USE OF THE STOCHASTIC SIMULATION TECHNIQUE FOR ESTIMATION OF THE ICE COVER STRENGTH BY INTERACTION WITH SHIP HULL

Vladimir YAKIMOV, Vladimir TRYASKIN
State Marine Technical University, Saint Petersburg, RUSSIA

ABSTRACT
The research into simulation of ice cover strength characteristics is performed within the scope of significant project on probabilistic simulation of ice loads on ship hull which was started at the Ship Structure and Technical Operation Department of SMTU in 2009. The paper covers the statement of problems connected with the simulation of ice and hydrometeorological conditions parameters, physical and mechanical characteristics of ice cover which are accepted as stochastic quantities. The main theoretical propositions as well as the general algorithm and the basic sequence of process implementation are adduced in respect to suggested simulation method. A number of test examples are considered for certain ship operation lines of Northern Sea Route at different seasonal navigation periods. The obtained simulation results are analyzed. The special-purpose software developed by authors is used for solution of above-cited tasks.

INTRODUCTION

The interaction process between ship hull and ice cover has an explicit stochastic nature, hereupon it is reasonable to proceed from deterministic methods for estimation of ice loads acting on ship hull to probabilistic approaches. The up-to-date hardware and software tools are able to simulate complicated processes and to investigate them by specialized computer experimentation taking into consideration the influence of various random components. The use of probabilistic (statistical) simulation technique for determination of ice loads makes it possible to set and to solve a number of important practical problems. Moreover it is supposed that the adoption and application of simulation concept by analysis of interaction processes between ship hull and ice cover will further the qualitative raising of safety level for sea transport operation in ice conditions as a result (Tryaskin, Yakimov and Besse, 2012).

The research related to simulation of ice loads on ship hull was initiated at the Ship Structure and Technical Operation Department of Saint Petersburg State Marine Technical University (SMTU) in the second half of the 1980s and this effort was primarily associated with Kurdyumov. He formulated the general problem statement, developed the theoretical basis of simulation method for stochastic ice loads and described the main principles for design and performance of probabilistic computer experiment. In the beginning of the 1990s all active works in this field were suspended. However the problem continued to remain urgent and significant in the course of time, and in 2009 the respective research was resumed at the mentioned department. At the present time it is brought on the whole to a completion the development of special-purpose methodology, knoware and software intended for solution of simulation problem for ice loads acting on ship hull.

The suggested approach provides for simulation of stochastic ice loads by the use of well-known statistical tests method (Monte-Carlo method), which is based on different limit relations of probability theory – on large numbers laws and limit theorems. The values of random input components considered within the framework of accepted physical model for ship and ice interaction are obtained by means of standard pseudorandom number generators and distribution law transformers. The key goal is to correctly select (using specified fitting criterion) the theoretical distribution laws approximating with the admissible precision the empirical distribution functions for stochastic ice loads parameters determined during the multiple performance of probabilistic computer experiment and to objectively estimate the statistical characteristics for concerned stochastic quantities. The process of practical implementation for simulation method in respect to ice loads on ship hull can be represented in general as a sequence of some stages (Tryaskin and Yakimov, 2012).

MAIN CONTENT

The distinctive feature of interaction process between ship hull and ice cover consists in a considerable number of various stochastic factors exerting the direct influence on ice loads magnitude. The actual ice characteristics are among them. Thereby the simulation of physical and mechanical properties for ice cover should precede the determination of ice loads parameters proper.

The forces arising in contact zone during the impact of ship side against the ice floe or ice field cause both the local crushing of ice edge and general deformation of ice cover. The crushing of ice edge to maximal depth is feasible only if the barrier strength is sufficient to stand the arising forces; otherwise the ice cover failure occurs earlier than the total contact force reaches its maximal value. The ice cover can fail owing to flexure by action of vertical component of total contact force or in consequent of stability loss by action of its horizontal component, and the stability loss is observed primarily for rather thin and level ice and for ice being in contact with the vertical or close to vertical ship side at that. It is known that the value of total contact force F is determined as an integral of contact pressures p taken throughout the actual area of contact zone between ship hull and ice cover A:

$$F = \int_A p dA. \quad (1)$$

For contact pressures determination it is proposed to consider the relationships of hydrodynamic model for solid body impact against the ice developed by Kheisin and Kurdyumov in the middle of the 1970s and used (in its original or modified variant) widely in the following years in Russia applied to design of hull structures for ice-going ships and icebreakers (Kurdyumov and Kheisin, 1976). Within the framework of hydrodynamic model it is provided to solve a problem for extrusion of relatively thin interlayer with the small-dispersed structure out of contact zone to free surface during the ship side penetration into the ice (see Figure 1). The existence of interlayer between penetrating solid body surface and uncrushed ice bulk has been ascertained by experimental way. Depending on volume of liquid phase in interlayer it can be considered as paste-like or powder-like substance possessing both viscous and plastic properties. The motion of specified medium is described by combined Hencky non-linear differential equations for viscous-plastic body which are reduced after linearization to simplified combined Reynolds differential equations defining the quasi-static squeezing for thin layer of viscous Newtonian liquid between two surfaces – penetrating solid body surface (ship side) and interlayer-uncrushed ice bulk interface (ice edge), hereat the plasticity of interlayer is neglected. Owing to existence of movable interlayer the pressures in contact zone could not remain permanent in process of ship side penetration into the ice, therefore in concerned model it is accepted that the pressures are changing in time proportionally to the interlayer thickness.

