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UNIFYING LOCAL AND GLOBAL ICE CRUSHING PRESSURES

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ABSTRACT
In the recent ISO 19906 standard on Artic Structures there are equations for design ice pressures for local loading and global loading for wide contact zones where crushing is the mode of ice failure at both the local and global scale. The two sets of equations do not match and there is inconsistency at contact areas in the 4.5 to 40m^2 range and regions where neither applies. In this paper we present a method to unify and extend the global and local pressure trends based on the observed geometry of so-called High-Pressure-Zones which can carry the majority of the load. Also reviewed are the data used to generate the ISO design pressures.

INTRODUCTION
There has been field scale data collected during dedicated tests using various indenters with contact area up to 3m^2. During these tests, loads, penetrations and contact area were known and/or calculated. These data are referred to as Local Loads. In contrast, there are load and pressure estimates from sensors placed on various Arctic Offshore Structures to record the data when the ice sheet fails against the structure sensors. These data are referred to as Global Loads and generally address loads and pressures over contact areas of greater than 10m^2. There is some overlap between the two categories in the nominally 3 to 40m^2 contact area range in terms of data sources.

Sanderson (1988) plotted all available pressure data in terms of contact area and Aspect Ratio defined as the ratio of ice width to ice thickness and did not make the distinction between Local and Global pressure. The pressure trend on contact area, expressed as a power law with an exponent of -0.5, seems to be the one most used in the ice community. Since then, various authors have proposed trend lines for the Local and Global pressures (eg Dorris and Winkler, 1989, Masterson and Spencer, 2000) and these various trends have been incorporated into codes and standards (CSA 2001, ISO 2010). In these codes and standards the Local and Global pressures have been treated differently with different functional dependence on area, aspect ratio and ice thickness.

When indentation rates are sufficiently high, phenomena such as spalling, high pressure zones (HPZ) and the generation of quantities of crushed ice have been observed at both Local and Global scale. Modelling of ice failure processes have used these high pressure zones to explain the observed pressure trends at both the Local (Palmer et al., 2009) and Global scale (Dempsey etal 2001, Jordaan 2001). What does not appear to have been done, is to use a single model that will address both Local and Global pressure trends. As an illustration of the problem, Figure 1(left) provides the deterministic Local and Global design pressure as a function of contact area using expressions provided in ISO (2010) where crushing is the mode of ice failure.

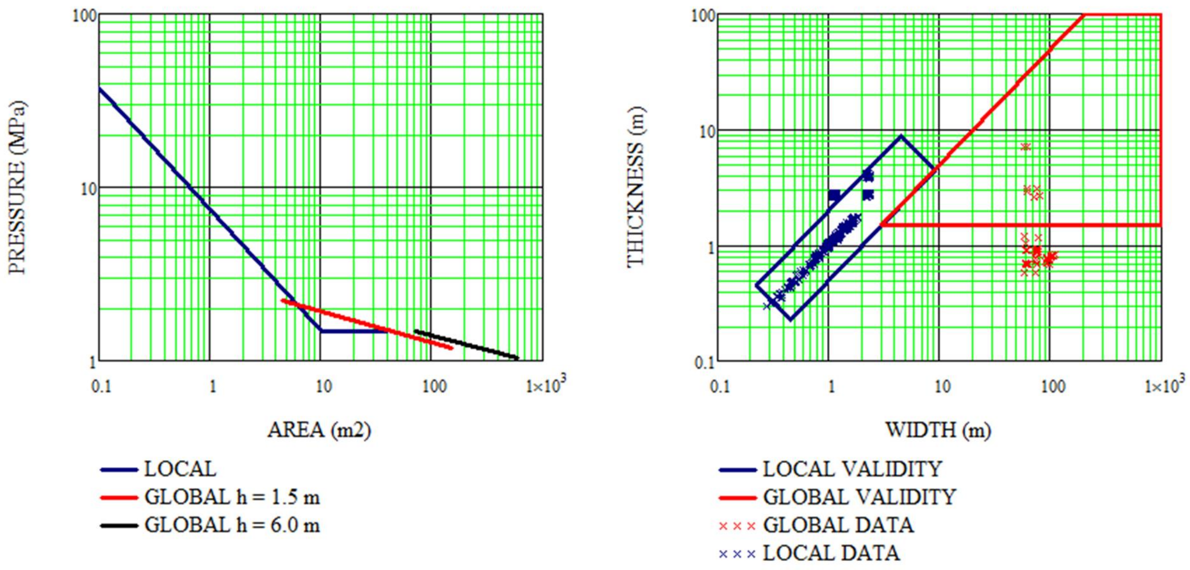

Figure 1. Local and Global Recommendations; left (Pressure); right (Applicability)

