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NUMERICAL STUDIES OF FLOATING STRUCTURES IN BROKEN ICE

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ABSTRACT
Numerical modelling of the interaction between broken ice and floating offshore structures requires new mathematical models capable of predicting both ice-structure and fluid-structure interaction processes. The aim of the present paper is to develop a concept and methodology of numerical simulation of a coupled fluid-solid system which represents a floater in ice-infested waters. In the model, a floating structure and ice pieces are assumed to be rigid bodies with 6 degrees of freedom. Ice failure such as bending and splitting is not included in the current study. A fluid medium is modelled under the assumptions of potential flow theory. A boundary element method is employed to calculate water flow induced by the hull and moving ice. Rigid-body equations of motion are solved using the fourth-order Runge-Kutta integration method.

INTRODUCTION
During the last century substantial research has been performed to study the icebreakers performance in level ice. Jones (2004) has made a historical review of the literature on ships' operations in ice published from 1888. However, there are very few papers focusing on modelling of the dynamic processes between floaters and either broken or level ice. Since the problem involves consideration of multi-body interaction, different material properties for ice and complex hydrodynamics, so far there is no general agreement how to model the behaviour of floating structures in ice-covered waters.
Most of early literature on ice-structure interaction has focused on the average resistance of an intact ice sheet to an advancing ship (e.g. Lindqvist 1989). This includes some more recent papers as well, which are partly described by Jones (2004). Empirical and semi-empirical formulas have been proposed to estimate average or maximum ice forces mainly for design purposes. Today for a wide range of ice-related problems, the averaged ice resistance is not sufficient and it is necessary to consider the time evolution of the interaction processes as well as the behaviour of ice and floating structures under dynamic loads. First of all, it is important for station-keeping and ice management. Among early works in this field was a discrete element approach developed by Hansen and Løset (1999) for simulating dynamic behaviour of moored structures in a broken ice field. Barker et al. (2000) and Sayed et al. (2011) also investigated floaters anchored in ice-covered waters using a particle-in-cell method based on a hybrid Lagrangian-Eulerian formulation. Later with the growing demand of accurate and efficient solutions by the industry sector and with the development of computing technologies, Lubbad and Løset (2011) proposed a numerical model for real-time simulation of a floater in ice employing physics engines to solve the problem of multi-body interaction. Their idea was also adopted by Metrikin et al. (2012a) for simulation of dynamic positioning (DP) in broken ice.
Operations in icy waters may involve different physical processes of the mutual interaction of water with ice, hull, risers, mooring lines, propellers, rudders and other marine systems. In general, fluid-structure interaction (FSI) problems are too complex and cannot be solved

analytically but often they can be analyzed by means of experiments or numerical simulations. Numerical methods for FSI usually combine computational fluid dynamics (CFD) and computational structural dynamics considering both problems together. Regarding FSI in ice-related problems, many authors admitted the importance of the hydrodynamic effect of water on ice and floaters dynamics including the following phenomena:

- resistance of a broken ice field to a ship;
- clearing of ice in the under-hull zone;
- rubble transportation and accumulation along the hull;
- ice loads on the hull due to friction;
- ice failure mechanisms.

However, no model is known to be capable of predicting such effects in real conditions and very few can provide any coupled solution for fluid and structure dynamics. Simulating both ice-structure and fluid-structure interactions are still at the threshold of practical applicability and therefore very simplified models of fluid are used to account somehow for the presence of water. When considering a dynamic ice load on the floater hull, the fluid is usually represented by a Winkler spring model (see e.g. Jebaraj et al. (1992); Lubbad and Løset (2011)). Thus, most models are purely hydrostatic, i.e. they account only for buoyancy neglecting the hydrodynamic damping and the water inertia or using empirical approximations for them (e.g. Liu et al., 2006). However, there are some rare purpose-build solutions related to the breaking of level ice which utilize more sophisticated models for the fluid. Dempsey et al. (1999) have solved two-dimensional elastohydrodynamic ice-slope interaction problems and emphasized the importance of truly hydrodynamic solutions. Valanto (2001) for simulating the response of a floating ice cover and the surrounding water to an advancing icebreaker used a method based on the theory of unsteady potential flow of a fluid with a plane surface. From another perspective, Kämäräinen (2007) developed a CFD code based on hydrodynamic lubrication theory with fluid inertia effects which allows calculating the flow in the gap between the hull surface and an ice floe. Wang and Derradji-Aouat (2010) and later Millan and Wang (2011) utilized a finite element method solving FSI problem where water was artificially modelled using dynamic viscosity and equations of state with a null material model. The research in development of the FSI methods for modelling floating structures in ice is ongoing and no generally applicable solution is available so far.