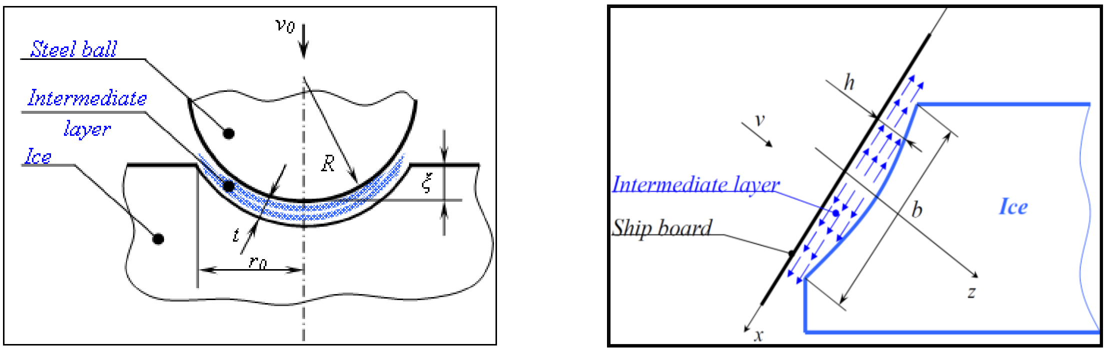


Figure 1. Hydrodynamic model for solid body impact against the ice.

In general case the solution for contact pressures can be obtained in the following state:

$$p = \sigma_c \cdot \left[\frac{n+3}{n} \cdot \frac{b^2}{t^2} \cdot \frac{\mu_1 \cdot w}{\sigma_c \cdot t} \cdot \varphi\left(\frac{x}{b}, \frac{y}{b}\right) \right]^{\frac{n}{n+3}}, \qquad (2)$$

where σ_c is compression strength of ice cover; μ_1 is coefficient of internal friction in interlayer; b is characteristic dimension (height) of contact zone; t is interlayer thickness; w is speed of solid body penetration into the ice; $n > 0$ is coefficient defining the properties of failure surface – its rigidity or compliance (in practical calculations it is allowed to use the constant value equal to one); $\varphi\left(\frac{x}{b}, \frac{y}{b}\right)$ is solution of Poisson equation $\nabla^2 \varphi\left(\frac{x}{b}, \frac{y}{b}\right) = -1$ for contact zone by zero boundary conditions presented in dimensionless form.

By impact of inclined ship side against the ice edge the contact zone has a form of segment extended considerably in longitudinal direction. Therefore it is reasonable to suppose that the displacement of crushed ice occurs primarily in vertical direction (across the contact zone). Its displacement in longitudinal direction (along the contact zone) can be neglected, that leads consequently to a certain overstating for contact pressures by simultaneous understating for depth of ship side penetration into the ice. Moreover it should be taken into account the initiation of ice chips along the contact zone edges owing to unrestricted increase of tangential stresses by approaching to its boundary. Thereby in case of inclined ship side impact the expression for contact pressures determination assumes the following final air:

$$p = \left[(2 \cdot \mu_1 \cdot k_p^{\,3}) \cdot w \cdot \left(\frac{b}{2 \cdot \alpha}\right)^2 \cdot (\alpha^2 - \bar{x}^{2}) \right]^{1/4}, \qquad (3)$$

where $\alpha > 1$ is dimensionless coefficient considering the presence of ice chips; $\bar{x}$ is relative coordinate for specified point of contact zone; k_p is empirical factor of proportionality between pressures in contact zone and interlayer thickness.

By impact of vertical ship side against the ice edge the contact zone has a form of rectangle and the crushing occurs throughout the ice cover thickness. The defined ratio of contact zone sides corresponds to each point of interaction time: during the impact the height of contact zone remains invariable and equal to specified ice cover thickness and the length of contact zone changes continuously depending on depth of ship side penetration into the ice at that. Thus the assumptions concerning incommensurability of contact zone sides and one-dimensional displacement of crushed ice justly done for inclined ship side are qualitative violated here. In case of vertical ship side impact the contact pressures are determined according to the following expression (with the consideration of ice chipping):

$$p = \left[(2 \cdot \mu_1 \cdot k_p^{\,3}) \cdot w \cdot \left(\frac{b}{2} \right)^2 \cdot \left(\frac{\xi}{1+\xi} \right) \cdot (1 - \bar{x}^2) \cdot (1 - \bar{y}^2) \right]^{1/4}, \qquad (4)$$

where $\xi = (l/b)^2$ is squared ratio of contact zone sides; b is contact zone height; l is contact zone length; $\bar{x}$ and $\bar{y}$ are relative coordinates for specified point of contact zone.