Figure 1 (left) illustrates that for Local Loads, the pressure is given by a power law on area with an exponent of -0.7 and a constant pressure region between 10 and 40m^2. For Arctic Global Loads the pressure is also essentially a power law on area with an exponent of -0.14 and only slightly different pressure for thin and thick ice sheets. Between 4.5m^2 and 40m^2 contact areas there are two different design pressures, one for Local design and one for Global design. Shown in Figure 1(right) is a map illustrating the combinations of width and thickness where the Local (blue box) and Global (red box) design recommendations apply. The thickness and width for the data points supporting the recommendations are also indicated. A 5% jitter has been added to these data points to allow visualisation of the number of data points. Inspecting Figure 1 (right) indicates that there are extensive regions where neither the Local nor the Global recommendations apply and the supporting data are limited to specific regions. Palmer (2011), has recently discussed some of these issues regarding the supporting basis of the ISO recommendations.

In this paper we review the design pressure guidelines contained in ISO (2010) for both Local and Arctic Global Loads and describe the model used to unify and extend these two different pressure guidelines.

LOCAL LOADS

The Local Loads guidelines contained in ISO (2010) come directly from work by Masterson et al., (2007). The guidelines are intended to be "deterministic" and have a low probability of exceedance. The method used in generating the trend lines consisted of putting the 351 data points into 23 bins by area, calculating the mean and the standard deviation of the binned data and then doing a regression on the mean plus 3 standard deviations against the midpoint of the bin area (R.M.W. Frederking pers com, 2013).

Table 1 illustrates the results from three different approaches to performing the regression. The first method is commonly used and is contained as a standard method in Microsoft Excel. It involves taking the logarithms of area and pressure and performing a linear regression on the transformed data. The second method uses a power law regression to the untransformed data with each point having equal weight. The third method also uses a power law regression to the untransformed data but each data point has a different weight. Note that in the binned input data, the standard deviations, in general, are different and there are a different number of data points in each bin. The weighting used the uncertainty in the pressure value of mean plus 3 standard deviations. From Table 1 and Figure 2 it can be seen that the three methods do not produce the same value for exponent and multiplier. Newman (1993) has indicated that the commonly used method of taking the logs (Method 1 in Table 1), introduces a bias in the estimated parameters.

Table 1. Local Pressure Regressions to Power function on Area

Regression Method	Multiplier (MPa)	Exponent ±1std
(1) Take logs, equal-weight linear fit	7.40±0.46	-0.704±0.065
(2) Equal-weight fit to power function	7.89±0.63	-0.652±0.059
(3) Weighted fit to power function	6.31±0.46	-0.624±0.065

Method 1 was used to generate the ISO (2010) Local Pressure parameters of 7.4MPa with an area exponent of -0.7. A review of methods to determine the most reliable assessment of a Local Pressure trend line is under way and will be published in the near future. We shall use the Method 3 values given in Table 1; 6.31MPa and exponent -0.624 for the current paper.

