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ON DEFORMATION MECHANISMS OF SEA ICE AND THEIR MATHEMATICAL MODELLING

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ABSTRACT

The focus of this paper is on the material modelling of ice for engineering purposes. The goal is to study equations for computer simulation of ice loads exerted on structures. Therefore, glacier flow, for example, is beyond the scope of this study. The following topics are investigated: The role of the von Mises operator in ice mechanics, the form of equations having quantities with a decimal number exponent, the role of time in models for creep, the dislocation creep model for ice, grain boundary sliding, the effect of microcracking on elastic properties of ice and failure locus.

INTRODUCTION

It is important from an economical standpoint to be able to determine the ice loads exerted on structures. Traditionally ice loads exerted on structures have been determined on the basis of experimental values obtained from full-scale field tests and/or model tests in ice tanks. Due to the lack of computer capacity mathematical models were simple and therefore their ability to predict ice loads was limited. Modern powerful computers allow the inclusion of many mathematically complicated phenomena in the mathematical models. Thus, today numerical simulation provides a potential tool for determination of ice loads exerted on structures. A key part of the numerical simulation of ice loads relies on the knowledge of deformation mechanisms of ice and their mathematical modelling. The focus of this paper is on the material modelling of ice for engineering purposes. Thus, the glacier flow, for example, is beyond the scope of this study.

Preparation of a material model for computer simulation has several stages. Historically it began by carrying out field tests on ice. Conditions on an ice field are usually so harsh that reliable material tests are too difficult to prepare. Ice samples are therefore transported to the laboratory where the experimental work is much easier to perform. Since sea ice has a rough internal structure, the ice samples need to be relatively large in order to produce statistically acceptable results. Thus laboratory grown ice has sometimes been used for the investigation of deformation of ice. Mechanical tensile, compressive tests can provide important results on the response of ice in the form of stress-strain curves, strain-time curves etc. These curves are an important part of the modelling task but they do not allow the preparation of a material model. Material modelling is not a pure curve fitting procedure; preparation of the macroscopic material model requires a physical background. Micromechanical investigation gives the physical background for a constitutive equation and therefore forms a solid foundation for the preparation of the material model – maybe first on the microscopic level and then extended to the macroscopic level. The curves obtained from mechanical testing are utilised with the macroscopic constitutive equation for determination of the values for the material parameters. This procedure is called curve fitting. Thus, micromechanical and the macromechanical investigation gives the form for the material model whereas curve fitting provides the values for the parameters.

Continuum thermodynamics is a tool for preparing the validation of a material model. Simplifying the validation of a material model by continuum thermodynamics means that the model is checked to follow the second law of thermodynamics. The tool for doing this is the Clausius-Duhem inequality. Validation of the material model should be a vital task in the preparation of the material model. Finally, unutilised curves obtained from experimental work are used for verification of the material model. After verification the material model is ready for numerical simulation.

There are many comprehensive papers on dislocations and other microscopic details in ice. The aim of the present study is not to try to compete with them but to take a different view and to study the macroscopic constitutive equations. An excellent and though evaluation of the micromechanisms in ice is given in the book by Schulson and Duval (2009).

When loaded, sea ice shows mainly elasticity, creep and damage. For elasticity and creep, traditional material models are applied. Damage mechanics, however, is a newer branch of science. Therefore, in ice mechanics the focus is on the description of damage. Although the 1980s was a very active time for modelling, the following recent papers should not be forgotten: Derradji-Aouat (2003), Pralong et al. (2006), Sain and Narasimhan (2011) and Duddu and Weissman (2012), just to mention some of the names working in this field. The comments on the following pages are not for these material models.

Due to the huge increase in computer performance today, the focus in ice mechanics lies on the modelling of ice-structure interaction. The papers by Lubbad and Løset (2011), Gagnon (2011), Paavilanen et al. (2011) and Zhou et al. (2013) give a good view of this activity.

INTRODUCTION TO MATERIAL MODELLING

Whether the researcher is selecting or deriving a material model, he/she will face the following questions: What is the appearance of an acceptable material model? What are the features of a reliable constitutive equation for creep? How can one check the quality of the model? How can one change this model to be better suited for computer simulation? This is the topic of the following study.

