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# EXPLICIT FINITE ELEMENT ANALYSIS OF COMPRESSIVE ICE FAILURE USING DAMAGE MECHANICS

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## ABSTRACT
The behaviour of ice in crushing is complex and of high relevance for safe design and deployment of structures in Arctic waters. To work towards understanding this important issue, a constitutive model for ice in compression was developed, using damage mechanics as its basis. The damage mechanics in the model utilizes Schapery's approach, which is in turn derived from nonlinear viscoelastic theory. The model is used to calculate a damage parameter for ice as a function of stress history which accounts for the effects of microcracking, pressure melting and softening, as well as viscoelastic aspects. The model was implemented using a user material subroutine (VUMAT) in the Abaqus/Explicit finite element modeling (FEM) software; explicit FEM offers a better capability for capturing high-speed failure events. For validation, FEM results were compared to medium-scale indentation test data from the Hobson's Choice program, as well as an earlier implementation of a similar constitutive model in Abaqus/Standard. Good agreement with the earlier results was achieved. Finally, code for element deletion was added to the VUMAT, to simulate the extrusion of crushed ice. Runs with this code modification showed that element deletion in the top contact layer (analogous to the extrusion of crushed ice) produced load drops consistent with the loading seen in compressive ice failure events.

## INTRODUCTION
Ice is a naturally occurring material that is unique and complex in its mechanical properties. Its mechanical behaviour can be described as viscoelastic, with rate-dependent responses to stress, including creep. Under certain conditions, sudden load drops are seen in experimental testing of ice in compression. Among other things, this is relevant for the phenomenon of ice induced vibration, which is of potentially high relevance to structure design for Arctic development. Proposed contributors to ice-induced vibration and load drops include the formation of high pressure zones (*hpz*s), pressure melting and recrystallization (Jordaan, 2001). These are small scale events that result in large-scale physical consequences, and so must be considered and understood for effective probabilistic risk assessment and safe design of structures subject to ice loading. The constitutive and numerical modeling discussed in the following is an effort to capture the observed behaviour of ice and work towards more accurate probabilistic modeling of ice behaviour for critical applications.

## THEORY
*Damage Mechanics*
The pioneering work in damage mechanics came from Kachanov (1958) as a way to express the effect of microcracks and voids on material strength. Kachanov's damage parameter is expressed in terms of nominal section area $A_0$ and damaged area A:

$$D = \frac{A}{A_0}, 0 \le D < 1 \tag{1}$$

giving an expression for effective stress $\sigma_a$:

$$\sigma_a = \frac{P}{A_0 - A} = \frac{\sigma}{1 - D} \quad (2)$$

for applied load $P$ and applied stress $\sigma$. Assuming the strain response is not otherwise modified, this implies an effective elastic modulus $E$ for undamaged modulus $E_0$:

$$E = E_0(1 - D) \quad (3)$$

Budiansky and O'Connell (1976) analysed the effect of microcracking on elastic moduli in terms of a damage parameter, a function of crack density and crack surface radius. Other work on crack-based damage mechanics was done by Horii and Nemat-Nasser (1983) and Kachanov (1993). Krajcinovic (1989) provides a comprehensive review of various damage mechanics formulations.

Schapery (1981, 1991) devised an unbounded damage parameter (as opposed to the bounded parameters starting with Kachanov (1958)) and a damage based model for viscoelastic materials. Schapery's model was based on the correspondence principle and the modified superposition principle. The modified superposition principle, for nonlinear materials in general, is given by Findley, et al. (1976) as:

$$\epsilon(t) = E_R \int_0^t D(t - \tau) \frac{\partial \epsilon^0(\sigma, \tau)}{\partial \tau} \partial \tau \quad (4)$$

for a linear compliance function $D(t)$ and reference elastic modulus $E_R$; $\tau$ is a dummy variable of integration. $\epsilon^0$, referred to as pseudostrain, can be defined in terms of complementary strain energy $W_c$:

$$\epsilon^0 = \frac{\partial W_c}{\partial \sigma} \quad (5)$$

so that

$$\epsilon(t) = E_R \int_0^t D(t - \tau) \frac{\partial}{\partial \tau} \left( \frac{\partial W_c}{\partial \sigma} \right) \partial \tau \quad (6)$$

and

$$\epsilon^0 = \frac{1}{E_R} \int_0^t E(t - \tau) \frac{\partial \epsilon(t)}{\partial \tau} d\tau = \frac{\sigma(t)}{E_R} \quad (7)$$

for relaxation function $E(t)$; the integral gives $\sigma(t)$ for the general case of linear viscoelasticity.