Within the framework of accepted model for contact pressures determination it is introduced additionally into consideration the conventional parameter $a_p = (2 \cdot \mu_1 \cdot k_p^{\,3})^{5/24}$ characterizing the dynamic strength of ice cover by crushing. Parameter a_p reflects in complex the typical conditions of ice navigation for specified ship and so it should be taken into account by justification of appropriate ice category. In this study it is suggested to calculate the parameter a_p depending on compression strength of ice cover σ_c:

$$a_p = A \cdot \left(\frac{\sigma_c}{B} \right)^n, \qquad (5)$$

where A, B and n are constant numerical coefficients.

The values of parameter a_p estimated by means of given empirical formula are in good agreement with the data obtained from analysis of side hull structures strength for existing transport ships operating in ice conditions (Appolonov, Nesterov, Stepanov et al., 1996).

Proceeding from foregoing it should be regarded as rated ice loads on ship hull either the values of forces causing the ice cover failure or the maximal values of total contact forces if the impact of ship side against the ice edge isn’t accompanied by ice cover failure. As it was shown above the analytical dependences used for determination of rated ice loads on ship hull include in this or another way the strength characteristics of ice cover – flexural strength and compression strength. Therefore the simulation of concerned quantities is an essential part of the whole process of stochastic ice loads simulation.

Simulation of ice conditions parameters

It is known that the ice cover thickness exerts the determinative influence on ice loads magnitude and so the available initial information on it should be taken into consideration by simulation process implementation in the first instance.

The modern monitoring facilities (such as radar and telemetry systems etc.) are able to take quick and high-precision measurements of ice cover thickness in automatic mode. The actual values of ice cover thickness got during the ice survey are used to plot the partial distributions bar chart which defines the probability (relative frequency) of existence for ice cover with the fixed thickness gradation at the specified ship navigation route in the specified seasonal period. Analytically the partial distributions can be represented by three number arrays: array of left-hand (lower) limits of thickness ranges; array of right-hand (upper) limits of thickness ranges and array of ordinates corresponding to the probabilities (relative frequencies) of falling for measured values of ice cover thickness within the specified range. In this study seven ranges for ice cover thickness are provided: 0.01÷0.10 m, 0.10÷0.30 m, 0.30÷0.70 m, 0.70÷1.20 m, 1.20÷1.80 m, 1.80÷2.80 m and 2.80÷5.00 m. Such division is in full conformance with the generally accepted classification of ice cover by age criterion: nilas, young ice, thin first-year ice, medium first-year ice, thick first-year ice, second-year ice and multi-year ice.

The random values of ice cover thickness are determined directly from the partial distributions bar chart plotted for specified ship navigation route and specified seasonal period. If the thickness is assumed to be distributed uniformly within each range, then the random values of ice cover thickness will be obtained by execution of the following action sequence:

– to simulate the first stream of random numbers ξ_1 distributed uniformly in the interval (0, 1) using pseudorandom number generator;

– to find the rated thickness ranges for which it should be valid the two-sided inequality $\Phi_{i-1} < \xi_1 \le \Phi_i$, where $\Phi_i = \sum_{j=0}^{i} P_j$; $\Phi_0 = 0$; $\Phi_N = 1$; P_j is probability (relative frequency) of falling for thickness value within the specified range; N is accepted number of ranges for ice cover thickness.

– to simulate the second stream of random numbers ξ_2 distributed uniformly in the interval (0, 1) using pseudorandom number generator;

– to calculate the random values of ice cover thickness using inverse transformation algorithm $H_{ice} = H_{1i} + (H_{2i} - H_{1i}) \cdot \xi_2$, where H_{1i} and H_{2i} are lower and upper limits of specified thickness range respectively.

By determination of certain ice cover characteristics (e.g. its temperature) it could be taken into account the existence of snow on ice. The total amount of snow on ice is commonly estimated by snow cover thickness. This quantity is notable for wide scatter of values depending on geographical and seasonal factors and, as a rule, shall be determined during the performance of direct measurements in natural conditions. The applicable ice survey data are used to plot the partial distributions bar chart for snow cover thickness. In this study the total number of thickness ranges and the numerical values of their lower and upper limits are chosen arbitrarily based on actual distribution of snow on ice in Arctic region. In respect of partial distributions bar chart for snow cover thickness the introduction of seven thickness ranges is found quite sufficient: bare ice (0.00 m), 0.01÷0.05 m, 0.05÷0.10 m, 0.10÷0.20 m, 0.20÷0.30 m, 0.30÷0.50 m and 0.50÷1.50 m. No correlations have been ascertained between ice cover thickness and snow cover thickness, therefore the separate (independent) formation of partial distributions bar charts for two concerned parameters of ice conditions does not contradict the essential principles of simulation method. The random values of snow cover thickness are obtained from the partial distributions bar chart plotted for specified ship navigation route and specified seasonal period as in the case of ice cover thickness by use of foregoing algorithm.

Figures 2 to 5 give the examples of partial distributions bar charts for ice cover thickness and for snow cover thickness plotted for two seasonal periods (February and April) and two ship navigation routes: Kolguev Island – Kara Gate Strait – Dickson Island (dashed lines) and Dickson Island – Vilkitsky Strait – Khatanga River Mouth (solid lines).

