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GROUNDED ICE PILE-UP. 2D DEM SIMULATION

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ABSTRACT
Studies of grounded ice rubbles formation aimed at the evaluation of their limit geometry and their potential effect on ice loads on shallow water platforms were performed. Based on observations and model studies the mechanisms for rubbles building up were developed. Numerical and analytical methods for modeling ice rubbles formation were suggested.
Scenarios of coastal ice rubble formation include the following stages: fracture of the ice cover edge and formation of ice block chains (rubble supply lines) advancing a coastal slope, accumulation of ice blocks due to changes in the inclination of the sliding surface or due to supply line loss of stability caused by resistance to the ice block motion, and formation of new supply lines. Then the cycle is repeated. Limit rubble height when the amount of advancing ice is unlimited is determined by supply lines breaking up due to loss of stability of an ice block chain under a longitudinal compressive load. Various scenarios of the supply lines formation, the shape and size of occurring rubbles are discussed.
Criteria for selected stages of particular scenarios were developed, including the ice edge breaking conditions, critical angle of supply line overturn, loss of stability critical load, as well as the modifications of limit states due to the geometry and strength of the obstacles.
Analytical estimates were compared with the results of numerical DEM simulations. In particular, the problem of grounded rubble formation against a slope structure due to advancing ice accounting for various parameters of the system was considered. A modified version of known ice load equations for slope shallow water structures is proposed.

INTRODUCTION
For the purposes of development designs of offshore hydrocarbons deposits in the Arctic it is necessary to predict such characteristics as limit height of the rubble pile-up in front of the obstacle, limit force acting on the obstacle in piling-up conditions, the possibility of penetration of broken ice onto the working zones of the offshore platforms and drilling islands etc. The requirements for accounting the aforementioned characteristics are given in the recently published Standard ISO 19906 (2010) developed on the basis of the available long-term studies. Nevertheless, the recommendations for quantitative estimations of the pile-ups and ride-ups characteristics are absent in the standard.
According to the Gazprom's strategy in the area of oil and gas production (Gazprom website, 2013) the Yamal Peninsula and water areas of the Russian northern seas, including the Ob and Taz Bays in the Kara Sea, are the regions of priority for the long-term prospect of gas production in Russia. Among the concepts for development of the deposits in Ob and Taz bays the fixed platforms and artificial drilling islands are considered (Tsybulskyi *et al.*, 2013). The region is characterized by difficult physical environmental conditions including harsh ice regime, namely, the duration of ice period is about 9 months, the average ice thickness is about 1.5 m (in extreme cases can reach 2 m and more), the conditions in the water area represent a combination of conditions typical both for a high sea and a river. The latter factor together with small water depths in the areas of the discovered deposits predetermines

the formation of the rubble pile-ups in front of offshore structures. So, these should be taken into account as one of the scenarios of ice-structure interaction in the design calculations.

The dangerous ice phenomena and lithodynamic processes in the water areas adjoined to the Yamal Peninsula are studied at the present time (Gafarov *et al.*, 2012; Onishchenko, 2008). Till now the available full-scale data about ice rubbles are extremely poor in this region, so the role of mechanical and mathematical modeling of the processes is increased.

The numerical examples considered below correspond to the case when the rubble is grounded on the sea floor. The updating of the approaches recommended in the ISO 19906 for this case is required. In particular, it concerns the scenarios of long-term motion of ice when the configuration of the pile-up and the distribution of internal loads become independent on the shape of the obstacle. Generally, the extremely high loads accompanying rubble formation act on different parts of the "pile-up – obstacle" system at different instants and also have different directions. Therefore, the estimation of the total contribution of these loads should be done in conformity with the scenario of the process.

GROUNDED ICE RUBBLES IN FRONT OF SLOPE STRUCTURES

One of the main issues needed to be solved for the design purposes is the potential influence of the rubble on the design values of ice loads and also the expected height of pile-ups, both floating and grounded. There exist several main approaches to the development of a model of ice pile-up formation. Representations on a generalized energetic equilibrium of a system are often used (Allen, 1970, Sodhi *et al*., 1983; Christensen, 1994; Marchenko, 2006, 2010 and others). Specific local kinematic schemes of ice mass motion are useful for the analysis of separate elements of a pile-up formation scenario (Croasdale, 1980, 2012). For floating pile-ups this was supplemented by numerical simulation methods for studying the process of block structures formation (Hopkins, 1997; Paavilainen *et al.*, 2006; Paavilainen & Tuhkuri, 2013).

The classical approach for the estimation of the ice actions on a slope structure in the presence of a rubble was proposed at first for 2D statement of the problem, describing the interaction of ice floes with wide structures (Croasdale, 1980), and then for 3D problem, clarifying the calculated values of the ice load for the case of platforms with conical hulls (Croasdale *et al.*, 1994). The resulting equation (it is included in ISO 19906, 2010 as well) obtained for the load acting on a slope structure in the presence of a rubble, F_0, is a sum of several terms:

$$F_0 = H_B + H_R + H_P + H_L \qquad (1)$$

where the term H_L, which is a prevailing one for a wide range of the parameters characterizing the ice floe – pile-up – obstacle system, presents the load component required to lift the ice rubble supporting on the advancing ice (denoted by $V^{(+)}$ on Figure 1,a), acting just before the breaking of the ice beam due to bending.