For materials subject to distributed damage, pseudostrain can be a function of a set of damage parameters $S_i$ representing multiple structural change mechanisms in the material, along with stress $\sigma$. Schapery (1981) used the following for quasistatic microcracking and a cracking rate following a power-law in stress:

$$\epsilon^0 = \sigma^r g(S) sgn(\sigma) \quad (8)$$

$$S = sgn(\sigma) \int_0^t \sigma^q f_1(\ldots) d\tau \quad (9)$$

with empirical constants $r$ and $q$, enhancement factor $g(S)$, and the function $f_1$ determined by crack tip material conditions.

*Ice-specific modeling*

A damage model based on Schapery's work, using two terms for distinct physical damage sources, was proposed by Jordaan et al. (1999):

$$S = S_1 + S_2 = \int_0^t \left\{ f_1(p) \left(\frac{s}{\sigma_0}\right)^{q_1} + f_2(p) exp\left(\frac{s}{\sigma_0}\right) \right\} dt \qquad (10)$$

for pressure $p$ and von Mises stress $s$ with reference stress $\sigma_0$; the power law exponent $q_1$ has an assumed value of 5 for the testing covered by this document, to be consistent with subsequent work (Xiao and Jordaan, 1996; Xiao, 1997; Jordaan et al., 1997, 1999; Li, 2002). $S_1$ corresponds to microcracking and dominates at low confinement pressures. $S_2$ relates to pressure softening, dynamic recrystallization and pressure melting, and dominates at high confining pressures. $S_2$ as an exponential term (as opposed to a second power law term) comes from experimental observations of runaway softening in ice (Jordaan et al., 1997, 1999). $f_1$ and $f_2$ are defined in terms of pressure $p$ as:

$$f_1(p) = \begin{cases} 0.712\left(1 - \frac{p}{37}\right), \ p < 37\ MPa \\ 0, \ p \geq 37\ MPa \end{cases} \qquad (11)$$

$$f_2(p) = 0.1\left(\frac{p}{42.8}\right)^r \qquad (12)$$

also from Jordaan et al. (1997, 1999). For the general case of a viscoelastic material, the strain tensor is given by:

$$\varepsilon_{ij} = \varepsilon_{ij}^e + e_{ij}^d + e_{ij}^c + \varepsilon_{ij}^v \delta_{ij} \qquad (13)$$

with strain rate given by:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^e + \dot{e}_{ij}^d + \dot{e}_{ij}^c + \dot{\varepsilon}_{ij}^v \delta_{ij} \qquad (14)$$

The four terms in the above equations correspond to elastic, delayed elastic, viscous (or secondary creep) and volumetric strain components, respectively; $\delta_{ij}$ in the volumetric term is the Kroneker delta.

For triaxial stress conditions, with a known total strain rate $\dot{\varepsilon}_{ij}$, the individual strain rate terms can be calculated as functions of the stress state and the accumulated damage as follows:

$$\dot{e}_{ij}^d = \frac{3}{2} \dot{e}_0^d \left(\frac{s^d}{\sigma_0}\right)^n \exp\left(\beta_d S\right) \frac{s_{ij}}{s} \qquad (15)$$

$$\dot{e}_{ij}^c = \frac{3}{2} \dot{e}_0^c \left(\frac{s}{\sigma_0}\right)^m \exp\left(\beta_c S\right) \frac{s_{ij}}{s} \qquad (16)$$

$$\dot{\varepsilon}_v = -\frac{f_3}{p}\left(\dot{e} - \dot{e}_e\right) = -\frac{f_3}{p}\left(\dot{e}_{ij}^c + \dot{e}_{ij}^d\right) \qquad (17)$$

$$\dot{\varepsilon}_{ij}^e = \dot{\varepsilon}_{ij} - \dot{e}_{ij}^d - \dot{e}_{ij}^c - \dot{\varepsilon}_{ij}^v \delta_{ij} \qquad (18)$$

