POAC'13
Espoo, Finland

Proceedings of the 22nd International Conference on
Port and Ocean Engineering under Arctic Conditions
June 9-13, 2013
Espoo, Finland

SYNCHRONISATION AND THE TRANSITION FROM INTERMITTENT TO LOCKED-IN ICE-INDUCED VIBRATION

Andrew Palmer[1] and Morten Bjerkås[2]

1 Department of Civil and Environmental Engineering, National University of Singapore, SINGAPORE.
2 Reinertsen AS, Trondheim, NORWAY

ABSTRACT

Previous research has shown that the force between ice and a structure is very far from uniformly distributed in space and highly variable in time. A detailed reexamination of field measurements on nine load panels on the Nörstromsgrund lighthouse in the northern Baltic displays the same non-uniformity. Before ice-induced vibration begins, the forces on individual panels are random and uncorrelated. The loads on individual panels then progressively synchronise in frequency and phase, and as they do so vibrations start and build up. A Kuramoto plot quantifies the synchronisation.

This is an instance of the entrainment phenomenon well known in other areas of physics and biology. It can be simulated by a simple mechanical model. The results throw light on the conditions under which ice-induced vibrations start, and on when they will stop. They can be compared with other instances of non-uniform loading from ice, such as those made by Sodhi with tactile sensors.

INTRODUCTION

Ice moving past a fixed structure can set the structure into oscillation. At a low velocity, the ice force increases slowly, and the structure moves with the ice and in the same direction, until the ice breaks, the structure rebounds and oscillates for several cycles, and then the sequence repeats. At a larger velocity, on the other hand, the ice fracture sequence 'locks in' to the structure oscillation, and the structure oscillates in nearly a steady state. Those oscillations are well known, have been observed in many parts of the Arctic, and have led to serious damage to structures in the Bohai Bay, the Gulf of Bothnia and the Beaufort Sea (Engelbrektson, 1977; Björk, 1981; Määttänen, 1978;Yue et al., 2000; Xu et al., 2006; Palmer and Croasdale, 2012, and many others).

An earlier paper (Palmer et al., 2010) applied dimensional analysis to 11 series of observations on widely different structures, and concluded that the transition to locked-in oscillations depends on a dimensionless group composed of the ice velocity, the ice thickness and the lowest natural frequency of the structure. In that paper, and in other papers about ice pressure/area relationships, it was argued that the contact breadth *D* would have to be less important than the ice thickness *t*, at least for large aspect ratios *D/t*, because the ice at one end of the contact could not be “aware” of what was happening at the other end of the contact.

That argument we now think to be incomplete, because of the phenomena of entrainment and synchronisation. Pikovsky et al. (2001) recount a famous example of entrainment. The astronomer and physicist Christian Huygens (1629-1695) was ill. He watched two pendulum clocks on his bedroom wall, and he observed that the pendulums of the two clocks moved in perfect synchrony, so that they kept identical time. However, when one clock was moved to another wall, the two clocks fell out of step, and a day later the times they showed were 5 s apart. Moved back to the first wall, the clock resynchronised within half an hour. The wall was almost certainly brick, and the mechanical coupling between the clocks hanging on the same wall must have been extremely weak, but it was nevertheless enough to synchronise the oscillations. He then carried out a test with two clocks suspended from a wooden beam, and found the same result. In his words:

> “…For a long time I was amazed at this unexpected result, but after a careful examination finally found that the cause of this is due to the motion of the beam, even though this hardly perceptible. The cause is that the oscillations of the pendula, in proportion to their weight, communicate some motion to the clocks. This motion, impressed onto the beam, necessarily has the effect of making the pendula come to a state of exactly contrary swings if it happened that they moved otherwise at first….”

Pikovsky cites Huygens’ writing at length: it is incidentally a model of scientific writing. There are many similar instances of synchronisation from biology, physiology, engineering and physics. The excitation by pedestrians of oscillations in the Millennium Bridge in London is one spectacular example: see also Strogatz (2003) and Kautz (2011).