Note, that the approach was initially developed for the case of pile-ups being afloat (Figure 1,a) and, in our opinion, cannot be used directly for pile-ups resting on the seabed. There are a number of arguments supporting this point. One of them is that due to much bigger height and weight of the sail of a grounded pile-up as compared to the case of a floating pile-up, one can expect that the advancing ice floe will be broken before the contact with the obstacle (Figure 1,b). The works dealing with the latter scenario for quantitative estimation of the resulting load on obstacles are not numerous till now. Below, this scenario will be illustrated by numerical examples. For simplicity, 2D statement of the problem is considered, where the ice floe is represented by a beam of unit width.

GOVERNING MECHANISMS OF COASTAL ICE PILE-UP FORMATION AND RELEVANT SCENARIOS

The available data on pile-up structure enable to identify its main structural elements (in particular, they are illustrated by examples of model calculations given in the paper):

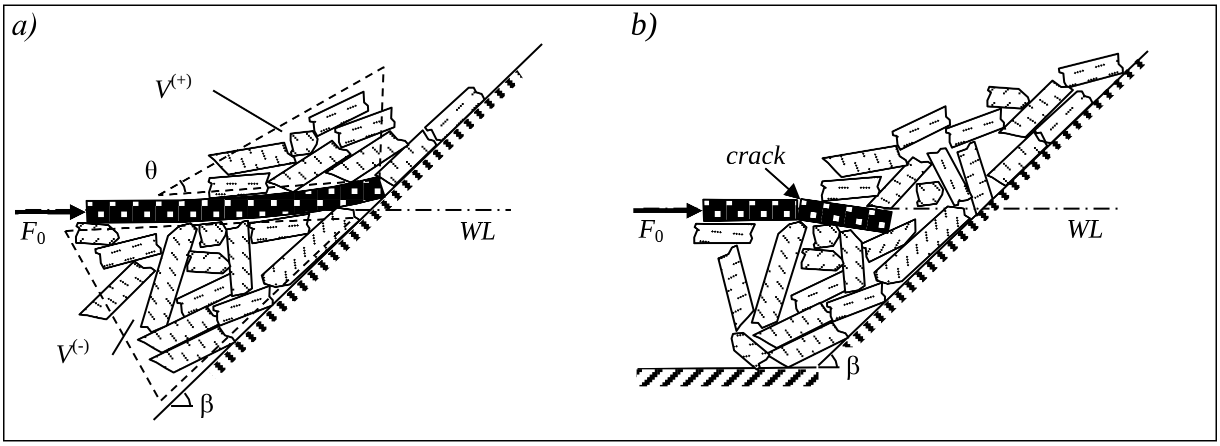


Figure 1. Alternative scenarios of oncoming ice floe penetration into the pile-up body formed in front of a slope structure

- supply line – a moving chain of ice blocks by means of which the broken ice comes into the pile-up;
- force skeleton – a set of blocks being in a certain sequence forming the trajectories of maximal tractions in the pile-up body and providing a transfer of the main part of loads from the approaching ice floe to the structure (note, that the term "force skeleton" emphasizes that this is a structure providing the bearing capacity of a rubble and is used instead of the term "force chains" commonly used in the mechanics of granular media, see also Paavilainen & Tuhkuri, 2013);
- stable set of ice blocks (self-assembling structures) with more strong inner bonds (forming, in particular, in the process of pile-up and ridge keel consolidation due to freezing of recently produced ice blocks);
- quasi-loose medium of weakly connected ice blocks.

At the same time it is necessary to point out remaining fragmentariness of description and incompleteness of the list of accounted scenarios for pile-up formation in whole, as well as the problems on choosing and justification of local conditions at the contacts of ice blocks within the pile-up and technical problems on modeling a multicomponent system. A set of relevant questions remains without answer. Among them are, for instance, the problems on pile-up formation in case of unlimited amount of incoming ice, on an influence of an ice motion regime on pile-up structure, etc. Some of these problems are considered in the next sections where the results of modeling are given.

Let us discuss in more details an approach for modeling the formation of coastal ice pile-up caused by moving of ice sheet along the horizontal plane on an inclined rectilinear obstacle. At first moving of ice floe leads to its edge break off and formation of a chain of ice blocks that rides-up on the slope obstacle due to the action of the advancing ice floe (Figure 2,a). Moving along the supply line the blocks are delivered to the accumulation area located at the top of the pile-up (Figure 2,b). Subsequent movement of the blocks and location of the accumulation area depend on which of the two alternative possibilities are realized: the chain of blocks on the supply line losses its stability (Figure 2,c) or the supply line turns-over on the top of the pile-up (Figure 2,d). In the case of unlimited amount of incoming ice the limiting height of the pile-up is determined by breaking of the supply line.

If the supply line is formed on a heap of blocks formed, in turn, after the loss of stability of the previous supply line, then the evolution of this line ends also with the loss of stability of a chain of blocks. Therefore, the height of the pile-up is limited by the height providing the critical load of loss of stability by a chain of blocks of fixed configuration. Finally, the pile-up has a form of a ridge with periodical hollows (Figure 3,a).