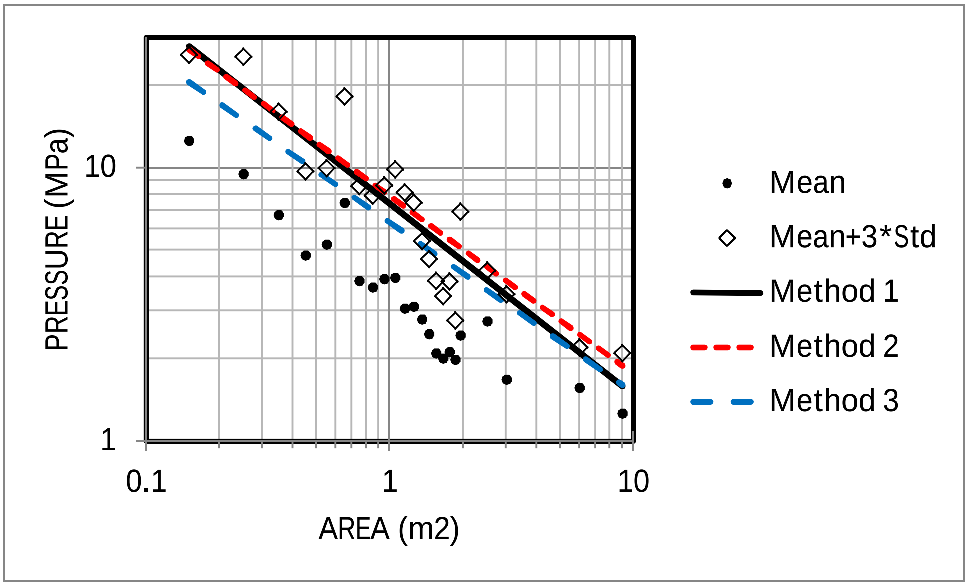

Figure 2. Local Pressure Data and Regression Lines

An additional consideration for the Local Pressure design pressure is that it is expressed as a function of only area. The data used in Figure 2 were all obtained from indenters with an Aspect Ratio of close to 1.0 (Masterson et al., 2007), however the Local Pressure may be a function of Aspect Ratio as well. Note that Spencer and Timco (2010) indicated that the ISO Local crushing pressure line is not necessarily the highest pressure loading case. Also note that the constant pressure region between 10 m^2 and 40 m^2 is indicative of an upper bound since there is an absence of Local pressure measurements in this region as illustrated in Figure 1 (right).

GLOBAL LOADS

The Global Pressure Design line for wide Arctic structures where the ice is failing by crushing is given in ISO (2010) as the following

$$P(MPa) = 2.8*(h/h_0)^{-0.3}*(w/h)^{-0.16} \quad (1)$$

Where h and w are the ice thickness and width respectively and the normalizing constant h_0 = 1.0m. The geometrical limitations on Equation 1 are that both h ≥ 1.5m and the Aspect Ratio (w/h) ≥ 2.0, thus equation (1) would apply to an area as small as $4.5m^2$. There is not a specified upper limit to either thickness or width in equation 1.

At first sight, it would appear that the Global design pressure is a function of both ice thickness and Aspect Ratio. However, equation (1) can be easily transformed to be:

$$P(MPa)\ \alpha\ (w*h)^{-0.14}*w^{-0.02} \quad (2)$$

Equation 2 illustrates that the design pressure is a power law function of Area, (w*h) and with a very weak power law dependence on width (w). The functional form of equation 2 has been illustrated in Figure 1.

The data that was used in the generation of equation 1 has been discussed in Kärnä and Masterson (2011). From these authors it appears that approximately 33 data points were used in the generation of equation 1 along with comparisons to Baltic data obtained at a thinner ice thickness. Given the small number of data points shown in Figure 8 of Kärnä and Masterson (2011), the method used in the Local pressure trend of binning the data and fitting the trend to the binned data would not appear to be applicable. In addition the Arctic data used in the generation of equation 1 is based mainly from measurements from Medof panels mounted on the Molikpaq structure (Jefferies and Spencer, 1989). The accuracy of Medof panel data has been discussed recently by Jordaan et al. (2011) and by Spencer (2013). It is thus difficult to assess the accuracy of equation 1. The small exponent on width (w) in equation 2 will likely be subject to large error.

PROPOSED MODEL

A model by Spencer and Morrison (2012) used statistical pressure averaging in combination with arrays of HPZ over the whole contact zone. This model matched the Global Load trends of equation 1 but did not reproduce the high pressures of the Local Loads at small areas. An earlier model by Spencer and Masterson (1993) used the geometry of HPZ observed in field scale indentation tests (Masterson et al., 1993). Figure 3 shows the geometry of the HPZ for square contact zones and the idealization for square and wide contact zones. Line-like HPZ have been observed during ship trials (Riska, 1991) and we have assumed that at a large Aspect Ratio, the HPZ would be as illustrated on the right hand side of Figure 3.

In the model of Spencer and Masterson (1993) the pressure in the HPZ was represented as a constant and without any statistical variation or fluctuation. In addition the pressure outside of the HPZ was set to zero. Using such a representation of the HPZ pressure, it can be shown that the pressure-area exponent for the whole contact zone cannot be any steeper than -0.5. Since the observed exponents at Local scale provided in Table 1 are in the range of -0.6 to -0.7, modifications to the model are required. Also note that because the Local pressure is represented as a power law on area will tend to infinity at zero area. Thus it is expected that the value of the pressure-area exponent will only be applicable down to a "small" non-zero contact area.