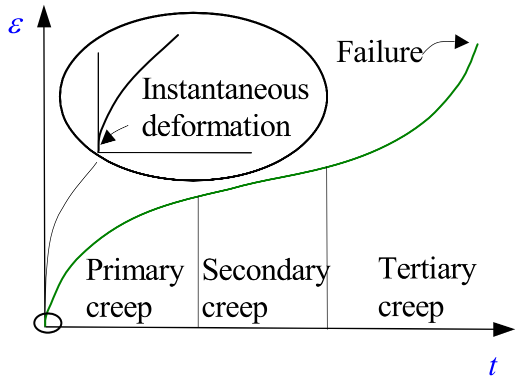

Figure 1. Typical strain-time curve for creep of a body at elevated temperatures under constant stress or constant load.

Creep is an important deformation mechanism of ice. Usually, the creep curve is constructed by carrying a uniaxial constant stress or constant load tension test. The exact form of a creep curve depends strongly on the material and on the temperature. However, many materials give the curve shown in Figure 1, which has three stages. The secondary creep is sometimes just a turning point of the curve.

THE VON MISES OPERATOR IS PRESENT IN CREEP MODELS AND SHOULD BE IN GBS MODELS

The micromechanical role of the von Mises operator $J_{\mathrm{vM}}(\cdot)$ for creep, plasticity and for grain boundary sliding (GBS) is studied. The square root of the mean square of the shear stresses through all directions of a material crystal is obtained by carrying out integration over all the planes of a unit sphere. This is

$$I(\tau_{\vec{n}}) = \left[\frac{1}{\Omega} \int_{\Omega} (\tau_{\vec{n}})^2 \, \mathrm{d}\Omega \right]^{1/2} . \tag{1}$$

In Equation (1) the notation Ω stands for the surface area of the unit sphere. Figure 2(a) shows half of the unit sphere and the shear stress $\tau_{\vec{n}}$ acting on it. The shear stress τ is the driving force for the dislocation creep [Figure 2(b)] and for the grain boundary sliding [Figure 2(c)]. According to Schulson and Duval (2009, p. 158), "It is worth noting that the viscoplasticity of polycrystalline ice can be described by invoking (only) four slip systems." This means that the effect of all shear stresses acting in four directions has to be taken into account. For an acceptable accuracy it can be replaced by an average value of the shear stress in all directions. In granular ice, there are grain boundaries in all directions. Thus, the quantity $I(\tau_{\vec{n}})$ is a good candidate for measuring the overall force for dislocation creep and for grain boundary sliding. However, it is time-consuming to compute the value for $I(\tau_{\vec{n}})$. Therefore, it should be replaced with something else that provides the same information but that is computationally much lighter.

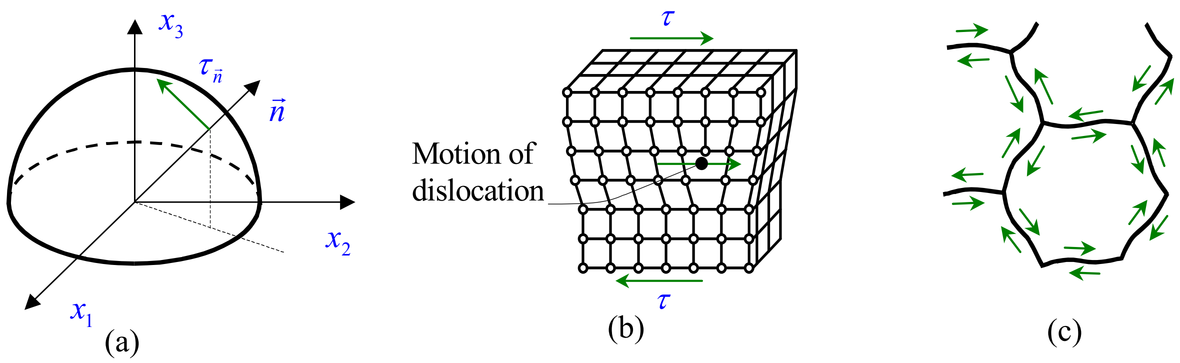

Fig. 2. (a) Shear stress on the (half of the) unit sphere. (b) Shear stress is the driving force for dislocation glide. (c) Shear stress induce grain boundary sliding.

