluating the probabilities for false positives (FP) and false negatives (FN) for a two-state variable. Another test is the model behavior analysis addresses the following question: does the model behavior predict the behavior of the system being modeled? It is usually done by an analyst who performs the scenario walk-throughs and checks whether the model prediction is consistent with the existing knowledge about the physics of the phenomena which is being analyzed and the general knowledge and understanding of the modeled system. The results of cross-validity analyses carried out for the models presented in this paper are shown in tables 2 and 3. The model A features good prediction power, which is expressed as the probability of delivering the right answer by the model, which for the variable *ship speed* varies between 0.75 and 0.93, and it depends on the state of the variable, see Table 2.

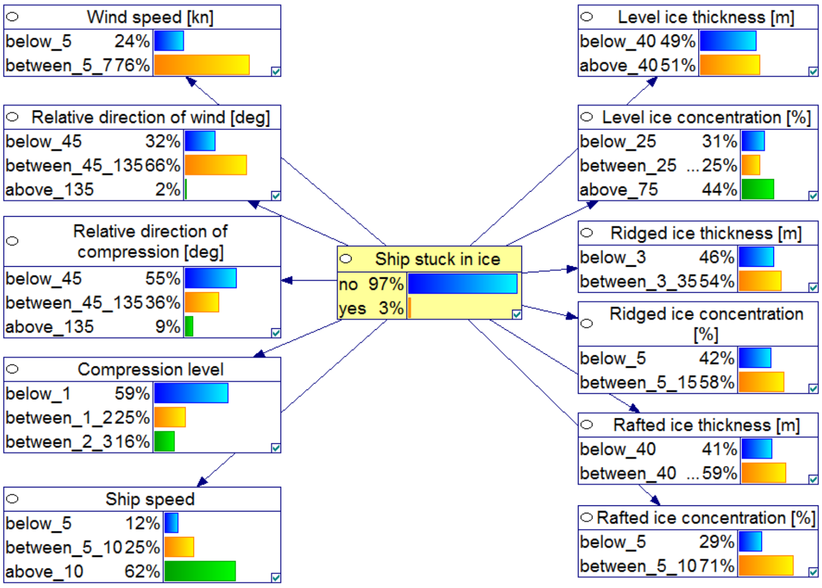

Figure 5 A probabilistic model for predicting a ship getting stuck in ice, developed with the use of *Naïve Bayes* learning algorithm (model B).

In the case of the model B, its predictive power is even higher, 0.9 and 1.0 depending on the hypothesis adopted, see Table 3, as the variable of interest (*ship stuck in ice*) has two states. The conclusions which can be drawn from the presented cross-validation analyses, consider the quality of prediction, which varies depending on the hypothesis, as presented in tables 2 and 3. Considering the model B, if we allow the following hypothesis: *the ship will get stuck in ice*, then the probability for the model to deliver the right prediction is 1 and the probability of false prediction (false positives – FP) in this case is 0. However, if the alternative hypothesis is adopted, meaning: *the ship will not get stuck in ice*, then the probability for the model to deliver the correct response is 0.9 and there are 10% chances for the incorrect prediction (false negative – FN). With this analysis the FP and FN are quantified, and the predicting power with the respect to given hypothesis is evaluated.

Table 2. Results of the cross-validation of the model A

		Ship's speed [kn]		
AIS data		Obtained from the model		
		Below 5	Between 5-10	Above 10
	Below 5	**0.78**	0.22	0.00
	Between 5-10	0.21	**0.75**	0.04
	Above 10	0.01	0.06	**0.93**

Table 3. Results of the cross-validation of the model B

Ship stuck in ice [yes/no]			
AIS data	Obtained form the model		
		No	Yes
	No	**0.90**	0.10
	Yes	0.00	**1.00**