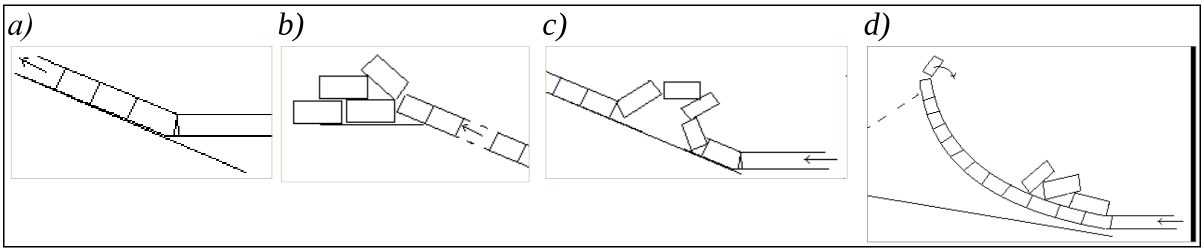


Figure 2. Elements of pile-up formation scenario. Schematic illustration

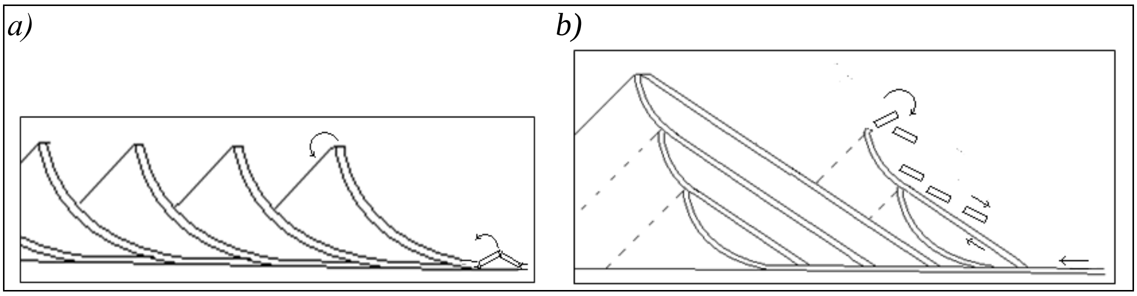


Figure 3. The developed structure of a shore pile-up

Alternatively, the supply line can attain the size at which it turns-over before the loss of its stability. In this situation the blocks are accumulated on the outer side of the supply line, so that the straight envelope contour to the blocks corresponds to the slope conditions of the quasi-loose medium. In the described scenario the supply line works in the buried regime. The blocks above the supply line induce an additional load preventing the moving of the blocks and resulting in loss of stability of the supply line apart from the pile-up. The slope forms a flat inclined surface on which a new chain of ice blocks rises. Since the natural slope angle is less than the limiting angle at which the supply line turns-over, the rectilinear part of a new supply line attends the same height on this slope without turning-over. Then, as before, the curvilinear part of the supply line is formed until attaining the conditions of turning-over. The cycle, during which the gradual increase of the pile-up height takes place, ends when the length of the straight part of the chain of blocks becomes sufficient to cause the loss of stability of the supply line at the pile-up base. This instant corresponds to the limiting height of the pile-up. The final shape of the pile-up is determined by a sequence of cycles ending with loss of stability of supply lines and is represented by a set of lags of maximal height separated by hollows (Figure 3,b). The section of each hollow may contain several buried contours of the supply lines with rectilinear and curvilinear parts.

The force skeleton can cause an additional resistance to the motion of the blocks along the supply line and, as consequence, influence on the conditions of loss of stability. So, the activity of force skeleton can lead to the correction of the pile-up parameters. Consider critical situations and corresponding conditions in a rubble development process, namely, the conditions of turning-over of supply line and conditions of loss of stability of a chain of blocks under the action of a longitudinal load. A condition at which the ride-up of a chain of blocks is replaced by a pile-up formation (kinematic loss of stability (Croasdale, 1980)) is related to the possibility of load transfer between the adjacent blocks moving through the jump of sliding surface inclination. Let us illustrate the criterion of turning-over of the supply line by an example of limit equilibrium of two blocks at the top of the supply line (Figure 4).
The limit equilibrium condition, representing the equality between the turning-over and restoring moments for the second block, has the form:

$$M\{F\} = M\{\gamma \ell h\} \tag{2}$$

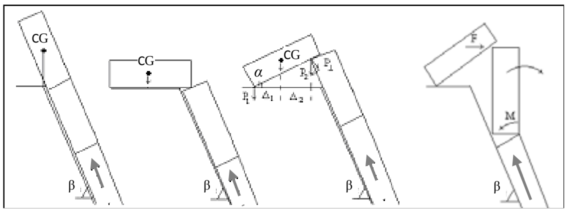


Figure 4. The kinematic scheme of turning-over of block at attaining the critical angle of the supply line orientation

or approximately, by neglecting the friction:

$$\frac{1}{2}\cos\left(\beta-\tan^{-1}\frac{h}{\ell}\right)\approx\frac{1}{1+(h/\ell)\tan\beta}\cos\left(\alpha+\tan^{-1}\frac{h}{\ell}\right)\cos\alpha\cos\beta\sin(\alpha+\beta) \qquad (3)$$

where β is the obstacle slope angle, h is the ice thickness, ℓ is the characteristic block length, γ is the ice specific weight, α is the first block turn angle. According to (3) the critical slope of the supply line substantially differs from vertical. The critical slope angle is $\beta = 74°$ for $h/\ell = 0.25$. This value agrees with the results of our model experiments.