METHOD

Initially we assume time domain harmonic oscillators on the form:

$$u_i(t) = A_i \sin(\omega_i t + \phi_i(t)) \qquad \text{for i = 1 to } N \qquad (1)$$

where A_i is the amplitude, ω_i the frequency and φ_i the phase lag of the *i*-th oscillator, *t* is time given a constant omega for all signals, we can illustrate the signals in the phase plane with polar coordinates. The vector length is then the amplitude A_i and the phase lag is the angle from a defined reference. From a collection of N = 10 signals, Fig. 1 shows the signals both in time domain and in the phase plane.

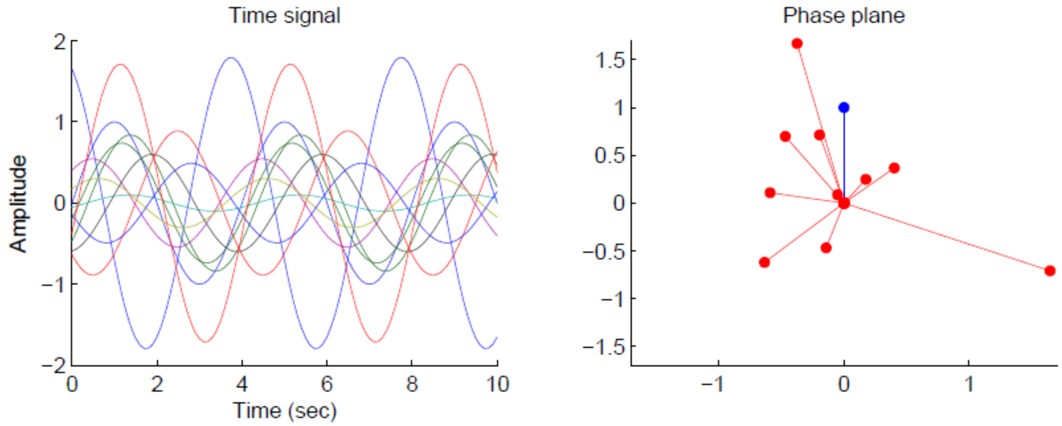

Figure 1. Ten harmonic time series with the same frequency but random phases and amplitudes.

We can also illustrate the behavior when the signals are synchronised in frequency and phase but still with a random amplitude. Fig. 2 shows ten signals where the phase difference between the signals is close to zero.

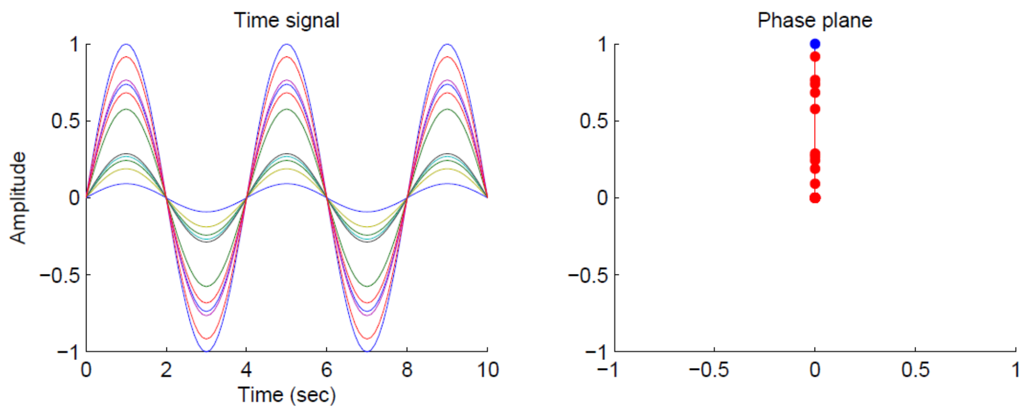

Figure 2. Ten time series with random amplitude and similarity in frequency and phase angle.

In the case of ice induced vibrations, we often go from a situation with N load signals which are out of phase to a locked-in situation where all the signals oscillate with a marginally small phase angle difference. This is in general a well known situation in physics. Kuramoto (1984) formulates a measure for the coupling between oscillators (Eq. 2), and puts forward a coupling parameter K that works on the time development of the phase angle as follows:

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i) \qquad (2)$$

For different phenomena, it is shown that a certain magnitude of *K* has to be present to obtain synchronisation between oscillators.