The aim of this paper is to show the importance of hydrodynamic solutions for some practical problems connected with operations in broken ice which can be investigated by means of numerical simulations. An approach for modelling ice and floater interactions coupled within a fluid will be proposed as well. The method itself can only account for the FSI processes and the elastohydrodynamics of ice is not considered. As a potential application, the concept can be used to complement the multi-body ice-floater models developed by e.g. Lubbad and Løset (2011), Metrikin et al. (2012a,b), Millan and Wang (2011) or other analogues enhancing their capabilities and providing more accurate results for the prediction of the floater and ice dynamics. Some basic aspects of these simulators will be repeated here in order to demonstrate how the proposed model can be integrated within a unified concept for numerical modelling coupled ice-fluid-structure interaction processes.

FORCES ON ICE FLOES

This section is devoted to the identification of the principal forces that act on submerged or floating ice pieces typically occurring in different scenarios of station-keeping in ice-covered waters. The focus was put, first of all, on the hydrodynamic forces which govern floes' motion in the under-hull zone of a floater, as well as their drift around the hull. Since no deformations or

breaking are considered here, the ice pieces and the floater itself will be treated as rigid bodies. The performed analysis will help further to define the most critical factors and parameters for the numerical model proposed in the next section.
First, let us consider an arbitrary body moving in an unbound fluid domain. In such a general case, the body may experience buoyancy, hydrodynamic force connected with fluid inertia, frictional drag force due to viscosity of the fluid, pressure drag and lift.
For a fully submerged body, calculation of buoyancy is quite straightforward, however for a free floating ice piece identification of the submerged volume may be rather challenging. The last is not completely addressed in this paper.
The fluid inertial force $\mathbf{F_{in}}$ is associated here with the added mass of the body which can be derived from potential flow theory. It should be mentioned that for a broken ice field modelled as a system of N rigid bodies with six degrees of freedom (6DOF) the added mass can be represented by a quadratic matrix with 6N rows and columns. This means that each body in the system will affect the dynamic properties of the others even without a mechanical contact between them. If the fluid domain has boundaries or fixed structures inside, it also will change the added mass of the bodies. In Fig.1, the added mass coefficient C_A for a cylinder moving parallel or normal to a fixed wall is illustrated as an example of the "boundary effect" in 2D.