The second criterion of changing the mechanism of the pile-up formation is related to the loss of stability of the supply line consisting of the chain of blocks compressed between the moving ice sheet and pile-up body. The condition of loss of stability is changed drastically at changing the scheme of loading and compliance of foundation. A number of useful variants are given by Kovach & Sodhi (1980), and can also be obtained on the basis of an analysis of statements of problems on stability of beams (Timoshenko, 1972). Note, that the critical force for two square weighty elastic beams resting on a rigid foundation has the following form:

$$F_{cr} = h\sqrt{\gamma\ell E/2} \qquad (4)$$

So, the factors adjusting the mechanism of the pile-up formation are approximately outlined.

For performing pile-up calculations, the parameters describing the strength of ice and properties of the effective loose medium of weakly connected blocks are also required.

Analytical modeling of the phases of pile-up development enables to identify the main scenarios of its structure formation both on the level of local interaction acts and on the global motion level. Such modeling is required for specifying the model parameters and defining the correct boundary conditions when numerical modeling is performed and also for interpretation of the full-scale data. Due to statistical heterogeneity of the pile-up the numerical simulation is preferable method for determination the quantitative characteristics of the pile-up. The results of such simulations performed using the DEM are presented below.

RESULTS OF NUMERICAL 2D MODELING

One of the obvious approaches for modeling a set of unfrozen ice fragments (for example, in the keel of hummock) consists in the modeling of the set of ice blocks by a loose medium with inner bonds. Granular materials theory (see, e.g. Pöschel & Schwager, 2005, and references inside) provides a wide class of relevant methods, usually called as discrete (or distinct) element method (DEM), which can be readily adopted for numerical study of the ice pile-ups modeling. The efficiency of DEM compared to the finite elements method in the dynamic or quasistatic description of the evolution of the loose medium structure is related to the following feature of DEM: prediction of the displacement of sufficiently stiff element of the loose medium during the small time interval dt by this method do not require the analysis of the stress-state inside the element. Instead, it requires the correct description of the contact forces occurring during the interaction of the element with other elements of the medium. So, the main efforts in application of DEM should be given to the modeling of the contact between elements, otherwise, the principle of conservation of energy can be violated. For instance, it was shown theoretically (see, e.g., Pöschel&Schwager, 2005) that the adoption of the proportionality between the normal contact force and the overlapping area at the contact of elements with angles leads to such kind of incorrect situation. Note, that known applications of DEM approach to study of pile-ups (Hopkins, 1997; Paavilainen *et al.*, 2006; Paavilainen & Tuhkuri, 2013) exploited the above mentioned proportionality rule. It seems that using of the obtained results requires a special analysis.

In all numerical examples considered below the process of formation of the pile-up resting on the seabed is modeled by using the demo-version of PFC2D simulation package developed by ITASCA (ITASCA website, 2013). The basic structural element of the package is a ball (or cylinder in 2D case). The advantage of the package is the availability of the contact bonds of special type, called parallel contact bonds (PC-bonds, Figure 5,a). This type of bonds describes the contact between the areas and is working in addition to the standard bonds describing point-type contacts. PC-bond realizes the distribution of the contact force on some contact area and, therefore, enables to transfer between the contacting elements not only the point forces, but also the moments. Due to this, the PFC2D package can be used for correct modeling of elastic ice beams accounting for longitudinal and bending stiffnesses and tension and shear strengths.

Before coming to the modeling of ice pile-up behavior, a number of tests problems have been solved, including: 1) lateral bending of a cantilever beam (Figure 5,a), 2) loss of stability of a beam under longitudinal compression (Figure 5,a), and 3) bending failure of a weighted beam resting on horizontal foundation (Figure 5,b). All the results of test modeling showed very good agreement with their theoretical counterparts, relevant details of this comparison are given in (Goldstein *et al*., 2013).

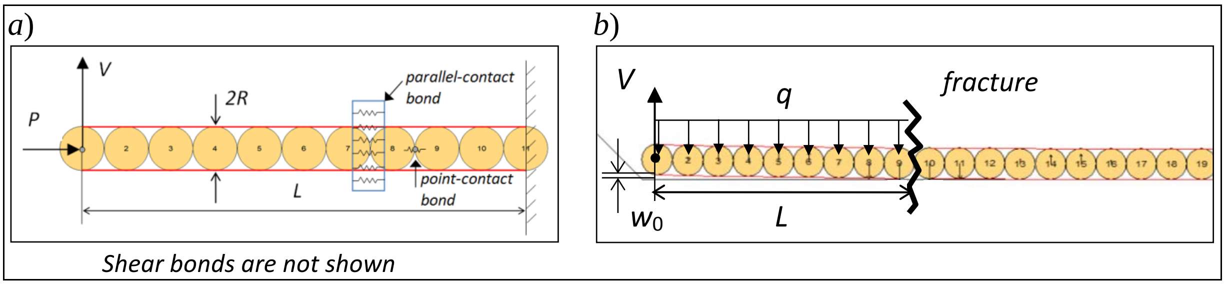


Figure 5. Examples of modeling the beams within the framework of PFC2D package

Modeling of the ice load on a slope structure for the case of the ice floe moving along the flat rigid foundation

Below the preliminary results of the simulation of the process of the rubble pile-up formation in front of the slope obstacle, which were performed by using the PFC2D package, are described. The problem on the pile-up formation is multiparametric, and at this stage of the study mainly the qualitative analysis of the scenarios of the process is given. Unless otherwise specified, in the all considered numerical examples the following values of the parameters are supposed: beam width $t = 1.0$ m, elastic modulus of ice $E = 3.0$ GPa, shear modulus of the ice $G = 1.0$ GPa, ice dencity $\rho_i = 920$ kg/m^3, ice bending strength $\sigma_b = 1.0$ MPa, stiffnesses of normal and tangential point-type contact bonds $k_n = 100$ MN/m and $k_s = 100$ MN/m, the ice – ice friction coefficient $\mu_i = 0.1$, the ice – obstacle friction coefficient $\mu_s = 0.2$, the velocity of the advancing ice floe is 0.1 m/s (is applied to the element farthest from the obstacle). The foundation and the obstacle are supposed to be rigid (this is modeled by assigning them very high stiffnesses).