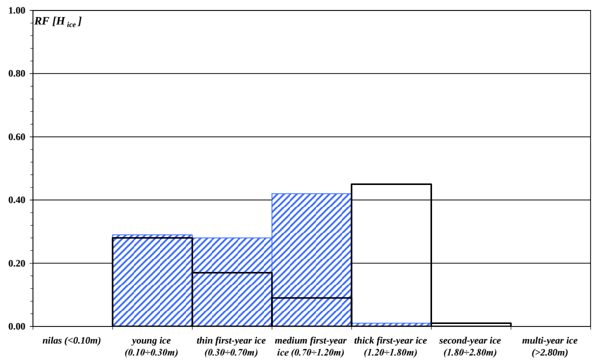


Figure 2. Partial distributions bar chart for ice cover thickness in February.

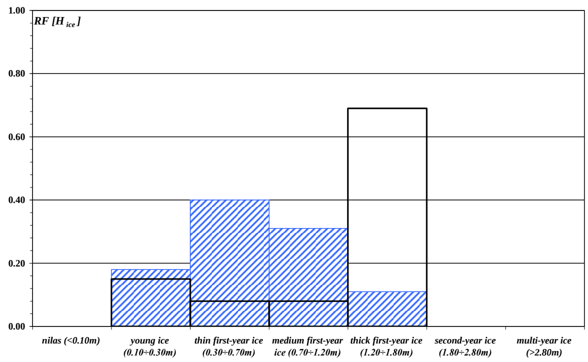


Figure 3. Partial distributions bar chart for ice cover thickness in April.

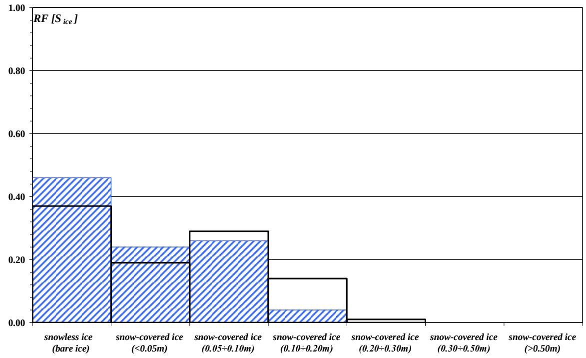


Figure 4. Partial distributions bar chart for snow cover thickness in February.

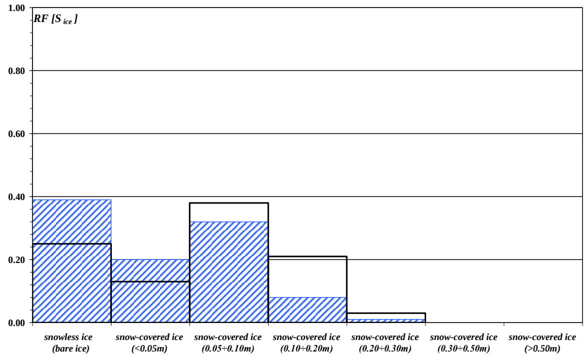


Figure 5. Partial distributions bar chart for snow cover thickness in April.

Simulation of hydrometeorological conditions parameters

In this study it is suggested to describe the hydrometeorological conditions at the specified ship navigation route in the specified seasonal period by the following parameters: ambient air temperature T_{air}, wind speed V_{wind} and sea water salinity S_{water}. For these parameters it should be input as initial data the appropriate ranges of values variation (e.g. -8.0…-4.0 °C or 3.0…7.0 m/sec or 24.0…28.0 ‰). Parameters T_{air}, V_{wind} and S_{water} are assumed to consider as stochastic quantities distributed uniformly in the specified ranges. The random values of concerned parameters are simulated using pseudorandom number generator and inverse transformation algorithm.

Simulation of physical characteristics for ice cover

It is known that in natural conditions the temperature of ice cover could vary over its thickness to a considerable degree, and the temperature of lower ice surface is found constant and close to the sea water freezing point (–1.8°C…–1.4°C depending on sea water salinity), while the temperature of upper ice surface follows approximately the ambient air temperature. The lower is the ice cover temperature the higher is the ice cover strength, which finally leads to increase of ice loads acting on ship hull by its interaction with the ice. Because the ship navigation in ice is accompanied by cold ambient air temperature as a rule, the temperature of upper ice layers doesn't exceed the temperature of lower ice layers. In this connection the temperature of upper ice surface could be accepted as the rated ice cover temperature with a certain margin to the safe side. Moreover the upper ice layers are mainly locally crushed in case of inclined ship side impact against the ice edge. For calculation of rated ice cover temperature it is used the following expression (Weeks and Assur, 1967):

$$T_{ice} = T_{air} \cdot \frac{H_{ice}}{H_{red}}; \qquad (6)$$

$$H_{red} = H_{ice} + 1.43 \cdot H_{snow} + \frac{2.0}{k_{wind} \cdot \sqrt{V_{wind} + 0.3}}, \qquad (7)$$

where T_{air} is ambient air temperature; H_{ice} is ice cover thickness; H_{red} is reduced ice cover thickness taking into account additionally the snow and wind effects; H_{snow} is snow cover thickness; V_{wind} is wind speed; k_{wind} is numerical coefficient.