Equations (15) and (16) are due to Li (2002) and Equation (17) is from Singh and Jordaan (1997, 1999). Here $n$, $m$, $\beta_d$, $\beta_c$ and $f_3$ are empirically determined constants, $\sigma_0$ is a reference stress, $\dot{e}_0^d$ and $\dot{e}_0^c$ are reference strain rates, $s$ is the von Mises stress, $s_{ij}$ is the deviatoric stress tensor and $p$ is the volumetric pressure. $s^d$ in the delayed elastic strain rate equation comes from the Burgers body representation of the material, as in Figure 1. Specifically, $s^d$ represents the effective stress in the Kelvin unit (upper portion of Figure 1) of the Burgers body. Following from the Burgers body representation, the viscoelastic strain rate terms can be expressed in terms of nonlinear viscosities $\mu_c$ and $\mu_d$:

$$\mu_d = \frac{2}{3\dot{e}_0^d}\frac{s^d}{\sigma_0}(s^d)^{-n} exp\,(-\beta_d S) \tag{19}$$

$$\dot{e}_{ij}^d = \frac{s_{ij}}{\mu_d} \tag{20}$$

$$\mu_c = \frac{2}{3\dot{e}_0^d}\frac{s}{\sigma_0}s^{-m}\, exp(-\beta_c S) \tag{21}$$

$$\dot{e}_{ij}^c = \frac{s_{ij}}{\mu_c} \tag{22}$$

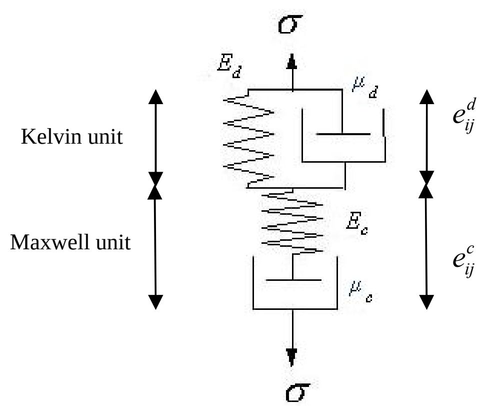


Figure 1. Burgers body representation of the ice constitutive model.

## MODEL IMPLEMENTATION

*Formulation for Finite Element Analysis (FEA)*

For the purposes of FEA in Abaqus/Explicit, the stress tensor is calculated by a material user subroutine using the separated strain terms and the stiffness matrix (Xiao, 1997):

$$\sigma_{ij} = K_{ijkl}\varepsilon_{kl} \tag{23}$$

$$K_{ijkl} = \begin{Bmatrix} K+4G/3 & K-2G/3 & K-2G/3 & 0 & 0 & 0 \\ & K+4G/3 & K-2G/3 & 0 & 0 & 0 \\ & & K+4G/3 & 0 & 0 & 0 \\ & & & 2G & 0 & 0 \\ & Symm. & & & 2G & 0 \\ & & & & & 2G \end{Bmatrix} \quad (24)$$

for bulk modulus *K* and shear modulus *G*. Incremental expressions are used to determine stress over time:

$$\delta\sigma_{ij} = K_{ijkl}\delta\varepsilon_{kl}^{e} + \delta K_{ijkl}\varepsilon_{kl}^{e} \quad (25)$$

$$\delta K_{ijkl} = \begin{Bmatrix} \delta K+4\delta G/3 & \delta K-2\delta/3 & \delta K-2\delta G/3 & 0 & 0 & 0 \\ & \delta K+4\delta G/3 & \delta K-2G/3 & 0 & 0 & 0 \\ & & \delta K+4\delta G/3 & 0 & 0 & 0 \\ & & & 2\delta G & 0 & 0 \\ & Symm. & & & 2\delta G & 0 \\ & & & & & 2\delta G \end{Bmatrix} \quad (26)$$