It is important to note that due to discrete nature of the PC-bonds the correct modeling of the elastic beam requires that the proper relations between the characteristics of PC-bonds and the size of structural element be satisfied. In the considered case these relations have the form:

$$k_n^{(pb)} = E/(2R), \quad k_s^{(pb)} = G/(2R) \tag{5}$$

where R is element radius.

The results of the simulation of the ice floe moving toward the slope obstacle are presented in Figure 6. The results show that the ice can ride up the obstacle on a very long distance even for sufficiently big values of slope angle and ice – obstacle friction coefficient (Figure 6,a), which is in full conformity with observation data (Kovach & Sodhi, 1980; Christensen, 1994). In Figure 6,a are shown a typical picture of block movement (red arrows, upper part of the figure) and scheme of load transfer via the supply line, which is directed along the foundation and then along the inclined obstacle (black color lines, bottom part of the figure).

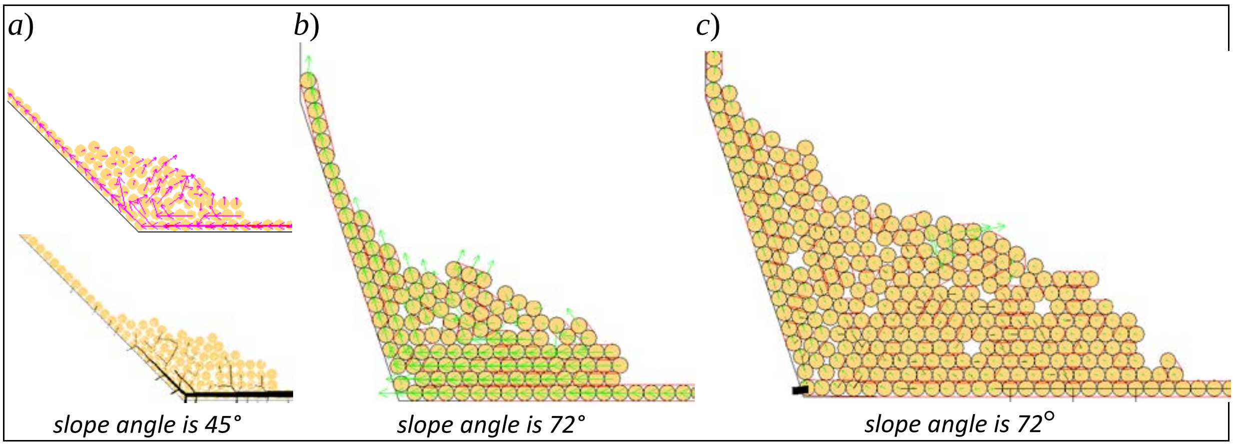


Figure 6. Typical scenario of pile-up development resulting from the ice floe advancing along rigid foundation

For modeling of the interaction between ice and sea platform it is important to consider the case of vertical (or almost vertical) obstacle. The results of simulation obtained for the slope angle equal to 72° and different values of ice – obstacle friction coefficient (0.1, 0.2 and 0.3) show that despite big obstacle slope angle the supply line, which usually corresponds to the chains of elements with the enhanced values of the contact loads, is still directed along the rigid foundation and then along the slope and is able to lift up the ice fragments on the substantial height (Figure 6,b and c).

The pile-ups formed at the moments of time t_1 and t_2 corresponding to the arrival of $n_{r,1} \approx 110$ and $n_{r,2} \approx 250$ elements into the pile-up body are shown in Figure 6,b and Figure 6,c, respectively. At time t_2 the supply line, located along the foundation, came to contact with the obstacle, while the ice beam is in bending state just before the breaking (the black rectangle at the base of the obstacle corresponds to the high contact load). The peak value of the load acting on the obstacle at time t_2 equals $H_2 = 2.67$ MN. The state at time t_1, as it can be seen in Figure 6,b, took place a little earlier than an event similar to that at time t_2. The peak value of the load at time t_1 equals $H_1 = 1.30$ MN.

The analysis of the results of the performed model calculations shows that the main factor that determines the scenarios of the pile-up formation and, probably, the peak values of the loads acting on the obstacle is the presence of flat rigid foundation. So, it seems that the described schemes of pile-up formation are more typical for the coastal pile-ups.

Modeling of the penetration of an advanced ice floe into the pile-up body

There are obvious reasons that in reality the scenario of stable motion of the ice beam along the horizontal foundation that was considered above may not be realized when the ice floe edge penetrates into the pile-up body. Therefore, other situations in which the horizontal part of the rigid foundation ends at some distance from the obstacle and a free space exists initially in front of the obstacle were also modeled (Figure 7,a). First, the initial pile-up of rather small size was formed (Figure 7,b). Next, the ice beam penetrated into the pile-up. The results of a number of calculations performed for a thin ice beam ($h = 0.2$ m) show that the beam is always broken up against the rubble. At the same time the supply lines consisting of broken beam fragments of different length can attain the obstacle (Figure 7,c) or go through the pile-up body in form of jets (Figure 7,d).