According to Novozhilov [1952, Eqs (1.2) and (2.13)], the quantity $I(\tau_{\vec{n}})$ is related to the second deviatoric invariant of the stress tensor $J_2(\boldsymbol{\sigma})$. Thus, $I(\tau_{\vec{n}})$ is also related to the von Mises value of the stress tensor $J_{\mathrm{vM}}(\boldsymbol{\sigma})$. The relation is

$$I(\tau_{\vec{n}}) = \sqrt{\frac{2}{15}} \, J_{\mathrm{vM}}(\boldsymbol{\sigma}) . \tag{2}$$

Based on the above discussion, the von Mises value of the stress tensor $J_{\mathrm{vM}}(\boldsymbol{\sigma})$ is a good quantity for measuring the driving force for dislocation creep and for grain boundary sliding. Therefore, the von Mises value of the stress tensor $J_{\mathrm{vM}}(\boldsymbol{\sigma})$ is present in material models for creep and plasticity and should be present in constitutive equations for grain boundary sliding. When expressed in the rectangular Cartesian coordinate system (x_1, x_2, x_3) the quantity $J_{\mathrm{vM}}(\boldsymbol{\sigma})$ takes the form

$$[J_{\mathrm{vM}}(\boldsymbol{\sigma})]^2 = \frac{1}{2}\left[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2\right] + 3\,(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2) . \tag{3}$$

In Equation σ_{ii} (no summation) is the normal stress in the x_i-direction and σ_{ij} is the shear stress caused by a x_j-directional force acting on the surface the normal of which is in the x_i-direction.

UNITS CANNOT HAVE DECIMAL NUMBER EXPONENTS

The secondary creep is modelled by Norton's law, which in ice mechanics is called Glenn's law. For uniaxial tension it is often written in the following form:

$$\dot{\varepsilon}^{\mathrm{v}} = \dot{\varepsilon}_{\mathrm{re}}\, \sigma^{n}, \tag{4}$$

where $\dot{\varepsilon}_{\mathrm{re}}$, n are material parameters. If Model (4) was used, the parameter $\dot{\varepsilon}_{\mathrm{re}}$ would take strange unit that would be dependent on the value of the exponent *n*. The following simple example displays the problem. The exponent *n* is assumed to have the value 3.427 and the stress σ was represented in MPa's. This leads to the conclusion that the unit for the parameter $\dot{\varepsilon}_{\mathrm{re}}$ should be $(1/\mathrm{MPa})^{3.427}$. If the value for the parameter *n* was known within a tolerance (as it should be), what would be the unit for the parameter $\dot{\varepsilon}_{\mathrm{re}}$? This implies that Form (4) is not an acceptable one.

The constitutive equation called Norton's law should be written as

$$\dot{\varepsilon}^{\mathrm{v}} = \dot{\varepsilon}_{\mathrm{re}} \left(\frac{\sigma}{\sigma_{\mathrm{re}}} \right)^{n} \qquad \text{or} \qquad \text{in 3D} \quad \dot{\boldsymbol{\varepsilon}}^{\mathrm{v}} = \tfrac{3}{2}\, \dot{\varepsilon}_{\mathrm{re}} \left(\frac{J_{\mathrm{vM}}(\boldsymbol{\sigma})}{\sigma_{\mathrm{re}}} \right)^{n} \frac{\mathbf{s}}{J_{\mathrm{vM}}(\boldsymbol{\sigma})}. \tag{5}$$

In Material Model (5) σ_{re} is a parameter the value of which can be fixed before determination of the values for the parameters $\dot{\varepsilon}_{\mathrm{re}}$ and *n*. It is important to note that Form (5) does not have more parameters to fit by a curve-fitting procedure than Form (4). In principle, the value for σ_{re} is an arbitrary one. The role of σ_{re} is to make the quantity $\sigma/\sigma_{\mathrm{re}}$ dimensionless. However, in order to reduce the error caused by the inaccuracy of the value for the exponent *n*, the value of σ_{re} should be chosen so that the ratio $\sigma/\sigma_{\mathrm{re}}$ takes values close to unity. This means that if there were three data curves with the stress values 40 MPa, 60 MPa and 80 MPa, a recommended value for σ_{re} would be 60 MPa. It is worth noting that the parameters $\dot{\varepsilon}_{\mathrm{re}}$ and n have a similar effect on the viscous strain rate $\dot{\varepsilon}^{\mathrm{v}}$. Increasing values for $\dot{\varepsilon}_{\mathrm{re}}$ and n lead to an increasing value for the viscous strain rate $\dot{\varepsilon}^{\mathrm{v}}$. Only results obtained by different values of the stress σ can display the different roles of parameters $\dot{\varepsilon}_{\mathrm{re}}$ and n in Creep Model (5).

TIME IS NOT A VARIABLE IN THE CREEP EQUATION

The creep model introduced by Costa Andrade (1910, p. 11) is studied next. The original appearance, viz.

$$\dot{\varepsilon}^{\mathrm{v}} = A\, t^{-2/3} + \text{other terms} \tag{6}$$

is replaced by

$$\dot{\varepsilon}^{\mathrm{v}} = A \left(\frac{t}{t_0} \right)^{-2/3} + \text{other terms}. \tag{7}$$

In Equation (7) t_0 is a parameter the value of which can be fixed before the determination of the values for the parameter *A*. This is the same as was discussed when evaluating Norton´s law.