The sea ice, apart from freshwater ice crystals, includes a considerable amount of liquid phase represented by cells with the brine as well as air bubbles, solid hydrated salt sediments and other mixtures of various origins, and it should be noted that the proportion of these components changes continuously in time. Just the existence of cells with the brine in ice causes the salinity of ice cover, which depends on sea water salinity, its mixing intensity, ice generation rate, ice age, etc. The ice cover salinity is several times less than the salinity of sea water frozen to produce the ice, on average 2 to 10‰. As the ice cover melts the brine being heavier than the ice flows down gradually through the cracks, while the spaces between crystals are filled with the air bubbles. For this reason the sea ice which was melted at least during one summer season features a lower salinity in comparison with the young and first-year ice, and the multi-year ice is almost completely desalinated.

The salinity of first-year ice cover (with the thickness from 0.30 to 1.80 m) is most commonly determined according to the following empirical formula (Ryvlin, 1974):

$$S_{ice} = S_{water} \cdot (1-b) \cdot e^{-a\sqrt{H_{ice}}} + b \cdot S_{water}, \qquad (8)$$

where S_{water} is salinity of sea water frozen to produce the ice; H_{ice} is ice cover thickness; b is constant non-dimensional factor equal to the ratio between ice cover salinity at the end of its winter season growth cycle and average sea water salinity.

Non-dimensional parameter a allows for influence of ice growth rate on ice cover salinity and varies in the range from 3.0 (if the ice generation rate is over 4.0 cm per day) to 6.0 (if the ice generation rate is less than 0.5 cm per day). In this study parameter a is assumed to consider as stochastic quantity distributed uniformly in the specified range. The random values of concerned parameter are simulated using pseudorandom number generator and inverse transformation algorithm.

The specific content of brine in sea ice averaged per its thickness could be estimated with sufficient accuracy for practical purposes depending on ice cover temperature T_{ice} and ice cover salinity S_{ice} according to the following expression (Frankenstein and Garner, 1967):

$$\nu_b = 0.001 \cdot S_{ice} \cdot \left(\frac{49.185}{|T_{ice}|} + 0.532 \right). \qquad (9)$$

Figures 6 to 9 show the simulation results for ice cover temperature and for ice cover salinity obtained for two seasonal periods (February and April) and two ship navigation routes: Kolguev Island – Kara Gate Strait – Dickson Island (dashed lines) and Dickson Island – Vilkitsky Strait – Khatanga River Mouth (solid lines).

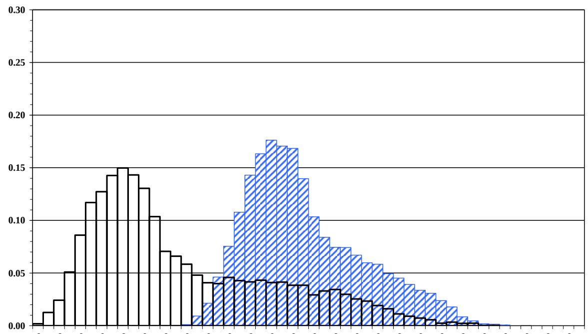


Figure 6. Ice cover temperature, °C: simulation results for February.

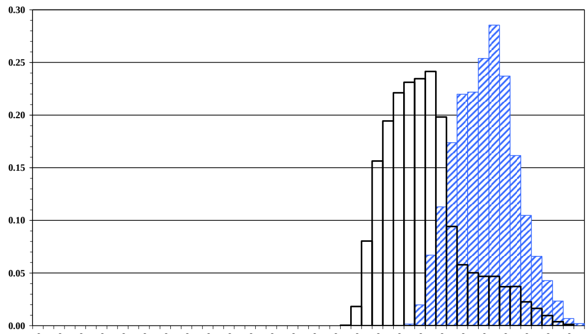
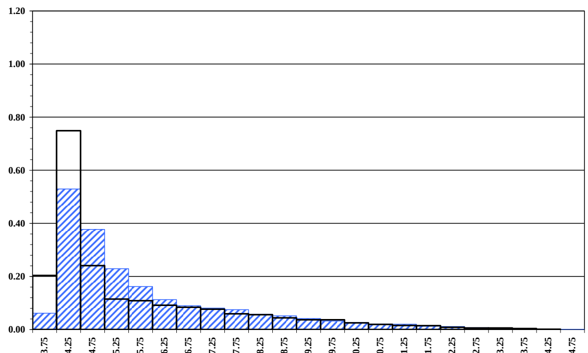


Figure 7. Ice cover temperature, °C: simulation results for April.



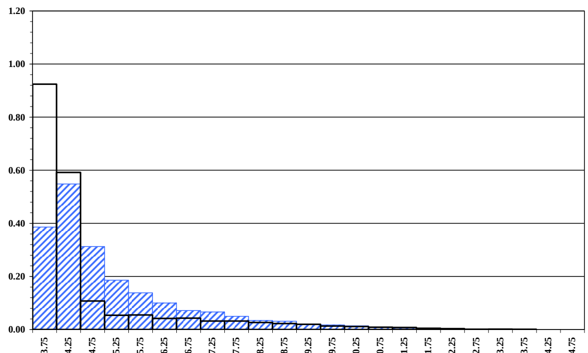


Figure 8. Ice cover salinity, ‰: simulation results for February.

Figure 9. Ice cover salinity, ‰: simulation results for April.