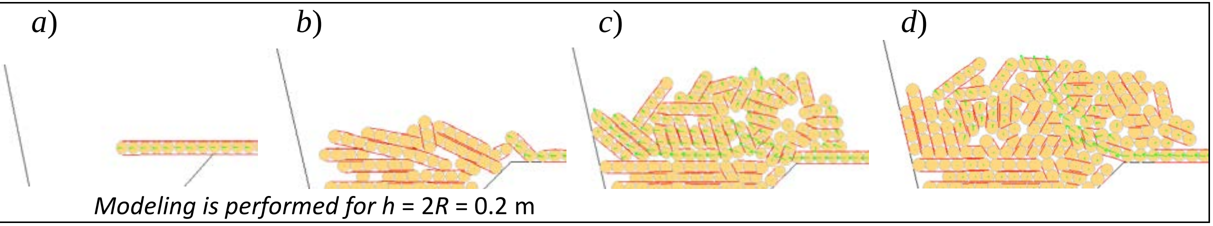


Figure 7. The penetration of the ice beam into the existing pile-up

The typical scenarios of the development of the process of ice beam penetration into the pile-up body are presented in Figure 8 (the number of structural elements is $n = 500$, ice thickness $h = 0.5$ m). These scenarios clearly illustrate the cyclicity of the pile-up formation process, namely 1) formation of a stable configuration of the force skeleton which blocks the supply lines and transfers the load from the advancing ice floe to the obstacle; 2) increasing of the forces in the elements of the force skeleton accompanied by the growth of the total load acting on the obstacle up to a peak value (at this phase the slowdown of the elements motion inside the rubble is usually observed); 3) "explosive" fracture of the force skeleton leading to redistribution of the forces inside the rubble followed by step-wise decreasing of the load acting on the obstacle and emission of a huge number of ice blocks (but not the whole pile-up) in the form of jets directed upwards or to the base of the pile-up; 4) almost free penetration of the supply lines into the rubble followed by consolidation of ice blocks and formation of a force skeleton in a new configuration. The last stage is characterized by the relatively low level of the total load acting on the obstacle.

Phases 1 and 2 are illustrated by Figure 8,a where maximal velocity of the ice blocks is 0.04 m/s). Phase 3 is shown in Figure 8,b. The peak value of the total load acting on the obstacle at this state attains the global maximum and equals 0.776 MN; two emission jets are observed, and the maximal velocity in the streams equals 0.64 m/s. The state which is intermediate between phases 3 and 4 is illustrated by Figure 8,c. Phase 4 is shown in Figure 8,d where an interesting scenario including two opposite flaws of ice blocks moving along the temporary fault can be observed.

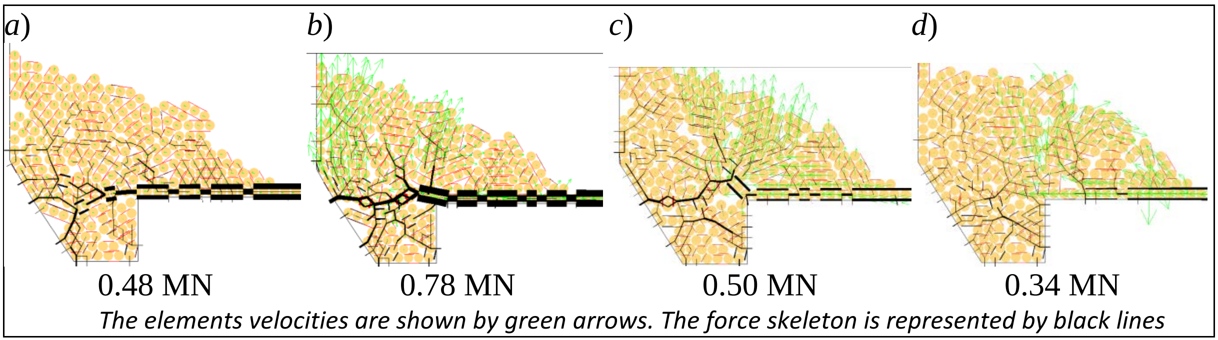
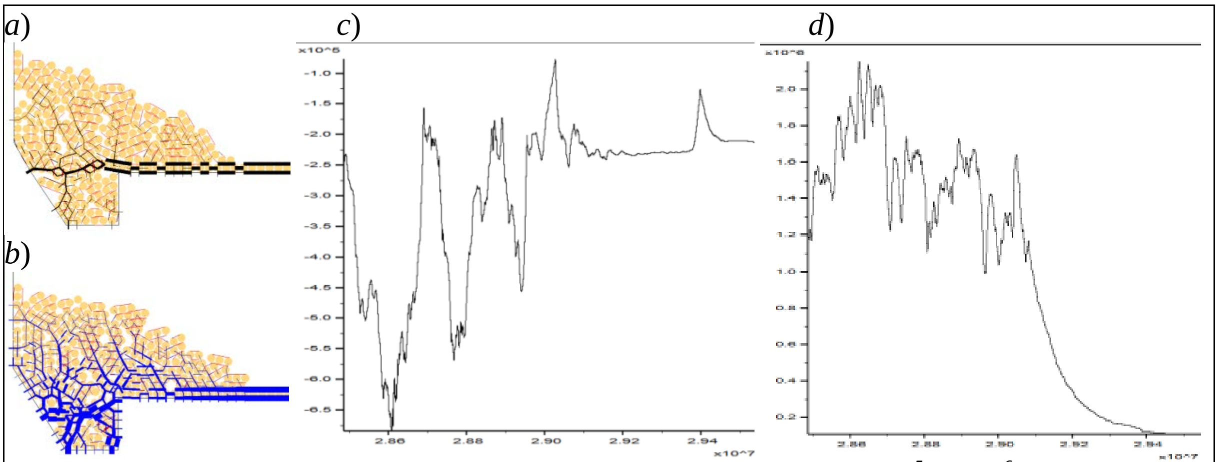