Model (7) can be generalised by writing (without 'other terms')

$$\dot{\varepsilon}^{\mathrm{v}} = A(\sigma) \left(\frac{t}{t_0} \right)^{-k}, \tag{8}$$

where *k* is a positive number less than 1, $A(\sigma)$ is a stress-dependent function and the other terms are dropped out. The modifications of Form (8) are widely used today.

One of the problems in Model (8) is that at the moment $t = 0$, the value for the viscous strain rate $\dot{\varepsilon}^{\mathrm{v}}$ tends towards infinity. However, the quantity $(t/t_0)^{-k}$ $(0 < k < 1)$ has a weak singularity at the point $t = 0$. This means that $(t/t_0)^{-k}$ is integrable and therefore the viscous strain ε^v is bounded (i.e. has a finite value). The problem arises when finite element or finite difference codes are used. Such programs carry out the time integration numerically. Usually numerical methods utilise the derivative of the integrand at the beginning of the time step. For the first time step (starting from $t = 0$) this derivative is unbounded and therefore no numerical integration is possible. This problem can be solved through careful modelling.

The second problem of Equation (8) is the starting shot to measure time t, i.e. the definition of the moment $t = 0$. The natural time scale is the one that started with the Big Bang. It is not for creep models. The other possibility is to measure time from the moment the load was applied. The difficulties related to this interpretation are discussed next.

Because of the nature of the creep process, the function $A(\sigma)$ is a monotonically increasing function. This means that if the stress σ tends toward zero, the value of the function $A(\sigma)$ approaches zero and, according to Equation (8), the viscous strain rate $\dot{\varepsilon}^v$ approaches zero as well. Two different loadings denoted by 1 and 2 shown by Figure 3(a) are studied. Load 1 immediately takes the value σ_1 whereas load 2 is extremely low up to the moment $t = t_1$ and then takes the value σ_1. Time-Hardening Material Model (8) gives strain-time curves with the forms sketched in Figure 3(b). The paradox in Figure 3(b) is that the shapes of the strain-time curves differ from each other. They should be equal, since the extremely low loading 'equals' no loading at all, therefore it can have virtually no effect on the strain-time relationship. This paradox can be solved by the following interpretation: Equation (8) is prepared for constant stress cases, i.e. for σ = const., and $t = 0$ is the moment of application of the constant stress. This means that the varying stress must be described by a sequence of constant stresses, i.e. by a staircase function, as shown in Figure 4. It is not a big problem for computational mechanics, since in the numerical integration procedures the stress-time curve has to be approximated in any case. On the other hand, the interpretation of Model (8) for varying stress-time dependence is a difficult task. This problem is discussed following the introduction of a strain-hardening model.

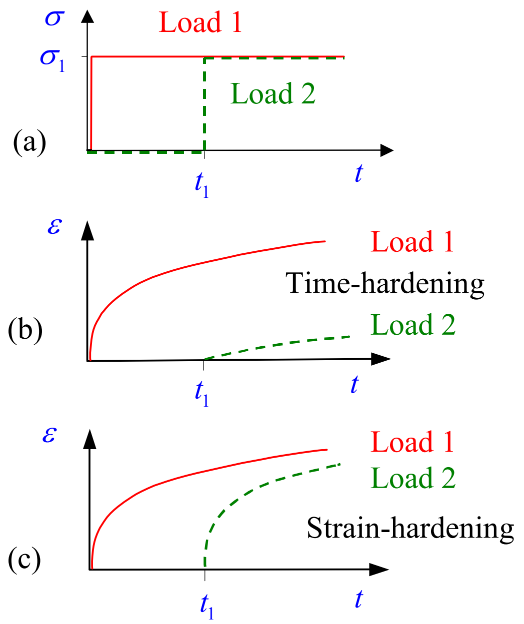

Figure 3. (a) Two loadings and the response of the material according to (b) Time-hardening Model (8) and (c) Strain-hardening Model (9).

A material model using strain-hardening formalism can be expressed as follows:

$$\dot{\varepsilon}^v = B(\sigma)\frac{1}{(\varepsilon^v)^m}, \tag{9}$$

where $B(\sigma)$ is a stress-dependent function and m is a non-negative number. As Figure 3(c), shows the strain-hardening model gives the correct $\varepsilon - t$ curve for both loads. Material Model (9) has a much stronger micromechanical foundation than Model (8), since the accumulated creep strain ε^v may be the source for hardening but time t can never cause hardening. The values of the material parameters for the time-hardening model determined from a constant uniaxial stress σ test can be mathematically converted to the values for a strain-hardening model [e.g. Santaoja (2012, pp. 227-228)].