SYNCHRONISATION OF ICE LOAD PANEL FORCES ON THE NORSTRÖMSGRUND LIGHTHOUSE

The Norströmsgrund lighthouse is located in the Gulf of Bothnia in the northern Baltic, some 75 km south of Luleå in Sweden. The water depth is 14 m; the tidal range is small. As part of the LOLEIF and STRICE EU projects, the lighthouse was fitted with nine load panels, covering 162 degrees of the circumference, and measurements were made in five winters 1999 to 2003. There were two biaxial accelerometers 16.5 and 37.0 m above the seabed, and two biaxial inclinometers at 22 and 37.0 m. Ice thicknesses were measured by an electromagnetic device. The details are described in Bjerkås (2006) and Bjerkås et al. (2013).

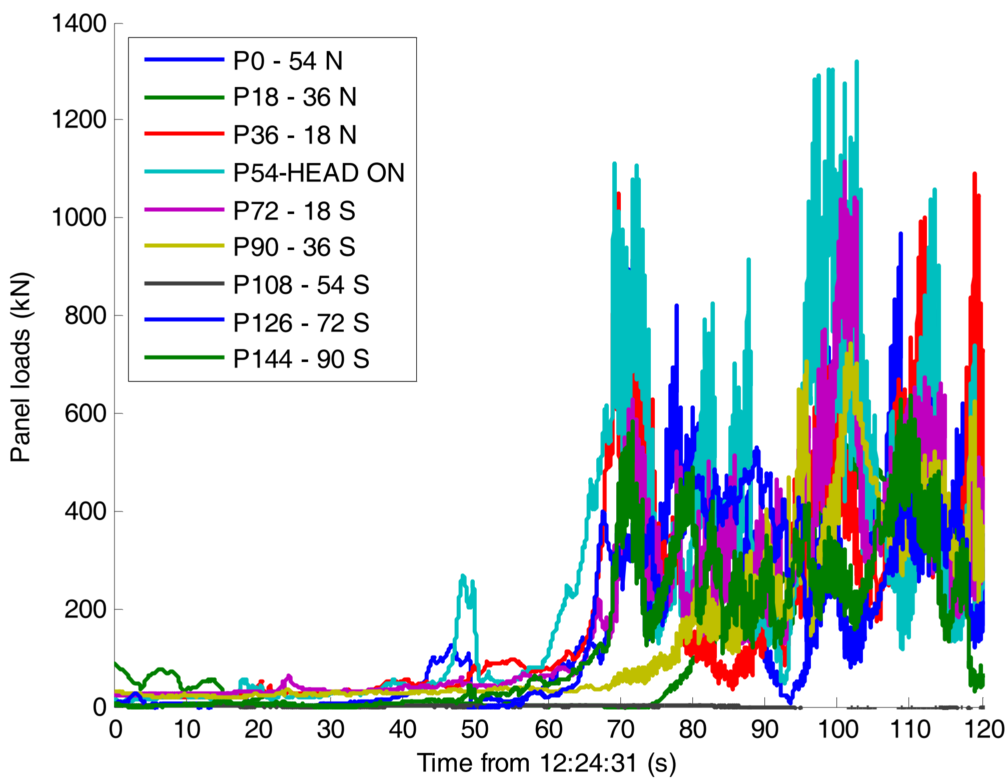

Figure 3 Panel forces measured at Norströmsgrund

Key: P0-54N is panel 1, P18-36N is panel 2, P36-18N is panel 3, P54-HEAD ON is panel 4,P72-18S is panel 5, P90-36 S is panel 6, P108-54S is panel 7, P126-72S is panel 8, P144-90S is panel 9

Fig. 3 redraws parts of the data presented by Bjerkås et al. (2013). It plots data from the nine load panels and shows the onset of steady-state vibrations after 12:24:31 on 30 March 2003. The diagram plots the forces from nine load panels against time: the inset diagram shows the panel positions. In what follows all times are counted from 12:24:31, so that 102.57 s refers to 12:26:13.57. The sampling rate was 87 Hz, so the interval between panel load measurements was 0.011494 s. The lowest natural frequency of the lighthouse structure is estimated to be 2.8 Hz (Björk, 1981), corresponding to a natural period of 0.36 s. When the lighthouse is surrounded by ice, the frequency must to some extent be influenced by the added mass of the ice and by the heavy damping its presence adds.