The incremental stiffness matrix terms are derived from the damage and strain equations:

$$\delta G = G' - G = \frac{G_0}{1+C(S+\delta S)} - \frac{G_0}{1+CS} \quad (27)$$

$$\delta S = \left[f_1(p)\left(\frac{\sigma}{\sigma_0}\right)^{q_1} + f_2(p)exp\left(\frac{\sigma}{\sigma_0}\right)\right]\delta t \quad (28)$$

$$\delta\varepsilon_{ij} = \left(\dot{\varepsilon} - \dot{e}_{ij}^{d} - \dot{e}_{ij}^{c} - \dot{\varepsilon}_{ij}^{v}\delta_{ij}\right)\delta t \quad (29)$$

The proportional constant *C* is assumed to be equal to 1 to be consistent with earlier work (Xiao, 1997; Li, 2002).

*FEA Implementation*

A user subroutine (VUMAT) was written to implement the ice damage model for Abaqus/Explicit. The relatively new VUMAT capability for the explicit solver offered a means of more accurately capturing high-speed dynamic events, typical of ice crushing/recrystallization phenomenae.

Besides several interface changes, two features of VUMAT stand out. The first is that the calculations are vectorized, so that the stress state of multiple elements are calculated with a single function call. Vectorization hence gives some increase in efficiency to offset the more complex problem of forward Euler FEM calculations, as compared to backward Euler in Abaqus/Standard.

The second feature of interest for Abaqus/Explicit user-defined material subroutines is that user-defined element deletion is available. On the code level, this requires a state variable in the subroutine to be assigned as a deletion flag, which is tested to switch the flag to false when deletion criteria are met. When a deletion flag is triggered for an element, Abaqus will assign it a zero stress and strain for the remainder of the simulation.

## TESTING

*Initial Validation*

For initial validation, the VUMAT code was used for a single-element triaxial loading run, using a 1m cube with 40 MPa hydrostatic pressure and 5 MPa compression on a single axis, and a 20 s creep/relaxation cycle. Material parameters used for test runs are listed in Table 1, with values consistent with Xiao (1997) and Li (2002); sensitivity analysis of the parameter values will be a focus of future research. Results were compared to those generated using an older UMAT implementation of a similar ice damage model. A suitably close match was obtained; see Figure 2.

Table 1. Summary of parameters used in modeling.

| Symbol | Description | Value | Units |
|---|---|---|---|
| $E$ | Undamaged elastic modulus of ice | 9500 | MPa |
| $E_k$ | Stiffness in Kelvin unit | 9500 | MPa |
| $\nu$ | Poisson's ratio | 0.3 | |
| $\sigma_0$ | Reference stress, damage calculations | 15 | MPa |
| $s_0$ | Reference stress, strain rate calculations | 1 | MPa |
| $\dot{e}_{ij}^{c}$ | Reference secondary creep strain rate | 1.76 | 1/μs |
| $\dot{e}_{ij}^{d}$ | Reference delayed elastic strain rate | 100 | 1/μs |
| $\beta_c$ | Constant for secondary creep calculation | 1 | |
| $\beta_d$ | Constant for delayed elastic calculation | 1 | |
| $m$ | Exponent for secondary creep calculation | 4 | |
| $n$ | Exponent for delayed elastic calculation | 2 | |
| $q$ | Exponent for microcracking damage calculation | 5 | |
| $f_3$ | Constant for volumetric strain calculation | 1 | |