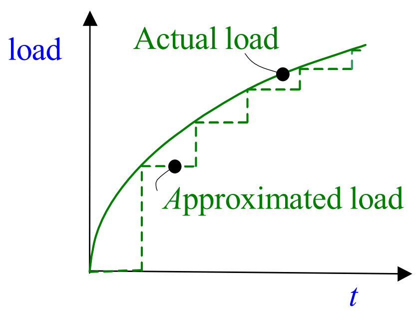

Figure 4. The load is approximated by a stair-case function.

The time-dependent response of materials can display a viscoelastic and/or viscoplastic character. Both deformations are time dependent, but viscoelastic strain is recoverable whereas viscoplastic strain is permanent. Usually both theories describe the response which is dependent of the deformation history of the material. Therefore, the term "time-dependent" should be replaced by the term "path-dependent", but the generally used terminology is difficult to change.

As discussed above, the real load can, for example, be approximated by a staircase function. However, for a path-dependent material the responses of the load increments are not mutually independent. In the classical theory of viscoelasticity, for example, the expression to determine the value for creep strain ε^{v} takes the following appearance:

$$\varepsilon^{\mathrm{v}} = \int_{\tau = 0}^{t} \ldots \, \mathrm{d}\tau \tag{10}$$

A method to find a solution (even with the staircase load) takes considerable computing time, since at every time increment the time integration has to be started from the first time increment and the integration has to be performed over all time steps. Thus, this kind of an approach is not for computation of ice loads exerted on structures. This is discussed in greater detail by Santaoja (1987), (1988), (1990, pp. 142-150).

A way out from the above problem is that the introduction of a quantity describing the history of the response should be included in the material model. This formalism is a vital part of continuum thermodynamics with internal variables. It is the topic of the next section. Prior to that a brief discussion is given of the following material model.

Sometimes material models are given in the form

$$\varepsilon^{\mathrm{v}} = \mathit{Something} \times t\,, \tag{11}$$

where *Something* is a function of appropriate quantities. Form (11) is possible, if it has been obtained by time-integration of a material model

$$\dot{\varepsilon}^{\mathrm{v}} = \mathit{Something\ else}\,. \tag{12}$$

Unfortunately, for a realistic material model the closed-form time-integration is not usually possible even in the case of constant stress σ. Norton's law may be the only exception.

The above discussion does not apply when experimentalists express their creep data. The creep research community has become used to reading creep curves when they are given in time scale. Furthermore, researchers working on material modelling can easily apply such creep curves for their own curve fitting and other purposes.

DISLOCATION CREEP MODEL FOR ICE

Le Gac and Duval (1980) have proposed a constitutive equation for ice for modelling the response of ice when the dislocation creep is dominant. The model is formulated for isotropic ice when the deformations are small. However, the type of anisotropy, which depends on the deformation history of the material (excluding Bauschinger effect), is described in the model. Both isotropic and kinematic hardening are modelled. The author has given it the following appearance (Santaoja, 2012, p. 232):

$$\dot{\boldsymbol{\varepsilon}}^{\mathrm{v}} = \dot{\varepsilon}_{\mathrm{re}} \left[\frac{\left\langle J_{\mathrm{vM}}(\boldsymbol{\sigma} - \boldsymbol{\beta}^{1}) - \beta^{2} - \sigma_{\mathrm{tr}} \right\rangle}{\sigma_{\mathrm{re}}} \right]^{n} \frac{\mathbf{s} - \mathbf{b}^{1}}{J_{\mathrm{vM}}(\boldsymbol{\sigma} - \boldsymbol{\beta}^{1})}\,. \tag{13}$$