It can be seen that the magnitudes of the panel forces were all low until about 60 s, but that after that most of the panel forces markedly increased. They did not increase together, and the panel 2 and panel 7 forces remained small. The 16.5 m accelerometer measured almost no acceleration until 67 s (12:25:38), but the acceleration then increased, at first slowly and then much more rapidly, reaching maxima of 2 m/s^2 at 81 s.

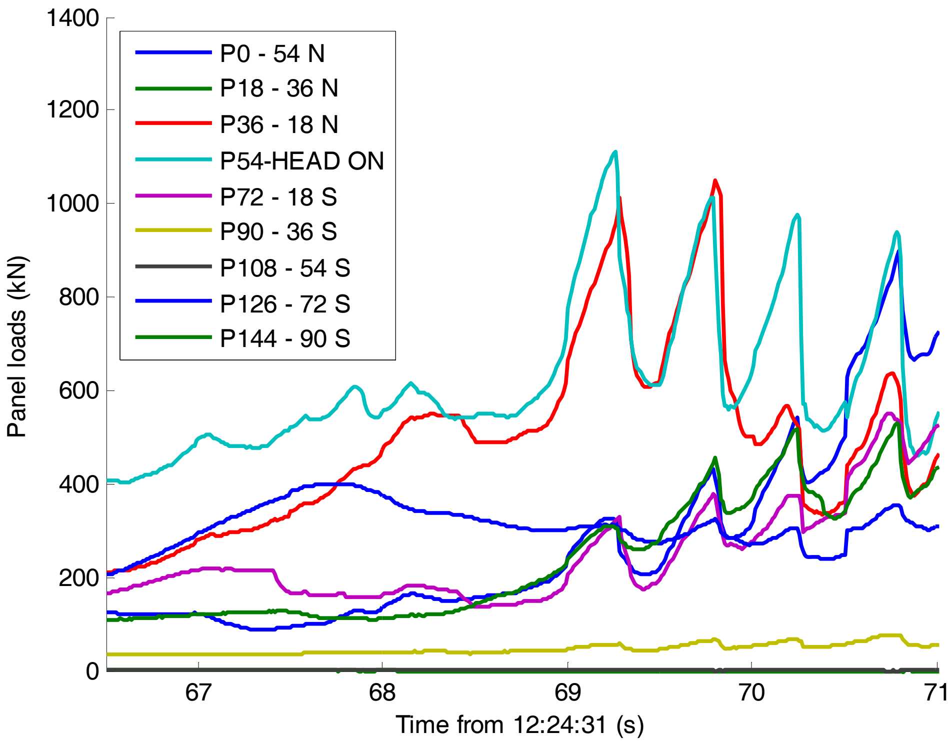

Figure 4 Expanded section of Figure 3

Figure 4 redraws the data in the period 65 to 69 s to a much larger time scale. At the start of that interval, around 65.5 s, the panel 4 force is just beginning to oscillate, though not with a steady frequency, and the remaining eight panel forces are varying slowly but not oscillating. Things change rapidly after 67.1 s. First panel 4, and then shortly afterwards panel 3 begin to show the characteristic asymmetric sawtooth force history, in which the force increases relatively slow and then rapidly falls, and the period of the oscillation is the natural period. Panels 5 9 and 1 follow slightly later, initially with much smaller forces, and panel 8 later still. The cyclic forces on panel 4 and panel 3 diminish slightly, and the forces on panels 5 6 8 and 9 increase until they are only a little less than the forces on panels 4 and 3. After 67.4 s, panel 6 shows the same pattern to a limited extent.

It appears from Figure 4 that the forces on the different panels are not initially synchronized in phase, but that they later become synchronized. Researchers into synchronisation have devoted much effort to quantitative measures of the phenomenon, and they define the phase of an arbitrary signal using the analytic signal concept and a Hilbert transform (Gabor, 1946; Pikovsky, 2001). It unambiguously gives the instantaneous phase and amplitude, though Pikovsky points out that there is a clear physical meaning only if the signal is narrow-band. Those researchers are fortunate in that they almost always have to deal with weak coupling but long time intervals, with oscillations that persist through many hundreds of cycles. Here, however, we have synchronisation in a very few cycles. It seems to us better to apply to the panels on the Norströmsgrund lighthouse the crude but easily understood Kuramoto method described earlier. A simple way of identify the relative phases is to pick out the time at which each of the nine force signals reaches a maximum, to convert the time differences into phase differences by using the natural frequency, and to assign the length vector to the force when the force is a maximum. This is done in Figure 5. All the diagrams are drawn at the same length scale. Panel 1 is arbitrarily chosen as the reference phase 0, and so the vector that corresponds to panel 1 is plotted vertically upwards: any other choice would give the same diagram rotated.