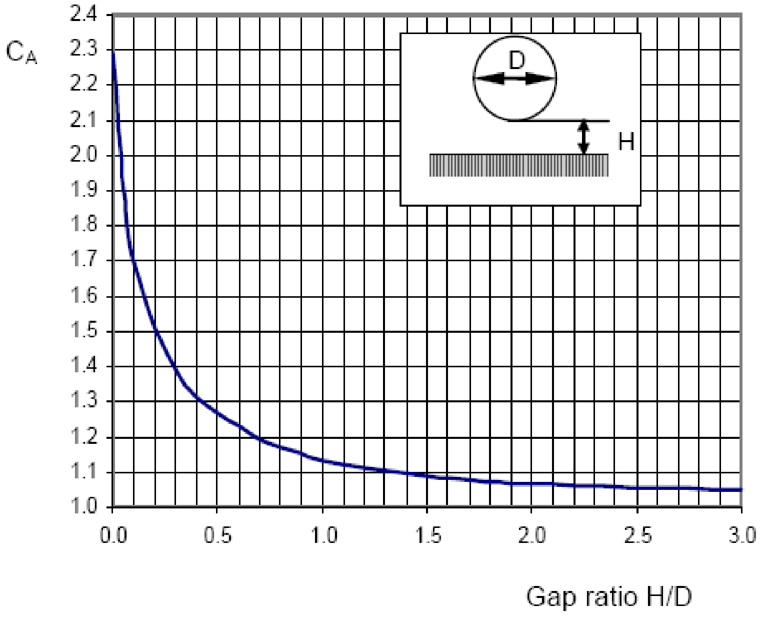

Figure 1. The added mass coefficient of a cylinder in the vicinity of a fixed boundary (DNV-RP-C205, 2010).

According to Newman (1977), the drag on a general body in a steady fluid flow with the high Reynolds number (Re >10^5) can be decomposed into two terms as follows:

$\mathbf{F} = \mathbf{F_F} + \mathbf{F_P}$,

where $\mathbf{F}$ is the total drag on the body, $\mathbf{F_F}$ so called skin friction and $\mathbf{F_P}$ pressure drag. Both components can be expressed similarly:

$$\mathbf{F_F} = -\frac{1}{2} A\rho C_F(Re)|\mathbf{U}|\mathbf{U} \quad (1)$$

$$\mathbf{F_P} = -\frac{1}{2} S\rho C_P|\mathbf{U}|\mathbf{U} \quad (2)$$

where *A* is the surface area of the body over which viscous shear stresses act, *S* the frontal (with respect to the flow) area of the body, ρ the density of the fluid, C_F and C_P are drag coefficients, and **U** is the relative velocity of the body. According to Newman (1977), for streamlined three-dimensional bodies with maximum thickness less than one fifth of the length, the frictional drag is dominant and C_F can be predicted from the flat-plate drag coefficient given in the literature. Relatively large free floating fragments of level ice (Fig. 2) satisfy these conditions and the drag on such floes can be calculated explicitly.

Figure 2. A picture of the icebreaker *Oden* in the Greenland Sea (taken by Løset S., Sept. 2012).

When considering broken ice clearing underneath a floater (Fig. 3), the orientation of floes with respect to the fluid flow can be quite random and the flat-plate drag may not account for the total drag any more. However, it can be seen from the pictures that almost all the ice pieces have mechanical contacts with each other or with the hull. Therefore, their motion is mostly governed by mechanical friction, buoyancy and fluid inertia rather than by the drag, provided that the velocities are not too high. If propeller washing is present, this conclusion may be not valid in the propeller wake where the velocities are usually quite high. This was not investigated in the paper and requires further analysis.