The quantity σ_{tr} refers to the onset of creep. The second-order tensor $\boldsymbol{\beta}^{1}$ describes kinematic hardening and the scalar valued quantity β^{2} describes isotropic hardening. The form of the quantity $J_{vM}(\boldsymbol{\sigma} - \boldsymbol{\beta}^1)$ expressed in the (x_1, x_2, x_3) coordinate system can be derived from Equation (3). The notations $\mathbf{s}$ and $\mathbf{b}^1$ are the deviatoric parts of the stress tensors $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}^1$. The scalar component s_{ij} of the deviatoric stress tensor $\mathbf{s}$ reads

$$s_{ij} = \sigma_{ij} - \frac{1}{3}(\sigma_{11} + \sigma_{22} + \sigma_{33})\,\delta_{ij}, \tag{14}$$

where the Kronecker delta δ_{ij} is defined to be zero when i and j differ, and to be unity when $i = j$. The evolution equations for both kinematic hardening and isotropic hardening have hardening and softening behaviour and they have similar appearances. The evolution equation for isotropic hardening β^2, for example, has the following form:

$$\dot{\beta}^2 = n^1 J_{vM}(\dot{\boldsymbol{\varepsilon}}^{v}) - n^2 \left(\frac{\beta^2}{\sigma_{re}} \right)^m, \tag{15}$$

where n^1, n^2 and m are material parameters. The first term on the right side of Expression (15) gives the hardening, whereas the second term gives the softening of the material.

The model proposed by Le Gac and Duval (1980), Equations (13) and (15), is an excellent example of a constitutive equation which can be written in terms of continuum thermodynamics with internal variables.

GRAIN BOUNDARY SLIDING

Besides the pure elastic distortion of crystal lattices of ice, there is another recoverable deformation mechanism called delayed elastic deformation. This is a time-dependent and recoverable mechanism and therefore it is a viscoelastic deformation. Delayed elasticity is a consequence of shear stress-induced grain boundary sliding. It is an important deformation mechanism at high homologous temperatures. In engineering applications such conditions arise in sea ice.

Because grain boundaries are not flat, sliding generates incompatibilities near deviations from planarity. Moreover, in polycrystals sliding is blocked at triple junctions. As the sliding displacement increases, the accommodation of these incompatibilities begins to control the extent and rate of sliding (Raj & Ashby, 1971). Different accommodation mechanisms are possible. (Weiss & Schulson, 2000, p. 281)

According to Weiss and Schulson (2000, pp. 281 and 282), there are four different types of accommodation. They are: (a) Elastic accommodation. Sliding is accommodated by the elastic distortion of neighbouring grains. (b) Accommodation by diffusion. Diffusion of atoms or vacancies in either the bulk of the grains or the plane of the boundary, or both, accommodates further sliding. (c) Accommodation by power-law creep. This mechanism is based on dislocation glide and climb. (d) Cracking. Finally, when stress concentrations resulting from grain boundary sliding are strong enough and accommodation mechanisms are too slow and/or too limited, either hole growth in the boundary plane or crack nucleation at blocking sites may occur.

The most well-known material model for the grain boundary sliding of ice is that proposed by Sinha (1978). It is for constant stress σ and it reads

$$\varepsilon^{g} = c \left(\frac{\sigma}{E} \right)^s \left[1 - e^{-(a_T t)^b} \right], \tag{16}$$

where ε^{g} is the grain boundary sliding strain, a_T is the inverse of the relaxation time and c, s and b are material parameters. For varying state of stress, the utilisation of Model (16) takes a lot of computing time, as pointed out by Santaoja (1987, 1988) and Evgin et al. (1991). Later Sinha modified his expression, but this is not studied here.

Since grain boundary sliding is a recoverable process, it has to be connected to a mechanism that gives a recoverable nature for grain boundary sliding. The author proposes that the recoverable mechanism is the distortion of crystal lattices not very close to the grain boundaries but on the ice crystal scale.

EFFECT OF MICROCRACKING ON THE ELASTIC PROPERTIES OF ICE

The formation of microcracks initiates the weakening of ice (Frost, 2001, p. 1823). The effect of microcracking on the elastic properties of ice can be determined analytically. Based on stress intensity factors K_{I}, K_{II} and K_{III}, Basista (2002, p. 227) derived expression of the specific Gibbs free energy g^{ud} for a Hookean material with rectilinear non-interacting microcracks in a two-dimensional domain. His equation is based on the expressions for the stress intensity factors K_{I}, K_{II} and K_{III}. The author modifies the expression by Basista and gives it the following appearance for the plane stress:

$$g^{\mathrm{ud}} = \frac{1}{2\rho_0}\left[\frac{1}{3(3\lambda+2\mu)}[\mathbf{1}:\boldsymbol{\sigma}]^2 + \frac{1}{2\mu}\,\mathbf{s}:\mathbf{s}\right] + \frac{\pi h}{E}\sum_{r=1}^{M} Q^r (a^r)^2 \times \left\{\vec{n}^r\cdot\boldsymbol{\sigma}\cdot\boldsymbol{\sigma}\cdot\vec{n}^r - [1 - H(\vec{n}^r\cdot\boldsymbol{\sigma}\cdot\vec{n}^r)]\,(\vec{n}^r\cdot\boldsymbol{\sigma}\cdot\vec{n}^r)^2\right\}, \qquad (17)$$