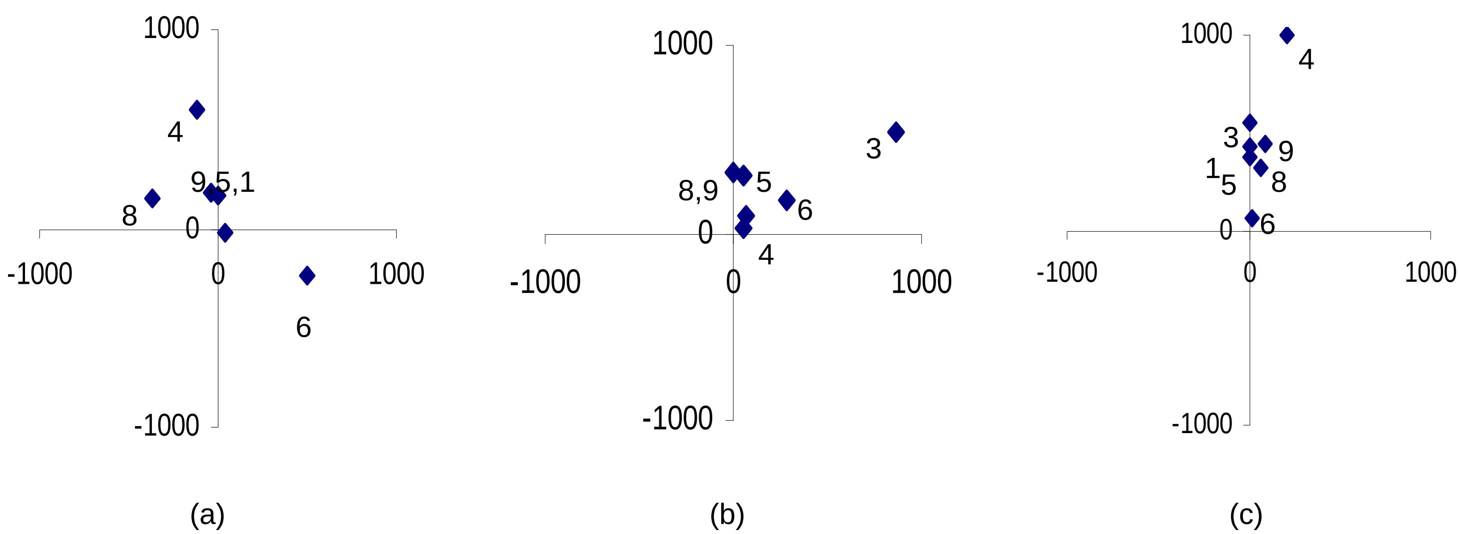

Figure 5: Kuramoto circle diagrams from Norströmsgrund

The circle diagram on the left of Figure 5 represents conditions when the time is about 66.3 s: recall that the maxima are at different times. It can be seen that the vectors point in completely different directions, and that that fits in with the observation in Figure 4 that at that time the different panels do not correlate. The forces on panels 2 and 7 remain small throughout, and are not plotted. The diagram in the middle represents conditions when the time is about 67.3 s. The vectors then occupy a smaller arc, the maximum phase difference is about 60º, and the panel forces are beginning to fall into step. The diagram on the right represents conditions when the time is about 67.9 s. Now the vectors occupy a very much smaller arc, the maximum phase difference is 11.6º , limited by the time resolution of the recording system, because the time interval between one measurement cycle and the next corresponds to 11.6º . The forces on the seven panels that have significant loads are now synchronised.

Is this what we ought to expect?

When the force of the ice pushing against one of the load panels reaches a critical level, that ice fractures, and the force on that panel drops precipitously. The reduction of the force on that panel immediately reduces the total force between the ice and the structure. The structure responds to that reduction in ice force, and its elasticity drives the structure back towards the ice. The forces on at least some of the other panels increase. The ice in contact with those panels in turn becomes more likely to fracture. The effect is to advance the fracture at the other panels, so that those fractures occur earlier than they would have occurred if the first fracture had not happened. The fractures at different panels tend to pull forward in time and to become synchronised. That is what we observe at Norströmsgrund.