The model used here to merge the Local and Global Pressure trends has the following essential features. The HPZ are restricted to regions of the contact area as illustrated in Figure 3 (right). Within the HPZ are an average number of "hot-spots" proportional to the gross area of the HPZ. The actual load provided by the HPZ is given by mean + 3*stdev of the "hot-spots". Note that the local pressure line illustrated in Figure 1 was also given as mean + 3*stdev. The number of "hot-spots" in the HPZ is represented as a uniform spatial Poisson process (Lewis and Stadler, 1978) and thus the standard deviation is given by $mean^{0.5}$. Outside of the HPZ is a constant non-zero "background" pressure or Low Pressure Zone. This last generalization is based on data obtained during ice crushing (eg Gagnon 1999, Masterson et al., 1990) plus the reasonable assumption that confinement will be required in order to generate the large pressures in the HPZ.

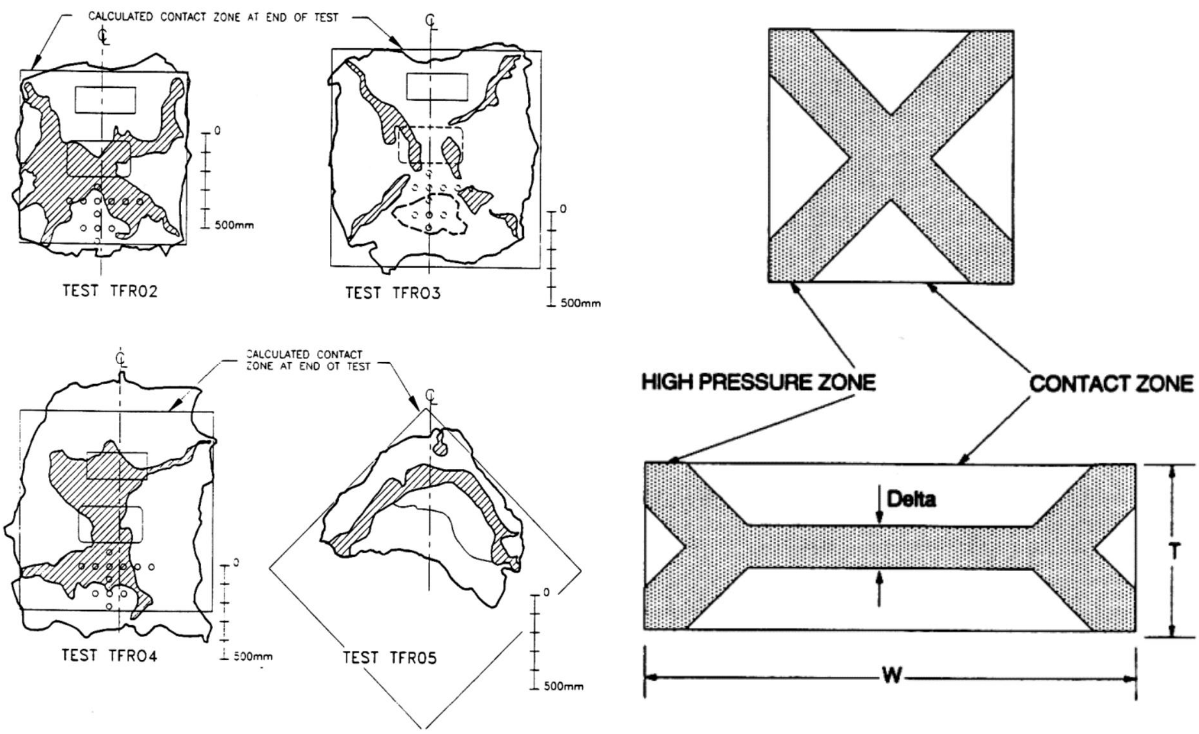

Figure 3. Geometry of HPZ in Ice Crushing; observed (left); idealized (right)

In the proposed analytical/deterministic model, the pressure at the Local scale is then dominated by the "hot-spots" within the HPZ whereas at the Global scale, the pressure is dominated by the assumed value of the background pressure. In this way the pressure-area exponent can vary from a value of about -0.6 to -0.7 at Local scale to a value of about -0.14 at Global scale.