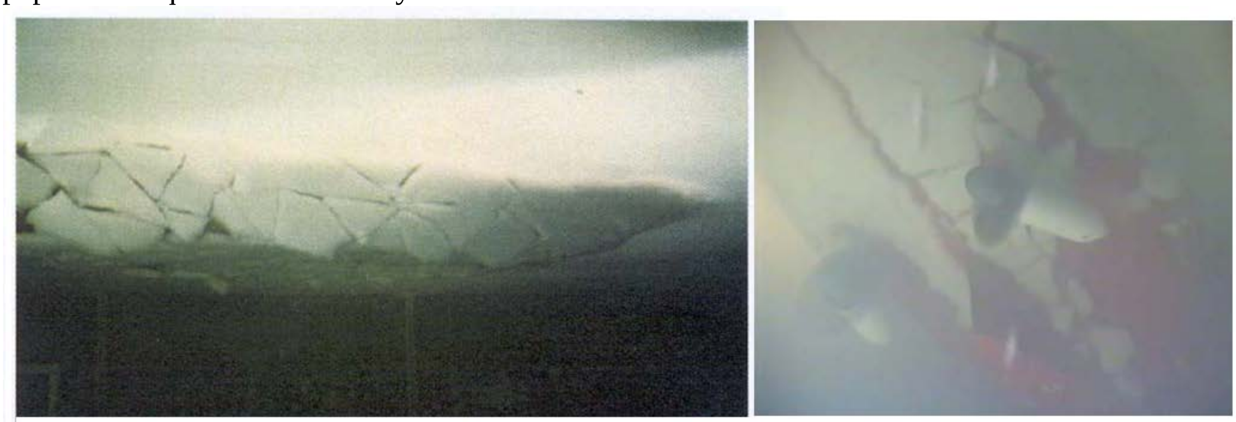

Figure 3. Left: an underwater picture of a ship model advancing in level ice (Valanto, 2001). Right: the bow propellers clearing ice drifting under the vessel hull, ice basin testing (Bonnemaire et al., 2010).

The lift which may act on bodies in a fluid flow was neglected here because usually it acts on ice or floating structures in the same direction with buoyancy, but the last is much higher. However, according to Kämäräinen (2007), this additional lifting force may somewhat increase the total frictional load on an icebreaker moving in ice when the underwater hull surface is fully covered by floes (Fig. 3, left).
All the introduced forces acting on floating ice are schematically illustrated in Fig. 4 with intent to further show their relative importance in different areas around and underneath a floater keeping its position in calm water possibly with a slow current. Basically, the fluid domain has been divided into three major parts: calm water, wake and inertial zones. At some distance from the floater, in the calm zone, the water is assumed to be not affected by the vessel (surface waves are not taken into account here). This fact has been confirmed by the numerical test described below. Consequently, flat ice floes there mainly will be driven by the current, or the frictional drag $\mathbf{F_F}$, and hence their positions relative to each other will change slowly. In contrast, the inertial zone is a highly dynamic area in the vicinity of the hull where the ice can easily change its drift direction because of the interaction processes, mechanical friction $\mathbf{F_\mu}$, buoyancy $\mathbf{F_B}$ and the hydrodynamic forces. The floater's motion and the floes themselves will determine the unsteady flow pattern in the zone accelerating the water around and thus increasing the added mass effect on the ice. Some representative values of the size of this inertial zone solely determined by the floater can be obtained from the simulations performed in the next section. For more accurate estimates, the ice concentration should be also taken into account. Finally, the wake zone is mainly associated with the propeller flow which is also in place when considering DP and propeller clearing (Fig. 3, right). For such a case, the fluid velocities may be relatively high and the pressure drag $\mathbf{F_P}$ will play an important role in the ice washing. But the flow is also transient and a simple formula for this force (e.g. one introduced above) can hardly be applicable here. Simulation of the ice behaviour in the wake may require a more rigorous hydrodynamic analysis.