MODEL

Fig. 6(a) illustrates a simple and heavily idealised model. The ice is on the left and moves to the right, and is treated as undeformable except at its contacts with nine load cells. The structure is connected to the foundation by an elastic spring, which represents the stiffness of the structure itself and the foundation it is built on, and a linear dashpot, which represent various kinds of damping within the system, and generates a force proportional to the structure velocity and opposing the movement of the structure. The natural frequency is determined by the mass of the structure and the stiffness of that spring. The load cells are connected to the structure by springs, but those springs have a high stiffness, so that the frequency of oscillation of a cell relative to the structure is very high, and the relative displacements are negligible.

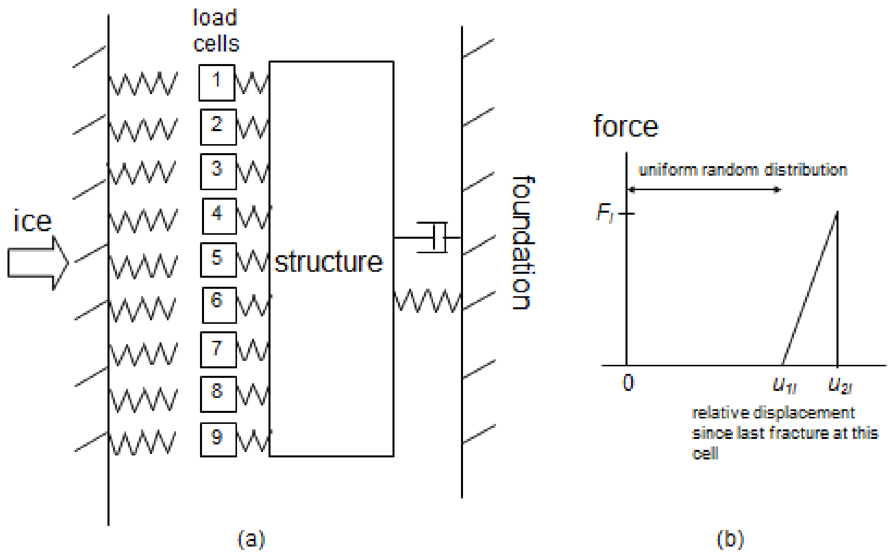

Figure 6. Idealised model

The springs drawn attached to the ice simulate the mechanics of the contact between the ice and each individual load cell. Figure 6(b) is the relationship between the ice force and the relative displacement between the ice and the load cell i. Immediately after a fracture at cell i, the force at that cell is zero. It remains zero until the displacement of the ice relative to the structure reaches u_{1i}. It rises linearly with relative displacement until the relative displacement reaches u_{2i}, and then drops suddenly to zero. The maximum force on that cell is then F_i. The process then repeats. The parameter u_{li} is not fixed between different fractures at the same cell, but instead is reset after each fracture, as a random variable with a uniform distribution between 0 and u_{l1}. The relative displacement u_{l2}-u_{l1}, is the displacement between the instant at which a cell begins to pick up load and the instant at which fracture occurs at that cell. In the present implementation, u_{l2}-u_{l1} is the same for all cells and all fractures, and F_i is fixed and the same for all cells, but they too could be made random variables.

Fig. 7 plots the nine cell forces against time for a typical run. At the start, the cell forces are relatively small and are not synchronised, because the structure is initially stationary. Later the cell forces synchronise in the way described earlier, although synchronisation is not perfect because of the random variables built into the model.