Using the model as described, Figure 4 shows the matching to the Local and Global Pressure trends of ISO (2010) over an area range between nominally 0.1 and $1000m^2$.The matching was done visually given the uncertainties in the ISO trend lines and that the ISO trend lines are likely to be incomplete since they involve only area and not thickness or width or Aspect

Ratio. For the Global part of the match, the model shows an effect on ice thickness over the 1.5m to 6.0m range whereas the ISO Global trend as per equation 2 is only weakly dependent of thickness. For the pressure at areas between $2m^2$ and $10m^2$, the Model produces higher pressures than either of the ISO trend lines.

The parameters used in the Model are given in Table 2 and are now discussed. The pressure in the "hot-spot" has been set to 60MPa based on observations at field scale (Masterson etal 1993). The coverage factor relates to the calculation of the average number of "hot-spots" within the HPZ. Since the extreme number of "hot-spots" is greater than the mean number, then the mean coverage has to be less than unity. For determining the number of "hot-spots" an assumed area for each "hot-spot" was used. Using the "hot-spot" pressure and area, the load per "hot-spot" is then equal to 1.2MN. Alternatively, these parameters can be combined into an average number of "hot-spots" per unit area of HPZ. The maximum pressure that the Model can produce is then given by the "hot-spot" pressure when the complete interaction surface is covered by "hot-spots". Thus the model can only produce pressures up to 60MPa and it can be expected to reproduce pressures of 25 to 30MPa at an area of $0.1m^2$, the minimum area given in Figure 2. In addition, the width of the HPZ has to take into account the $0.1m^2$ minimum area used in the comparison. The value of the low pressure or background pressure is not well defined and has been chosen to match the Global part of the plot.

Table 2. Model Parameters Used for Matching to ISO expressions

HPZ Width (m)	Hot-Spot Area (m^2)	Hot-Spot Pressure (MPa)	Hot-Spot Coverage	Low Pressure Zone (MPa)
0.025	0.020	60	0.20	0.90

The fraction of the total load that is carried by the HPZ is shown in Figure 5. At an area of less than $13m^2$, over half of the load is carried by the HPZ. As the area increases, a smaller fraction of the load is carried by the HPZ the value depending on the ice thickness. In the Global part of the curve the fraction of the load carried by the HPZ is a function of area and additionally of thickness as also shown in Figure 5

As may be seen from Figure 4, the Model does not produce pressure trend lines that are exactly power laws. On a log-log plot a power law would be a straight line. However, over certain area ranges, the power law is a reasonable approximation of the trend. Analysis of the power law exponents that the Model produced was done and these are provided in Table 3. As expected, given the matching of the Model to the ISO lines, the exponents on area at the Local scale and the Global scale are a reasonable match to the exponents from the ISO expressions. In addition, the thickness and width exponents generated show that they vary depending on the value of the other constraint parameter. As can be seen from Table 3, the thickness and width exponents do not match the Global ISO values. In particular the dependence on thickness is stronger than from ISO and the dependence on width is weaker than from ISO. Also indicated in Table 3 is that at the Local scale, the pressure is essentially independent on Aspect Ratio of the interaction at a fixed area.

In Table 3 are also given the pressure exponents calculated from the Spencer and Morrison (2012) model that used pressure averaging. The dependence on area for the Global situation is the same in these two models although the other exponents differ. The relative trend on thickness and width are similar in both models. The main difference between the current model and that of Spencer and Morrison (2012) is that the HPZ have now been restricted to a small part of the contact area. This then allows for the high pressures at Local scale to be produced while still trending to the lower pressure values at Global scale. The background pressure in the current model is essentially the same as the average pressure used in the model of Spencer and Morrison (2012).