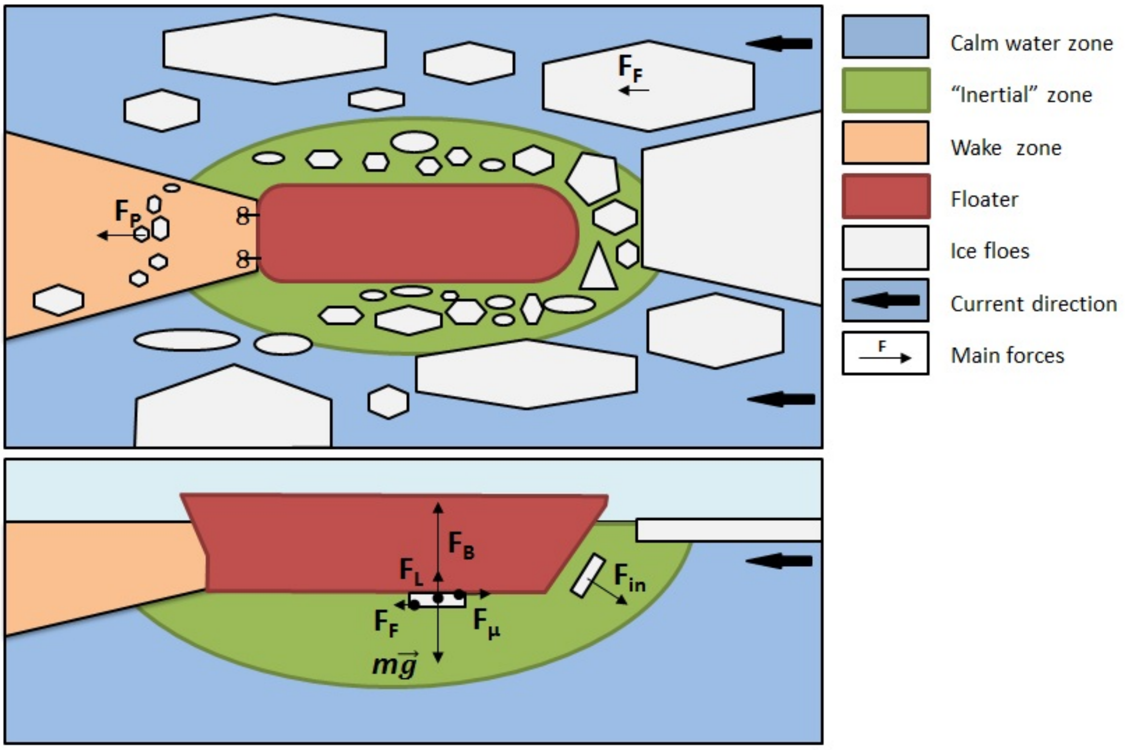

Figure 4. Principal sketch of the main forces on broken ice near a floater and conditional zones of their influence.

MATHEMATICAL MODEL

In this section, a mathematical model of a system of multiple bodies floating in a fluid is proposed. The main focus was put on the hydrodynamic aspects and the rigid-body dynamics of the system. The hydrostatic problem is not completely solved here. Potential flow theory was adopted to calculate the velocity field in the fluid and the inertial forces on the submerged ice (bodies). Newman's (1977) formulation for a single body was used here as the mathematical basis. This purely inviscid hydrodynamic approach was extended to multiple bodies and complemented, where it is necessary, by introducing the viscous friction drag defined in the previous section. All the bodies are assumed to be non-deformable and are governed by the equations of motion of rigid bodies with 6DOF expressed as the following (Fossen, 2011):

$$
\begin{aligned}
& m[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q})] = X \\
& m[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(qr - \dot{p}) + x_g(qp + \dot{r})] = Y \\
& m[\dot{w} - uq + vp - z_g(p^2 + q^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p})] = Z \\
& I_x\dot{p} + (I_z - I_y)qr - (\dot{r} + pq)I_{xz} + (r^2 - q^2)I_{yz} + (pr - \dot{q})I_{xy} \\
& \qquad + m[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] = K \\
& I_x\dot{q} + (I_x - I_z)rp - (\dot{p} + qr)I_{xy} + (p^2 - r^2)I_{zx} + (qp - \dot{r})I_{yz} \\
& \qquad + m[z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)] = M \\
& I_x\dot{r} + (I_y - I_x)pq - (\dot{q} + rp)I_{yz} + (q^2 - p^2)I_{xy} + (rq - \dot{p})I_{zx} \\
& \qquad + m[z_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] = N
\end{aligned}
\tag{3}
$$

where m is the body mass, (x_g, y_g, z_g) center of gravity relative to body-fixed coordinate system; [I_{xx}, I_{xy}, I_{xz}; I_{yx}, I_{yy}, I_{yz}; I_{zx}, I_{zy}, I_{zz}] the inertia matrix; (u, v, w) are linear velocities in surge, sway, and heave; (p, q, r) are angular velocities in roll, pitch and yaw; X, Y, Z, K, M and N denote the generalized external forces and moments. In this paper, only the hydrodynamic components of X, Y, Z, K, M and N were considered, but it is possible to introduce additionally other forces associated, e.g. with contact interactions as it was done by Lubbad and Løset (2011).