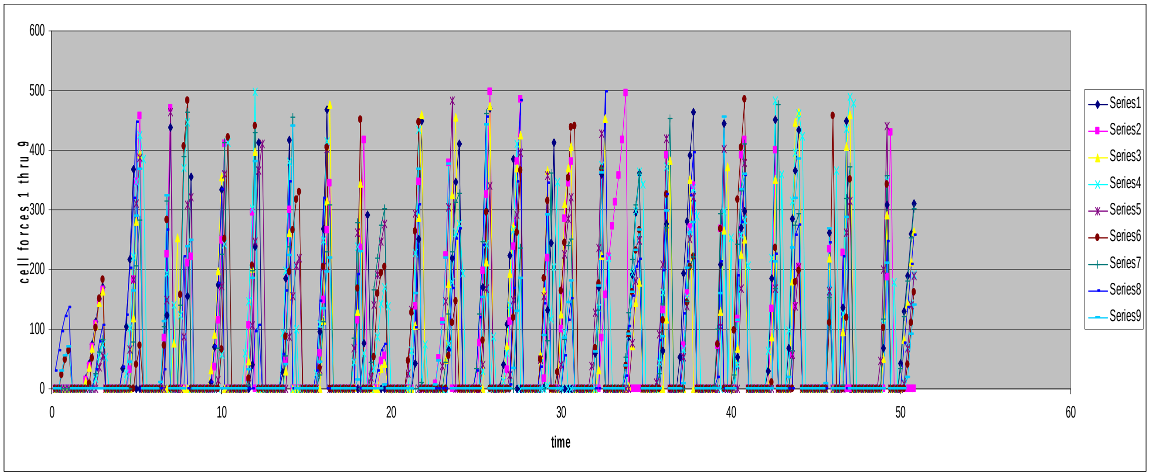

Figure 7: Typical results from simple model in Figure 6

The model has two independent natural frequencies. The first is the natural frequency of oscillation of the structure on the foundation. The other corresponds to the average time interval between a fracture at a cell and the next fracture at the same cell, which is the average displacement interval (u_{l2}-(1/2)u_{l1}) divided by the ice velocity. In the Figure 7 results, the second frequency is much higher that the first, which is why the interval between the synchronised cell forces corresponds to the second natural frequency.

There has to be some damping, represented by the dashpot component in Figure 6(a), because without it the amplitude of the structure oscillation increases indefinitely.

Figure 8 plots some results from the simple model as Kuramoto circle diagrams. The natural period of oscillations of the structure on the foundation is 4.4 s. In Figure 8, (a) represents conditions after about 7 s, less than two natural periods from the start. The load maxima are beginning to synchronise, but the phases are still distributed over more than a quarter period, and the ratio between the largest and smallest force amplitude is about 2. Figure 8(b) represents conditions much later, after about 49 s, more than 11 natural periods. The oscillations have almost completely synchronised in phase, but the instantaneous amplitudes vary over about the same range.

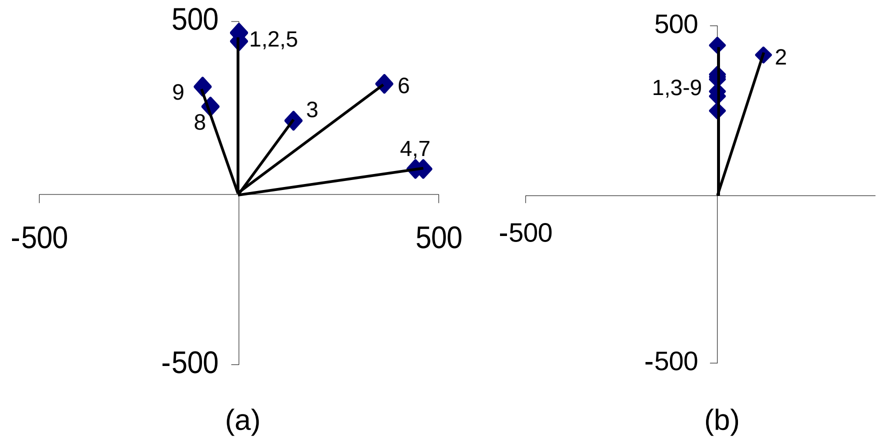

Figure 8: Kuramoto circle diagrams for simple model
Numbers indicate cells in the model in Figure 6

DISCUSSION

The idealised model generates results that display the synchronisation observed at Norströmsgrund, and encourage confidence in the simple explanation. The system is sychronised by the structure, which has the effect of increasing the load on other cells when a fracture occurs at one cell. It plays the same role as the wall in Huygens' observations.

The model is obviously severely idealised. The choice of nine load cells was prompted by the nine load panels at Norströmsgrund, but is essentially arbitrary. The real system has distributed flexibility and distributed mass, in the ice, the structure and the foundation, and its damping is far from linear.