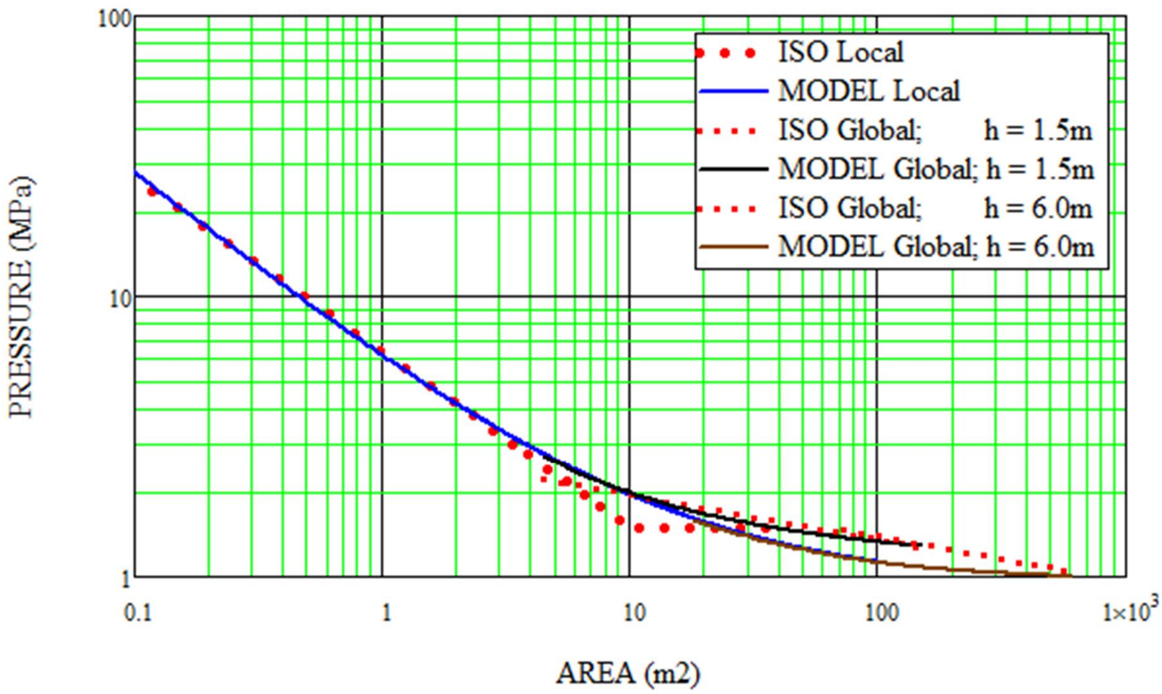

Figure 4. Matching Model to ISO

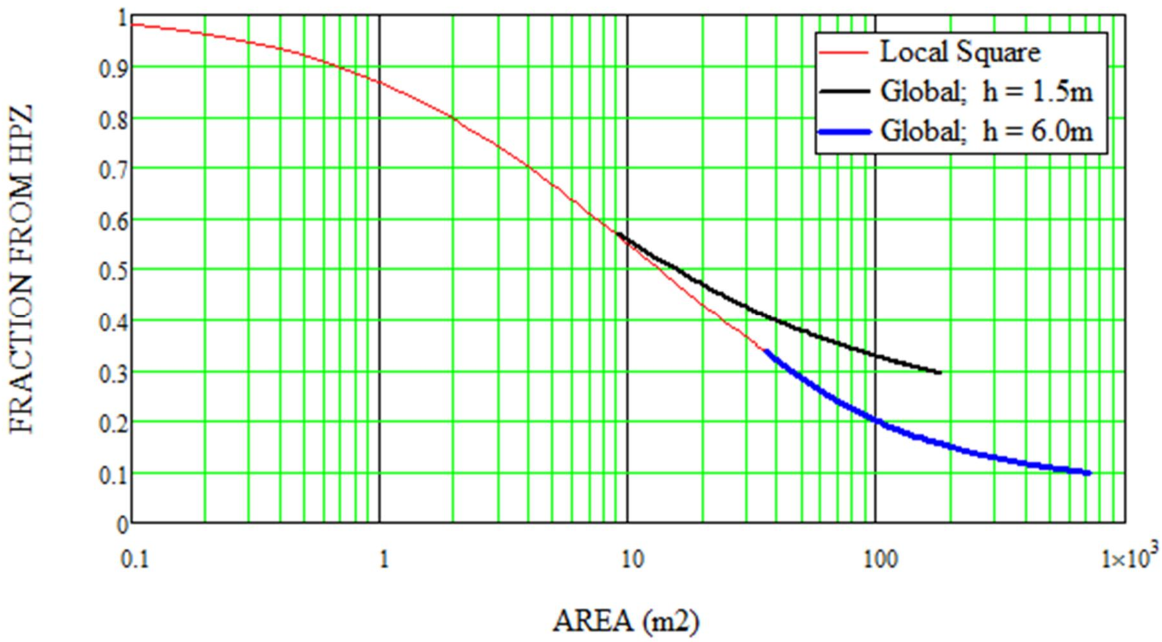

Figure 5. Fraction of Load Carried by HPZ

Table 3. Pressure Exponents Generated by Model

Variable	Range	Constraint	Model	ISO	Spencer Morrison
Area	0.1 to 3.0 m^2	Width = Thickness	-0.653±0.006	-0.70	n/a
Aspect Ratio	0.5 to 8.0	Area = $2.0m^2$	+0.030±0.007	-0.00	n/a
Area	45 to $720m^2$	Width = 20*Thickness	-0.130±0.004	-0.14	-0.132±0.004
Thickness	1.5 to 6.0m	Width = 30m	-0.219±0.005	-0.14	-0.140±0.002
Thickness	1.5 to 6.0m	Width = 150m	-0.159±0.004	-0.14	-0.085±0.002
Width	20 to 120m	Thickness = 1.5m	-0.110±0.005	-0.16	-0.176±0.002
Width	20 to 120m	Thickness = 6.0m	-0.053±0.003	-0.16	-0.125±0.002

Table 4. Pressure(MPa) from Model and ISO

Thick\Width	0.3 (m)	1.0 (m)	3.0 (m)	10.0 (m)	30.0 (m)	100 (m)	300 (m)
30.0 (m)	3.5, na	1.7, na	1.2, na	1.0, na	0.96, na	0.93, 0.83	0.92, 0.70
10.0 (m)	4.8, na	2.2, na	1.4, na	1.1, na	1.0, 1.2	0.96, 0.97	0.95, 0.81
3.0 (m)	7.6, na	3.3, na	2.0, 1.6	1.4, 1.7	1.2, 1.4	1.1, 1.1	1.1, 0.96
1.0 (m)	13.2, na	6.2, 7.4	3.3, na	2.2, na	1.7, na	1.5, na	1.4, na
0.3 (m)	29.9, 39.9	13.2, na	7.6, na	4.8, na	3.5, na	2.8, na	2.4, na

In Table 4 are pressure values calculated from the model and from ISO for various combinations of thickness and width. When ISO is outside the range of validity na is provided. At the Local scale note that the model was matched to a different set of parameters as from ISO, see Table1.The model provides pressure generally larger than ISO at the large contact area. Given the lack of field data from this region, such conservatism is reasonable. From Table 4 it can be seen that the model can provide pressure values over a wide range of thickness and widths in contrast to the limited set given by ISO.