Figure 8. The typical scenarios of the development of the process of
ice beam penetration into the pile-up body

Several numerical experiments aimed at modeling the stop of an ice floe were performed. As a result of these simulations the equilibrium states of the system beam – pile-up were obtained. In the equilibrium state the external compression load applied to the beam is balanced by the obstacle reaction and by the loads responsible for keeping the pile-up on the seabed. The force skeleton typical for the equilibrium state is shown in Figure 9,a,b (in both cases the pile-up height is about 7 m). In the first example (Figure 9,a) a most loaded part of the force skeleton transferring the load from the ice floe to the obstacle is clearly seen; the value of the load acting on the obstacle equals 0.4 MN in this case. In the second example (Figure 9,b) the force skeleton is much more homogeneous inside the pile-up, the most loaded parts of the force skeleton are located at the bottom of the pile-up and transfer the load which is caused not by the motion of the ice sheet but mainly by the weight of the ice blocks. The load acting on the obstacle is much less in this case and equals 0.2 MN.

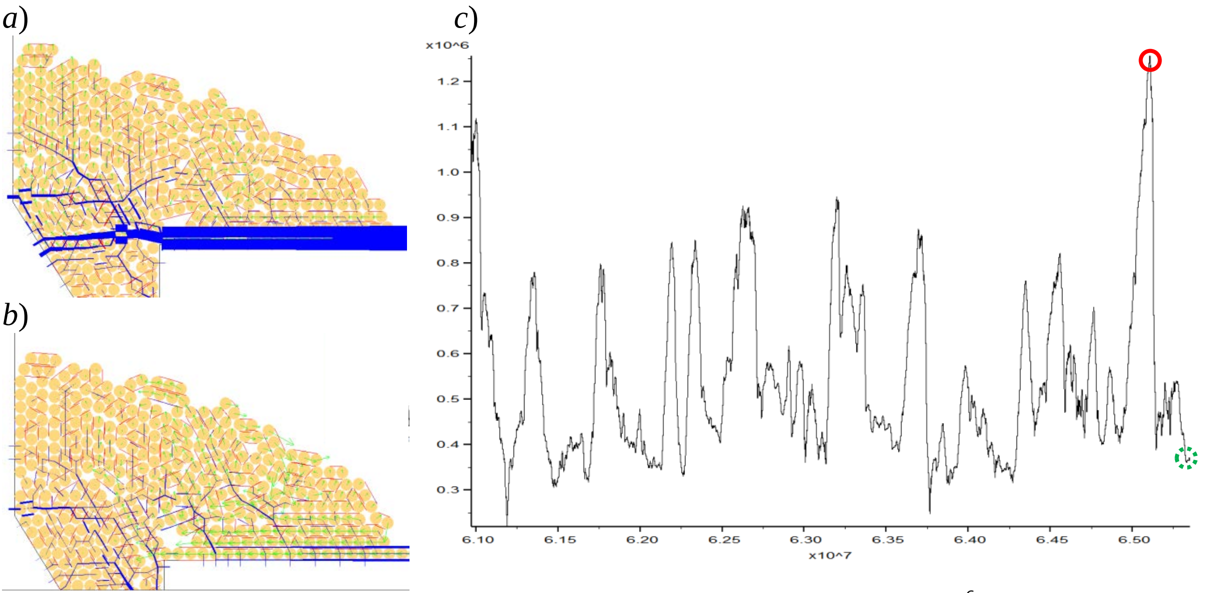
The graph presented in Figure 9,c corresponds to the second example (Figure 9,b) and shows the changing of load during the time preceding the stop of the ice floe and also during the transition of the system to the equilibrium state. The peak values of the load before the stop and at the equilibrium state are 0.68 MN and 0.25 MN, respectively. The graph presented in Figure 9,d relates to the same example and shows the changing in time of the load applied to the moving ice floe. Note, that the values of this load are much bigger than the values of the load acting on the obstacle (Figure 9,c). In particular, the peak value here equals 2.16 MN. The observed difference in the values of the load acting on the obstacle and load applied to the moving ice floe indicates that even for the low velocity (0.1 m/s in the considered numerical experiments) of the advancing ice floe sheet the process of the pile-up formation is inherently dynamic. It is obvious that the main part of the kinetic energy of the advancing ice floe is dissipated at the expense of work of friction on multiple contacts between the blocks and also between the blocks and the obstacle.