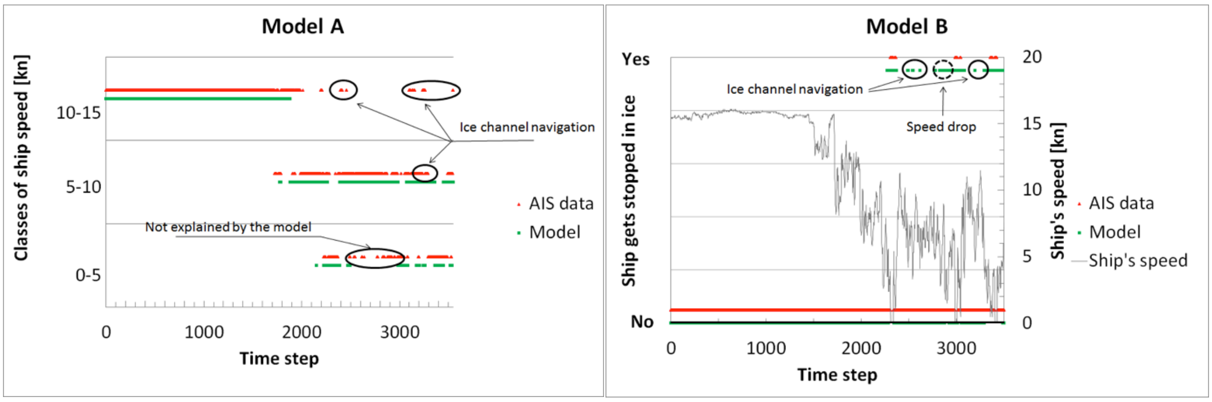

Figure 6 Models' predictions versus AIS data.

DISCUSSION

In Figure 6 the results of cross-validation are visualized, and the following is found:

Model A

It tends to overestimate the modelled parameter for the certain classes of output variable. For the lowest speed category (*below 5kn*) the model classifies wrongly 22% of the cases, assigning it to the higher speed category (*between 5-10kn*), see Table 2. This means that the model may deliver results that are too optimistic for a ship. In other cases the model tends to underestimate slightly the ship's speed. The accuracy of the model is 78%, 75% and 93% for the lower, medium and highest speed categories respectively. The model has problems with proper estimation of the speed category in the locations where ship speed fluctuates significantly, whereas the ice conditions do not change much. This may bring to conclusion that in certain areas the ship was slowing down deliberately, or the ice data obtained from HELMI model significantly differs from the real conditions. For the presented set of variables the accuracy higher than 78% for the lowest speed category cannot be attained.

Model B

The predictive power with respect to hypothesis stating that the ship gets stuck, is very high whereas in the case of alternative hypothesis it is burdened with some inaccuracy, meaning that the model predicts more cases of ship being stuck in ice than obtained from the records. We found two reasons for this, first is that the model tends to classify the cases in which the ice conditions are challenging and the ship's speed drops below 4 kn as "*ship stuck in ice*", which in the reality is not always the case. According to the recorded data, in some instant of time the ship's speed dropped dramatically from 8 kn to 3 kn, marked in Figure 6 as "speed drop", however the ship managed to speed up and continue for some time before she eventually stopped. The second reason is the effect of ice channels on ship's speed, meaning that the model predicts the ship being stopped in ice where she is still underway. However this type of navigation was rather minor, as a significant level of compression was observed, and the ice channels could not remain open for a long time.

CONCLUSIONS

In the presented paper we have introduced two probabilistic models predicting ship performance in ice, meaning ship's speed and the situations where a ship is probable to get stuck in ice. The models feature two novelties, first they considered the joint effect of various ice features on ship performance, and second ice compression has been taken into account. The models were developed with the use of the techniques of Bayesian learning from real world data.

Notwithstanding all the assumptions, the results obtained from the presented model are promising as they can help to understand the joint effect of ice features on ship speed in addition to the process and conditions associated with ships getting stuck in ice. The models developed can be further used for ship track analysis. They could be used to determine the probability that a ship will attain a certain speed class or to specify the areas where she may get stuck in ice, especially important in the case, where assistance of an ice-breaker is not available immediately. The models can also quantify the effect of each single component describing the ice cover on ship speed, pointing out the most relevant ice features.

It should be noted, that the existing models are valid only for a specific ship type (ice going bulk carrier), and the specific ice hindcasting model (HELMI). Further work should focus on incorporating ships of various types and ice classes to the model.

ACKNOWLEDGMENTS

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