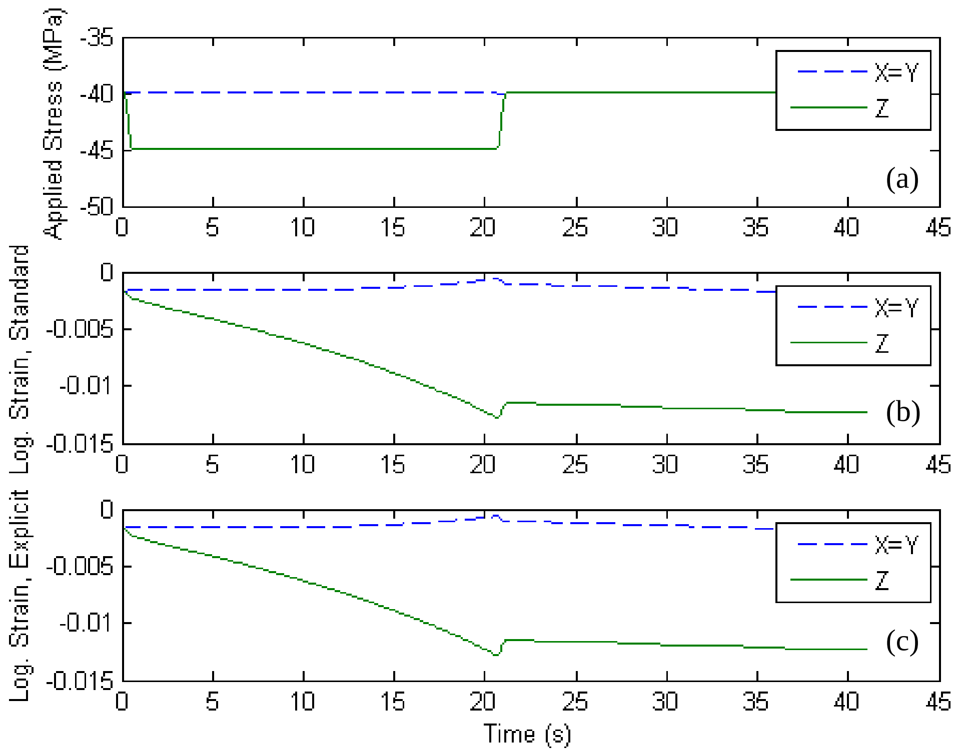


Figure 2. Simulated triaxial loading of a single cubic element. (a) Applied stress (negative for compressive stress); (b) Abaqus/Standard result; (c) Abaqus/Explicit result.

*Validation with experimental results*

Next, FEA geometry was implemented to simulate the Hobson's Choice experiments detailed in Frederking et al., (1990a, b). That program included medium-scale indentation of *in situ* sea ice. The test apparatus is shown in Figure 3(a) with detailing of the indentation in Figure 3(b). Specifically, Test 7, with a 68 mm/s indentation speed, of this experimental program was used; this was the test run used by both Xiao (1997) and Li (2002) for FEA verification of the earlier ABAQUS/Standard ice damage model. Figure 4(a) shows the load-time trace for this indentation test, with the region of interest, the initial load drop, shown in Figure 4(b).

For the old and new analyses of the Hobson's Choice test, the simulated geometry was two-dimensional (2D) (axisymmetric (Li, 2003) and plane strain (Xiao, 1997) analyses were executed with a similar mesh). The geometry assumes idealized spalling has occurred prior to the run, with a rigid indentor extending past the width of the contact surface, on top of the wedge. An example mesh used for the present modeling is shown in Figure 5; the mesh has 290 triangular linear elements and an average node spacing of about 15 mm for an area of approximately 1000 mm x 1000 mm.. The analysis was run as an axisymmetric 2D geometry. The geometry and derived mesh were verified using a closed-form solution for a flat indentor on a semi-infinite plane and an elastic material, from Timoshenko and Goodier (1934).

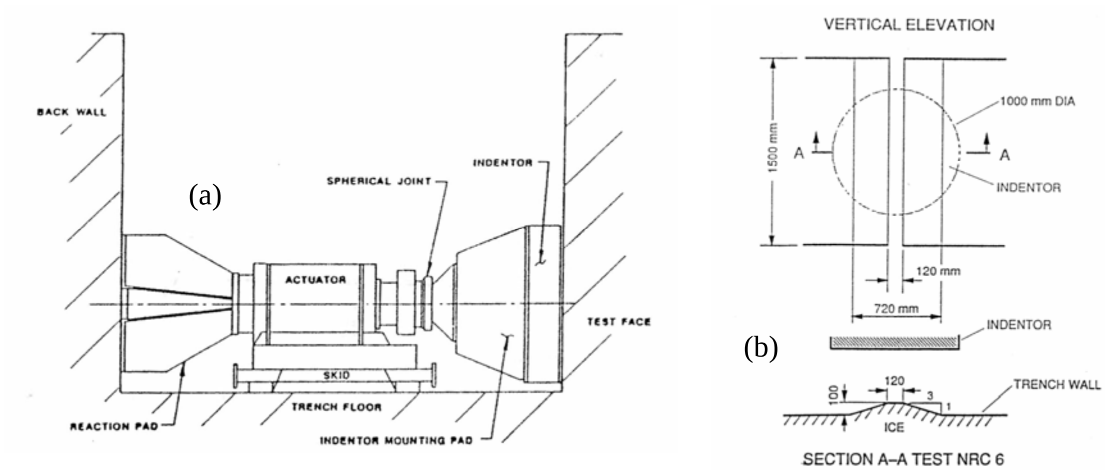


Figure 3. Hobson's Choice test configuration – (a) side view (b) plan view (Frederking et al., 1990a)