Simulation of strength characteristics for ice cover

In accordance with the experimental data the flexural ice strength σ_f varies within the rather wide limits in function of ice cover temperature and ice cover salinity, however its values are practically not dependent on load application time. The average values of parameter σ_f obtained during the performance of static tests for floating ice cover in natural conditions are from 0.50 to 0.70 MPa for strong winter-season saline ice and from 0.20 to 0.30 MPa for summer-season saline ice weakened by melting (Ryvlin and Kheisin, 1980). It should be admitted that at the present time the test method based on breaking of ice-cut beams and cantilevers afloat is most reliable among a whole number of experimental techniques proposed for determination of flexural ice strength. Unlike the tests on small-size ice samples the use of concerned test method ensures the gaining of practically identical results with a quite moderate values scatter for parameter σ_f by different researchers. Also the values of flexural ice strength got from tests on small-size ice samples (1.00 to 2.00 MPa) are significantly higher than the similar values observed by breaking of ice-cut beams and cantilevers afloat.

The compression ice strength σ_c is usually determined experimentally by crushing of standard-size ice pieces with cubic or close to cubic form using special test presses. The parameter σ_c is substantially dependent on velocity and direction of load application to specimen, on characteristic dimensions of specimen, its structure, temperature, salinity, orientation of crystals relative to the optical axes etc. In accordance with the experimental data the range of values variation for parameter $\sigma_{\tilde{n}}$ is from 0.50 to 1.00 MPa under static loading, while the compression ice strength increases several times under dynamic loading as compared to its static values (Popov, Faddeev, Kheisin et al., 1967). But the values of parameter $\sigma_{\tilde{n}}$ obtained during the performance of appropriate tests should be treated exclusively as certain measures of local crushing ice strength by ship hull and ice cover interaction. These values could not be used directly (without processing) to determine ice loads acting on ship hull, because the experimental pattern of ice cover failure is essentially different from the real pattern of ice edge crushing caused by ship side penetration into the ice. The main difference is that in case of tests on ice samples the uniaxial compression is observed, while the interaction process between ship hull and ice cover is accompanied by constrained deformation (biaxial compression). It should be noted that the compression strength of upper ice layers is quite low in the summer season due to melting under solar radiation, while in the winter season it proves to be considerably higher than the compression strength of ice samples under test due to constrained deformation. As a rule the constrained deformation is taken into account by introduction of appropriate proportionality factor. According to approximate estimates the constrained deformation increases the values of parameter $\sigma_{\tilde{n}}$ determined under uniaxial compression of ice samples in 2.50 to 2.70 times (Korzhavin, 1962). The compression ice strength σ_c changes within the rather wide limits owing to the effects of numerous and various factors on its magnitude: according to some sources up to 8.0÷10.0 MPa and even up to 25.0 MPa in case of triaxial compression tests (Nawwar, Nadreau and Wang, 1983).

In respect to engineering calculations it is asserted that among the multitude of factors exerting the direct influence on ice strength it is sufficient to consider only two main factors – the ice cover temperature and the ice cover salinity (under condition that the time of load application is given, i.e. static or dynamic loading). The strength of ice cover increases with reducing of its temperature and decreasing of its salinity and vice versa. The specific content of brine in sea ice ν_b is usually accepted as united parameter taking into account the simultaneous influence of ice temperature and ice salinity.

In this study the empirical expressions deduced on experimental data are used as an analytical basis for determination of ice cover strength characteristics σ_f and σ_c. These expressions setting the dependences between concerned strength characteristics and parameter ν_b have the same simplified construction and are obtained as a result of processing and generalization for data of ice tests in natural conditions done by Weeks and Anderson (for flexural ice strength) and Peyton (for compression ice strength) by means of least-squares method:

$$\sigma_f = \sigma_{f0} \cdot \left(1 - \frac{\sqrt{\nu_b}}{0.45}\right); \qquad (10)$$

$$\sigma_c = \sigma_{c0} \cdot \left(1 - \frac{\sqrt{\nu_b}}{0.55}\right), \qquad (11)$$

where $\sigma_{f0} = 0.75$ MPa and $\sigma_{c0} = 1.65$ MPa.

In case of increased values of parameter ν_b the values of flexural and compression ice strength are considered constant and equal to 0.20 MPa.

In addition it is suggested to adjust the rated values of ice cover strength characteristics σ_f and σ_c by introduction of appropriate allowances which allow for existing scatter (i.e. for deviation from formula-based values) of parameters values determined in the course of tests performance:

$$\sigma_f' = \sigma_f \pm \Delta\sigma_f; \qquad (12)$$

$$\sigma_c' = \sigma_c \pm \Delta\sigma_c. \qquad (13)$$

It is ascertained that the allowances values don't exceed 0.10÷0.15 MPa. In this study the allowances $\Delta\sigma_f$ and $\Delta\sigma_c$ are assumed to consider as stochastic quantities distributed uniformly in the specified ranges (from $-0.10 \cdot \sigma_{f0}$ MPa to $+0.10 \cdot \sigma_{f0}$ MPa for $\Delta\sigma_f$ and from $-0.10 \cdot \sigma_{c0}$ MPa to $+0.10 \cdot \sigma_{c0}$ MPa for $\Delta\sigma_c$). The random values of concerned allowances are simulated using pseudorandom number generator and inverse transformation algorithm.