where ρ_0 is the density, λ and μ are the Lamé elastic constants, h is the thickness of the two-dimensional domain and M is the number of microcrack groups. In each group the sizes and orientations of the microcracks are equal. The quantity a^r is the length of the microcrack and the unit normal vector for the microcrack surface is denoted by $\vec{n}^r$, as shown in Figure 5. The microcrack densities are $Q^r = m^r/(\rho_0 V^{\mathrm{rve}})$, where m^r is the number of microcracks within the r‘th microcrack group and V^{rve} is the representative volume element. The Representative volume element RVE of the material is large enough to be statistically representative of the material properties that will be modelled and small enough compared to the macroscopic structural dimensions. The latter conditions means that the RVE has to be small enough to be to be treated as a material point in the structural analysis. It is noteworthy that the indices in Expression (17) only take values 1 and 2.

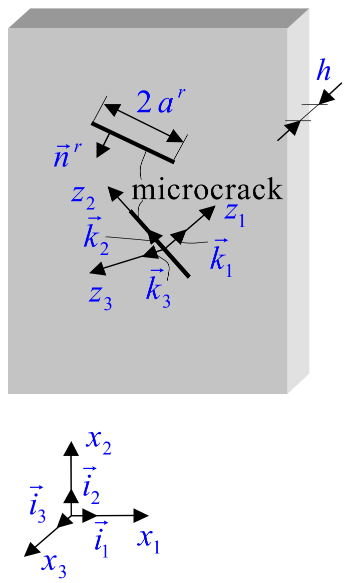

Fig. 5. Microcracks in a two-dimensional domain.

Since in the present model $g \propto g^{\mathrm{ud}}$ and

$$\boldsymbol{\varepsilon}^{\mathrm{de}} = \rho_0 \frac{\partial g}{\partial \boldsymbol{\sigma}}, \qquad (18)$$

Equation (17) gives for the damage elastic strain tensor $\boldsymbol{\varepsilon}^{\mathrm{de}}$

$$\boldsymbol{\varepsilon}^{\mathrm{de}} \propto \boldsymbol{\varepsilon}^{\mathrm{e}} + \boldsymbol{\varepsilon}^{\mathrm{d}}, \qquad \boldsymbol{\varepsilon}^{\mathrm{e}} = \mathbf{S}:\boldsymbol{\sigma} \qquad \text{and} \qquad \boldsymbol{\varepsilon}^{\mathrm{d}} = \mathbf{S}^{\mathrm{d}}:\boldsymbol{\sigma}, \qquad (19)$$

where $\boldsymbol{\varepsilon}^{\mathrm{e}}$ is the Hookean strain tensor, $\boldsymbol{\varepsilon}^{\mathrm{d}}$ is the damage strain tensor and $\mathbf{S}$ is the compliance tensor of the Hookean material. The z_1 -coordinate of the microcrack coordinate system (z_1, z_2, z_3) is parallel to the normal of the microcrack and the coordinates z_2 and z_3 are on the plane of a microcrack, as shown in Figure 5. For one microcrack group referred to as the microcrack coordinate system (z_1, z_2, z_3), the components of the compliance tensor $\mathbf{S}^{\mathrm{d}}$ read

$$S_{1111}^{\mathrm{d}=\circ} = 2\,\frac{\pi\,h}{E}\,(a^r)^2\,Q^r\,H(\sigma_{11}^{\circ}) \tag{20}$$

and

$$S_{1212}^{\mathrm{d}=\circ} = S_{1221}^{\mathrm{d}=\circ} = S_{2112}^{\mathrm{d}=\circ} = S_{2121}^{\mathrm{d}=\circ} = \frac{1}{2}\,\frac{\pi\,h}{E}\,(a^r)^2\,Q^r\,. \tag{21}$$

The compliance tensor $\mathbf{S}^{\mathrm{d}}$, referred to as the reference frame (x_1,x_2,x_3) for a multidirectional microcrack field, is obtained from Equations (20) and (21) first by carrying out coordinate transformation and then adding together the compliance tensors due to M microcrack groups.

The Heaviside function $H(\vec{n}^p\cdot\boldsymbol{\sigma}\cdot\vec{n}^p)$ in Equation (17) is an important detail beyond the work by Basista (2002, p. 227). It ensures that under compression the crack surfaces do not penetrate each other, as demonstrated in Figure 6.