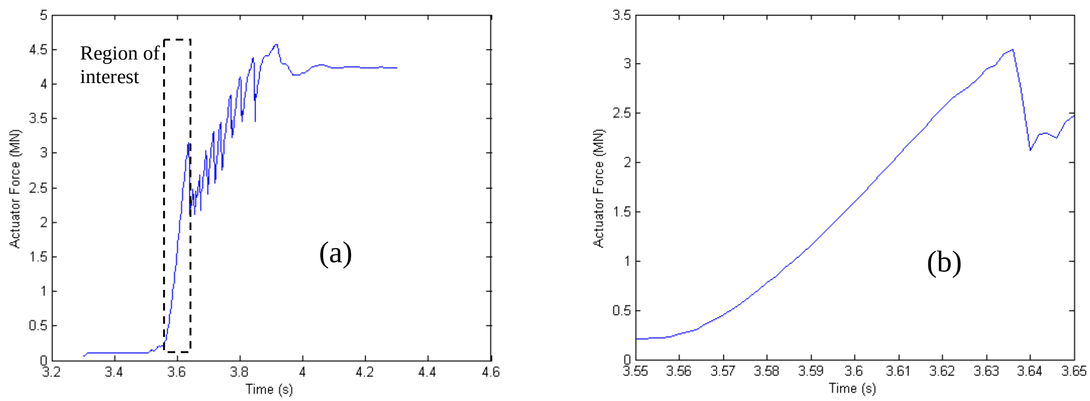


Figure 4. Hobson's Choice (1989) Test NRC07 – (a) Load time record, (b) Initial load drop (Frederking et al., 1990a)

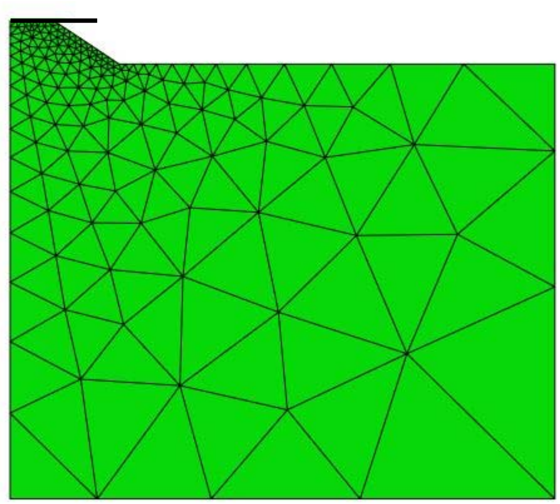

Figure 5. Example 2D FEA mesh for Hobson's Choice simulation. Rigid indentor is visible at the top of the ice wedge, protruding to the right.

**RESULTS**

With a simple element deletion criterion (damage $S > S_{crit}$ for a single element) added, load drops similar to the test result (Figure *4*(b)) were achieved. The results are shown in Figure 6 for $S_{crit}$ = 10, 20 and 30. Each sub-figure shows an initial gradual load drop due to softening and sharper load drops at time points where element deletion occurred. $S_{crit}$ = 10 is the best approximation to the pointed load drop of the test trace.