Synchronisation of this kind can be expected to occur at different length scales, and perhaps also over different time scales. If one of the nine load panels at Norströmsgrund were divided into nine smaller panels, if each of the smaller panels were again subdivided, and so on, we might expect to see signs of synchronisation at each scale, an instance of the fractal behaviour of ice that has been suggested in other contexts (Palmer, 1991, 2004). The whole question of scales in ice/structure interaction is one that needs further understanding, but lies beyond the scope of the present paper. The results also suggest that it is a mistake to focus exclusively on the total ice force on a structure: the way in which total force is distributed and the way its components change with time are perhaps more illuminating.

One question that remains open is what has to happen before the oscillations fall out of synchronisation and perhaps stop altogether. The same challenging issue arises with vortex-excited oscillations in water, where there is hysteretic behaviour and once oscillations have begun they persist until after the velocity drops substantially (see, for example, Figure 14-7 in Palmer and King (2008)). In a damped system, there has to be continued excitation for the oscillations to continue, and if the ice velocity drops the synchronization may break down. The broken ice moving past the structure can be expected to be an effective source of damping. This issue requires further research.

ACKNOWLEDGEMENT

The authors thank Reinertsen AS for permission to publish this paper, and thank Hagbart Alsos, Ken Croasdale, John Dempsey, Andreas Meese, Michael Perry, Dev Sodhi, Cynthia Wang, Xu Ning, Yap Kim Thow and Yue Qianjin for helpful discussions.

REFERENCES

Bjerkås, M. Ice actions on offshore structures. Doctoral thesis, Norwegian University of Science and Technology, Trondheim (2006).

Bjerkås, M. and Skiple, A. Occurrence of intermittent and continuous crushing during ice-structure interaction. Proceedings, Eighteenth International Conference on Port and Ocean Engineering under Arctic Conditions, Potsdam NY, 3 (2005).

Bjerkås, M. , Meese, A. and Alsos, H. Ice induced vibrations – observations to a full-scale ice lock-in event. Proceedings, Twenty-Second International Conference on Port and Ocean Engineering under Arctic Conditions, Espoo, (2013).

Björk, B. Ice induced vibrations of fixed offshore structures: part II; experiences with Baltic lighthouses. Marine Structures and ships in ice (ed. Carstens, T.), report 81-062 (1981).

Engelbrektson, A. Dynamic ice loads on a lighthouse structure. Proceedings, Fourth International Conference on Port and Ocean Engineering under Arctic Conditions, St. Johns, (1977).

Gabor, D. Theory of communication. Journal of the Institution of Electrical and Electronic Engineers, London, 93, 429-457 (1946).

Kautz, R. Chaos: the science of predictable random motion. Cambridge University Press. Cambridge, UK (2011).

Kuramoto, Y. Chemical oscillations, waves and turbulence. Springer-Verlag, Berlin (1984).

Palmer, A.C. Fracture mechanics models of ice structure interaction. Ice-structure interaction, Proceedings, IUTAM/IAHR Symposium on Ice-Structure Interaction, St John's, Newfoundland, 1989, 93-107, Springer-Verlag, Berlin (1991).

Palmer, A.C. Fractality of crushed brittle materials: geometry or fracture mechanics? Proceedings, Exploratory Workshop on fractals and geotechnical engineering, Innsbruck, 37-45, Logos Verlag (2004).

Palmer, A.C. and King, R.A. Subsea pipeline engineering. Pennwell, Tulsa, OK (2008).

Palmer, A.C., Yue, Q and Guo, F. Ice-induced vibrations and scaling. Cold Regions Science and Technology, 60, 189-192 (2010).

Palmer, A.C. and Croasdale, K.R. Arctic offshore engineering. World Scientific, Singapore (2012).

Pikovsky, A, Rosenblum, M. and Kurths, J. Synchronisation: a universal concept in nonlinear sciences. Cambridge University Press, Cambridge, UK (2001).
Strogatz, S. Sync. Hyperion, New York, NY (2003).
Xu, N., Yue, Q.,Shi, Z.,Guo,F. and Qu, Y. Ice load and structure vibration when ice acts on the up-down cone. Proceedings, Nineteenth International Conference on Port and Ocean Engineering under Arctic Conditions, Dalian, 1, 328-336 (2007).