But the values of compression ice strength calculated according to above-cited empirical expression based on experimental data could not be used for estimation of ice cover strength during the ship side penetration into it and thereafter for determination of ice loads acting on ship hull owing to constrained deformation occurring by real interaction process between ship hull and ice cover. As stated above the constrained deformation effect could be approximately taken into account by introduction of appropriate proportionality factor k_1, which leads to more than twofold increase for actual values of compression ice strength:

$$\sigma_c'' = k_1 \cdot \sigma_c'. \qquad (14)$$

In this study factor k_1 is assumed to consider as stochastic quantity distributed uniformly in the specified range (from 2.50 to 2.70). The random values of concerned factor are simulated using pseudorandom number generator and inverse transformation algorithm.

Another way to obtain the rated values of parameter σ_c with the account of constrained deformation consists in the use of linear dependence connecting directly the values of flexural and compression ice strength:

$$\sigma_c'' = k_2 \cdot \sigma_f. \qquad (15)$$

According to numerical estimates done by various researchers the proportionality factor k_2 could possess any values equal from 2.0 to 6.0. By analogy with the factor k_1 the concerned factor could be presented as stochastic quantity distributed uniformly in the specified range. But in view of considerable scatter of values for factor k_2 the use of proposed dependence between flexural and compression ice strength is less preferential in comparison with the above-cited approach.

SIMULATION RESULTS FOR ICE COVER STRENGTH CHARACTERISTICS

According to suggested stochastic simulation technique there were simulated in this study the strength characteristics of ice cover used subsequently for determination of rated ice loads on ship hull within the framework of accepted physical model for ship and ice interaction – flexural ice strength σ_f, compression ice strength σ_c and dynamic ice strength by crushing a_p. In test examples the simulation of concerned characteristics was performed for two lines of Northern Sea Route following one after another: Kolguev Island (Barents Sea) – Kara Gate Strait – Dickson Island (Kara Sea) and Dickson Island (Kara Sea) – Vilkitsky Strait – Khatanga River Mouth (Laptev Sea). February and April were selected as seasonal periods for ship navigation.

The random values of ice cover strength characteristics were obtained during the multiple implementation of probabilistic computer experiment: each specified simulation variant provided for ten thousands realizations. The special-purpose software developed by authors was used directly for experimentation (Dudal, Besse, Yakimov et al., 2011). The output empirical distributions for concerned stochastic quantities represented in the form of appropriate bar charts and accumulation curves were approximated by continuous two-parameter Weibull distribution having the following air of probability density for nonnegative random values:

$$f(x) = \alpha \cdot \beta^{-\alpha} \cdot x^{\alpha-1} \cdot e^{-(x/\beta)^{\alpha}}, \qquad (16)$$

where $\alpha > 0$ is shape factor and $\beta > 0$ is scale factor.

Parameters α and β for Weibull distribution were estimated on sample data by maximum likelihood method with the use of iterative algorithm (Thoman, Bain and Antle, 1969). The formal statistical checking of null hypothesis in relation to accepted theoretical distribution was done on the basis of modified Kolmogorov-Smirnov fitting criterion (Chandra, Singpurwalla and Stephens, 1981).

Figures 10 to 15 present the simulation results for flexural ice strength, compression ice strength and dynamic ice strength by crushing obtained for two seasonal periods (February and April) and two ship navigation routes: Kolguev Island – Kara Gate Strait – Dickson Island (dashed lines) and Dickson Island – Vilkitsky Strait – Khatanga River Mouth (solid lines). The found statistical estimates of parameters α and β for Weibull distribution as well as the values of mathematical expectation μ, standard deviation σ and variation coefficient cv for each specified simulation variant are put in Table 1.

Table 1. Summary statistics.

Strength characteristics of ice cover	Parameter	February		April	
		Route №1	Route №2	Route №1	Route №2
Flexural ice strength	α	8.076	9.024	5.510	7.324
	β	0.527	0.563	0.430	0.500
	μ, MPa	0.497	0.533	0.397	0.469
	σ, MPa	0.073	0.071	0.083	0.076
	cv, %	14.7	13.3	21.0	16.1
Compression ice strength	α	9.517	10.080	6.667	8.719

	β	3.271	3.439	2.801	3.139
	μ, MPa	3.105	3.273	2.613	2.968
	σ, MPa	0.391	0.391	0.459	0.406
	cv, %	12.6	11.9	17.6	13.7
Dynamic ice strength by crushing	α	15.23	16.13	10.67	13.95
	β	371.87	383.67	337.52	362.41
	μ^*	359.27	371.32	321.95	349.14
	σ^*	28.96	28.32	36.44	30.61
	cv, %	8.1	7.6	11.3	8.8
* – $mug^{5/6} \cdot m^{-35/24} \cdot sec^{-35/24}$					

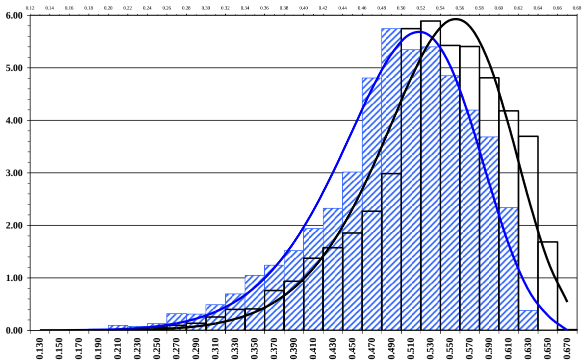


Figure 10. Flexural ice strength, MPa: simulation results for February.