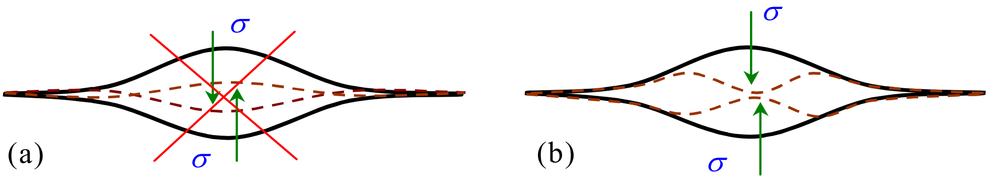

Figure 6. (a) Surfaces of a microcrack penetrate each other under compression. (b) Penetration is prevented by the Heaviside function $H(\vec{n}^r\cdot\boldsymbol{\sigma}\cdot\vec{n}^r)$.

A similar equation to Equation (17) is for penny-shaped microcracks [Santaoja (1989) & (1990)].

The above evaluation neglects the interaction between microcracks. It is very difficult or even impossible to formulate a general expression for microcrack interaction. Sevostonianov and Kachanov (2010) argued that the key parameter of reduction of strength is not the reduction of the average (over the specimen) stiffness (e.g. due to the microcracks), but the local minimal values caused by the formation of defect (microcrack) clusters.

Results (20) and (21) are based on the laws of nature, on Hooke's law and on the fact that there are microcracks in the material. This is a great benefit over the damage mechanics expression $\boldsymbol{\sigma} = E\,(1 - D)\,\boldsymbol{\varepsilon}^{\mathrm{de}}$ the foundation of which is usually in curve fitting.

FAILURE LOCUS

Failure of materials is sometimes expressed by a failure surface as expressed in Figure 7. A failure surface may cause problems. Let us assume that increasing the number of forming microcracks leads to failure of the material. Figure 7 sketches a failure locus within which there is a surface that causes the onset of microcracking. The failure locus was determined experimentally and state $\mathscr{E}_1$ was found to be a point of maximum stress, which was interpreted to be the point of failure. Unfortunately, if the material was loaded differently and the different path in the σ_1 and σ_2 space was taken, the same point in the σ_1 and σ_2 space would lead to the state $\mathscr{E}_2$. Two different states means two different numbers of microcracks. Thus, the state $\mathscr{E}_2$ would not be a point of failure and therefore no failure surface valid for every loading history can be drawn in the σ_1 and σ_2 space.

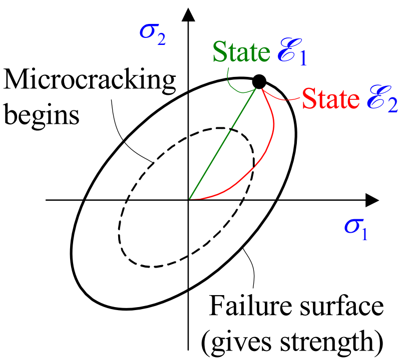

Figure 7. Failure surface and two responses.

Furthermore, computing requires a smooth stress-strain curve after the peak value of the stress σ independent of what happens in nature. This aspect may be forgotten when failure surfaces are introduced, although smooth curves can be combined with a failure surface.

Good fit of the failure surface with the experimental data does not guarantee the quality of the failure function. A rubber band, for example, attached to the measured data points by pins gives a perfect fit, but can not be a model, since it does not have a physical explanation and therefore it may give totally incorrect results outside the data points. The von Mises surface, however, has a solid physical foundation, as already pointed out in this paper.

DISCUSSION AND CONCLUSIONS

The main conclusions of this paper are: The von Mises operator describes the effect of shear stresses on deformation, time t is not a variable for material models, instead of traditional damage models, i.e. $\boldsymbol{\sigma} = E(1 - D)\,\boldsymbol{\varepsilon}^{\mathrm{de}}$, the effect of microcracks on the elastic properties of ice can be determined analytically and the failure locus in the σ_1 and σ_2 space may be impossible to determine. A model for dislocation creep was given.

In spite of the above discussion, experimentalists are encouraged to publish their results with the strain-time equations. By doing so they will not be taking a stand as to which is the correct material model. Furthermore, the strain-time equation or tabulated data allows material modellers to utilise the experimental data effectively.

It should be remembered that every material model has approximations or even flaws. The limitations of the model are not the main point, but the suitability of the model for engineering works. Earlier the computer capacity imposed restrictions on ice-structure interaction simulations. Today the lack of reliable material models is the main obstacle for computer simulations. The development of material models is an important part of the work for ice researchers.

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