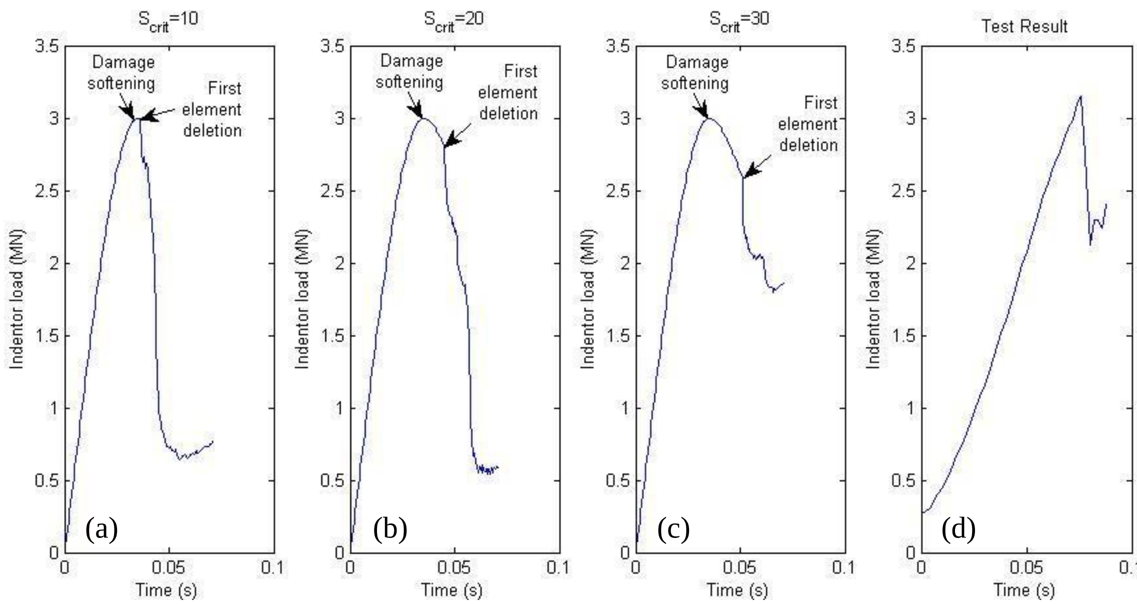


Figure 6. Explicit FEA test runs for Hobson's Choice geometry, element deletion at (a) $S_{crit}$ > 10, (b) $S_{crit}$ > 20, (c) $S_{crit}$ > 30; original test result in (d)

The resulting elastic slope is greater than that of the test loading, and several explanations for this can be advanced. An initial elastic modulus of 9500 MPa is assumed for the ice in this run. This value is typical of pure ice in situations of rapid loading; static or low speed tests would give a lower value due to the dominance of delayed elastic strain effects (Sanderson, 1988). Pre-existing damage in the field ice was almost certainly present, also lowering the elastic modulus of the ice below the theoretical maximum. Li (2002) discusses this and assumes a constant initial damage prior to the modeling runs in that work. Finally, it is likely that some compliance in the test apparatus, not presently accounted for in the FEA model, resulted in a less stiff load response for the test result. Generally though, the load drop behaviour in the model result is consistent with the test result, and the factors lowering apparent stiffness in the test can be added to the FEA model for future calibration and development, given reasonable assumptions to quantify them.

**CONCLUSIONS**

The new implementation of the damage mechanics model offers good agreement with test results and potential for high-fidelity simulation of rapidly occurring ice phenomenae. Additionally, the successful simulation of a load drop with element deletion suggests that material removal (extrusion) of crushed ice, as driven by softening and pressure melting processes, is a factor in creating the loading patterns (e.g., ice-induced vibration) observed in ice in field situations.

The availability of element deletion in Abaqus/Explicit offers several possibilities for controlling the simulation and increasing fidelity. A minimum viscosity limit is a quantitative indicator of softening; low viscosities also result in very high creep strains, which result in convergence issues. Element deletion criteria based on a singular stiffness matrix, which would normally invalidate the calculation and be used as a flag for run termination, can also be easily implemented. The development of improved deletion criteria is an ongoing area of research in this program. Additionally, accounting for the effects of changes in physical conditions (e.g., strain rate, temperature) on the chosen criterion is of interest for investigation.

Near future work will concentrate on finalization of the VUMAT code and more exact calibration to more sets of test data. Medium-to-long term development will be directed to adding thermal calculations to the model, and the simulation of refreezing of softened elements by heat-driven processes. Additionally, integration of damage modeling with macro-scale fracture modeling (along the lines of the FEA work in Taylor (2010)) is desirable.

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