In ISO (2010) there are also Global Pressure based on Baltic measurements. For ice of less than 1.0m in thickness, the pressure thickness exponent is a function of the ice thickness. At larger thickness the expression is the same as equation 1 except that the multiplier is 1.8MPa rather than 2.8MPa. Jordaan et al., (2011) have proposed that the Arctic design curve; ie equation 1, may be high by a factor of 2. The Model presented here can match these Global pressure alternatives by reducing the value of the background pressure and keeping the remaining inputs the same.

There are data at laboratory and field scale eg. Gagnon (1999) and Riska (1991), taken together suggest that the dimensions of the HPZ are a function of ice thickness and/or width. In the Model presented here, for simplicity, we have not included such scaling but kept the HPZ width as a constant. When additional data on pressures and HPZ dimensions becomes available, in combination with any re-assessment of the ISO design curves, then it may be possible to modify the simple model presented here to incorporate such scaling.

CONCLUSIONS

A review of the pressure data supporting the ISO Local and Global design pressures has been done. For the Local pressures, alternative data reduction methods have indicated that changes to the ISO design line may be needed. In addition the ISO Global curve is in actuality a function of primarily area.

The simple model presented here based on HPZ works surprisingly well in matching the general trends of ISO (2010) over contact area between $0.1m^2$ and 1,000 m^2 and can provide a framework for unifying and extending the range of applicability of the Local and Global design pressures.

The model predicts pressures that smoothly transition from the Local scale to the Global scale. From the model it is found that at Local scale, the pressure is almost independent of Aspect Ratio supporting the ISO design line that is a function of only area. For the Global design curve the Model predicts that the pressure can be expressed as a power function of width and of thickness with exponents different from those given in ISO. We view these differences as a result of the limited amount of data that were available for generating the ISO Global load trends.

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