Loads (Y-axis) are in newtons on graph c) and d) and should be taken with factors 10^5 and 10^6, respectively. The X-axis corresponds to time presented by step number with factor 10^7, while the step size is $1.743 \cdot 10^{-5}$ s.

Figure 9. The scenarios of ice beam stop

An example of the realization of an extreme scenario of penetration of the ice beam into the pile-up is shown in Figure 10. In can be seen that although the edge of the unfragmented (intact) beam (thickness of the beam equals $h = 0.5$ m) is located on some distance from the obstacle, the force skeleton is formed against the obstacle in the pile-up body such that its main part serves as a beam continuation (Figure 10,a). The load acting on the obstacle in this case attains the value 1.26 MN (denoted by the red circle on the top of the graph in Figure 10,c). At the moment of fracture of the force skeleton the common upwards motion of the ice fragments located under the main part of the skeleton playing the role of a pseudo-beam is observed. After fracture of the force skeleton the relaxation of the loads inside the pile-up body occurs, so that the load acting on the obstacle is decreased substantially to the value 0.36 MN (denoted by green circle on the bottom of Figure 10,c). Thus, the described scenario is qualitatively similar to the scenario realized when the edge of the ice beam comes directly to the obstacle (Figure 6,c).



The values of loads (Y-axis for graph c) are in newtons and should be taken with factor 10^6.
The X-axis corresponds to time step with factor 10^7, the step equals $1.743\cdot10^{-5}$ s.

Figure 10. The extreme scenario of penetration of the ice beam into the pile-up body

Modified equation for the ice load in the case of grounded rubble

Based on the additional theoretical prerequisites on the floating ice beam interaction with a slope structure (Goldstein *et al.*, 2005a, 2005b), the results of the 2D numerical modeling performed, and the parametric analysis of the components of equation (1) as well, the following compact equation may be proposed for the conservative estimate of the ice load acting on a slope structure in the presence of a grounded rubble pile in 2D case:

$$\widehat{H} = (V_0 + W_r)\tan(\beta + \varphi), \quad \tan\varphi = \mu_s \qquad (6)$$

where β is a slope angle; V_0 is the vertical load required to break up by bending the weighty ice beam resting on rigid foundation (not floating ice beam!), W_r is the total weight of the ice in the pile-up. The range of applicability of the equation is restricted by the condition of absence of ice jamming against the structure: $\beta + \varphi < \pi/2$ (see Goldstein *et al.*, 2013).

Let us consider an example of application of equation (6) for the model pile-ups presented in Figure 6,b,c for ice thickness $h = 0.5$ m. First, 2D simulation of a beam bending gives $V_0 \approx 1.8\cdot 10^4$ MN. Then, accounting for the size and the number of elements within the pile-ups shown on the figures one obtains: $W_{r,1} \approx 0.196$ MN and $W_{r,2} \approx 0.445$ MN. In the considered example $\mu_s = 0.2$, and thus $\tan(\beta+\varphi) \approx 8.5$. So, using equation (6) the following estimates for ice load can be obtained: $\widehat{H}_1 = 1.73$ MN, $\widehat{H}_2 = 3.88$ MN. These values exceed the values obtained in numerical modeling by 33 % and 45 %, respectively. Probably, this difference is due to the fact that at peak loading not the whole, but only a part of the pile-up body moves upward. Comprehensive verification of equation (6) requires a further series of the multiparametric calculations and comparison between the results of these calculations and the available full-scale data, and the data of model experiments as well.

CONCLUSION

The process of ice rubbling which can result in a pile-up formation is one of phenomena influencing the design ice loads and potentially causing the dangerous kinematic situations near offshore platforms. The main attention is usually paid to the pile-ups formed afloat. At the same time when a pile-up is grounded, as it can take place in front of platforms in shallow water or in coastal zone, the processes related to the motion of ice blocks occur mostly without floating effects.

An approach for 2D modeling of mechanisms and scenarios of a pile-up resting on the seabed formation is described. The main points resulted from the study performed are as follow:

- the process of the forming and growth of grounded pile-up differs from that being afloating pile-up, mainly due to the weighty sail;
- the possibility for an advancing floe to penetrate through the sail and subsequently to break in bending against the structure is questionable; much more likely is the scenario of ice buckling before the pile-up or floe breaking within the sail body accompanied in some cases with the emission of a huge number of ice blocks through the sail body in the form of jets;
- peak loads on the obstacle correspond to the events of breaking up of the force skeleton formed within sail body (this phenomenon is known from the general theory of granular matter and has been revealed earlier for floating ice pule-ups, see Paavilainen & Tuhkuri, 2013, and reference inside);
- the process of the supply line penetration into the sail body is dynamical even under low ice floe velocity (0.1 m/s) and can make essential effect on resulting ice loads on the obstacle;
- based on the analysis of a limited series of the numerical simulation of pile-up formation for ice thickness in the range 0,2 m to 1 m, a characteristic pile slope angle about 20–25 degrees seems reasonable;
- the maximal pile height in numerical simulations varied depends on ice thickness, friction conditions and obstacle geometry; within a limited series of the numerical simulation for ice with thickness 0.5 m typical values observed for pile height were about 7-8 m;
- based on well-known Croasdale's equation for the load on a slope structure and on the preliminary results of numerical simulation performed, a simple equation is proposed for the ice load in the case of the grounded pile-up formed in front of the structure; further analysis is required for its proper verification.

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