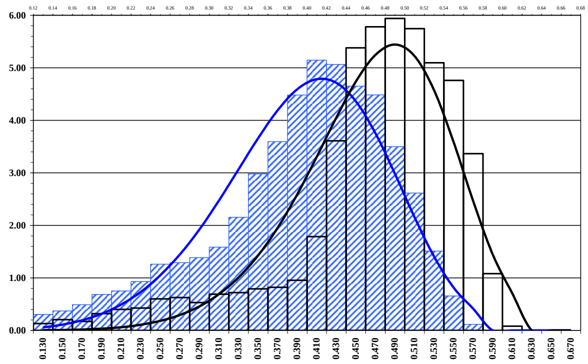


Figure 11. Flexural ice strength, MPa: simulation results for April.

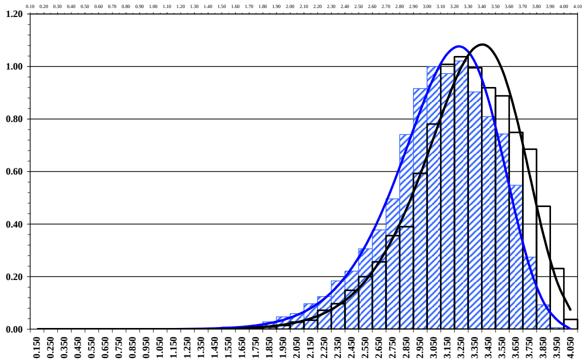


Figure 12. Compression ice strength, MPa: simulation results for February.

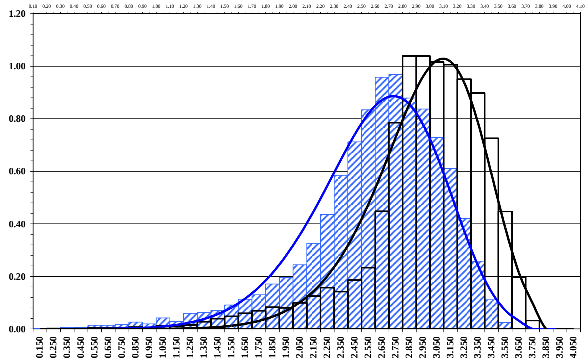


Figure 13. Compression ice strength, MPa: simulation results for April.

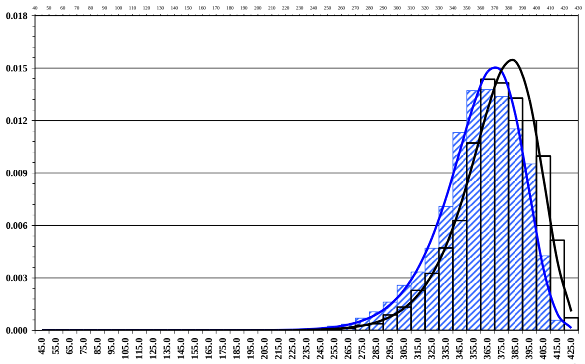


Figure 14. Dynamic ice strength by crushing*: simulation results for February.

* – $mug^{5/6} \cdot m^{-35/24} \cdot sec^{-35/24}$

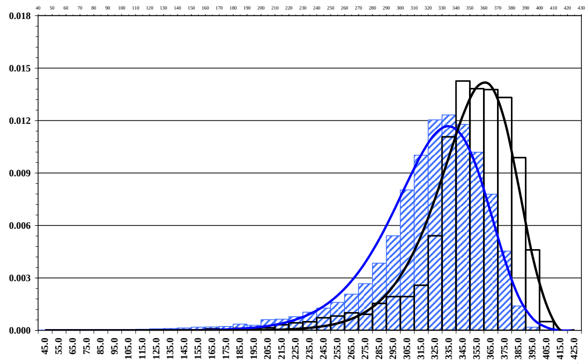


Figure 15. Dynamic ice strength by crushing*: simulation results for April.

* – $mug^{5/6} \cdot m^{-35/24} \cdot sec^{-35/24}$

CONCLUSION

In this study it was shown that the simulation of ice cover strength characteristics is an essential part of the whole process of stochastic ice loads simulation. There were suggested the practical simulation techniques:

– for stochastic ice and hydrometeorological conditions parameters using input bar charts of partial distributions plotted on data of ice survey and environment monitoring;

– for stochastic physical and mechanical characteristics of ice cover using simplified empirical expressions deduced on data of ice tests in natural conditions.

Both techniques are based on well-known statistical tests method (Monte-Carlo method) and provide for application of pseudorandom number generator and inverse transformation algorithm. The comparative analysis between obtained simulation results and existing experimental data for specified ship navigation routes and specified seasonal periods confirms in sufficient measure the adequacy and availability of suggested approaches. The main content of this study could be used by solution of various problems in probabilistic definition connected with the dynamic ship hull and ice cover interaction.

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