Now let us consider a body moving with velocity **v** in an ideal fluid. Under the made assumptions, there exists a potential Φ such that the fluid velocity **u** can be expressed as:

$$\mathbf{u} = \nabla\Phi\,. \tag{4}$$

The fluid continuity, incompressibility and the boundary conditions give:

$$
\begin{aligned}
& \Delta\Phi = 0, \\
& \mathbf{n}\cdot\nabla\Phi = \mathbf{v}\cdot\mathbf{n},
\end{aligned}
\tag{5}
$$

where **n** is the inward unit normal vector on the body surface. The fluid inertial force $\mathbf{F_{in}}$ on the body can be derived from Bernoulli's equation as follows:

$$\mathbf{F_{in}} = -\rho\frac{d}{dt}\int_{S_b}\Phi\mathbf{n}\,dA \tag{6}$$

where S_b is the area of the wet body surface. The corresponding moment $\mathbf{M_{in}}$ can be expressed likewise:

$$\mathbf{M_{in}} = -\rho\frac{d}{dt}\int_{S_b}\Phi(\mathbf{r}\cdot\mathbf{n})\,dA\,, \tag{7}$$

where **r** is the radius-vector from the centre of mass of the body.

To obtain the instantaneous added mass coefficients for a system of N submerged bodies, the total potential Φ can be expressed in the following way:

$$\Phi = \sum_{i}^{6N} U_i \varphi_i \qquad (8)$$

where φ_i is the potential due to the unit motion in the i-th mode (all the N bodies together provide the system with $6N$ degrees of freedom – modes); U_i the actual time dependent velocity in the i-th mode. Since U_i is independent on the integration path, substituting (8) into (6) gives:

$$\mathbf{F_{in}} = \sum_{i=1}^{6N} \left[-\rho \frac{dU_i}{dt} \int_{S_b} \varphi_i \mathbf{n} dA - \rho U_i \frac{d}{dt} \int_{S_b} \varphi_i \mathbf{n} dA \right]. \qquad (9)$$

The integral in this equation leads to a definition of an added mass tensor:

$$a_{ji} = \rho \int_{S_b} \varphi_i n_j dA \qquad (10)$$

where $j = 1..6N$ and n_j means the component of $\mathbf{n}$ corresponding to the j-th mode. Considering a single rigid body motion in an unbounded fluid or perturbations of a system of multiple bodies, when solutions for φ_i are time independent or can be assumed so, the added mass tensors are also independent on time (or quasi-static). In such a case, the hydrodynamic forces $\mathbf{F_{in}}$ and moments $\mathbf{M_{in}}$ on the bodies are solely defined by the instantaneous added mass coefficients, the bodies' accelerations and velocities.

The fluid inertial effect exhibits when bodies move with acceleration or rotate in the fluid and is important when considering the dynamics of broken ice in close vicinity to the hull. To simulate the ice drift at some distance from the floater, the viscous damping should be taken into account. For this, as it was shown in the previous section, the frictional drag $\mathbf{F_F}$ comes to play.

Now the full set of equations is provided and a numerical model can be developed to simulate the floater and ice dynamics. We have used a two-step scheme to solve the problem. At the first step, the velocity potentials should be obtained solving Laplace's equation subject to the boundary conditions (5) corresponding to the current velocities and positions of every object in the considered multi-body system. This is done using a three-dimensional boundary element method where only the floater's and the ice floes' surfaces are discretized and the wave-making at the free surface is neglected. To account for the boundary on the surface, a "double-body" potential flow method (see e.g. Bertram, 2000) has been adopted. In the method, the floater's hull and the ice floes are reflected at the waterline and the flow in an infinite fluid domain is computed. Then the lower half of the flow gives automatically the flow about the structure with a plane undeformed water surface. The velocity field is used to calculate the frictional drag $\mathbf{F_F}$ where it is needed. The obtained potentials are used to identify the actual added mass coefficients required for $\mathbf{F_{in}}$ and $\mathbf{M_{in}}$. At the second step, the ordinary differential equations of rigid-bodies motions (3) with the right-hand side already expressed through the positions and their first and second derivatives are solved by the fourth-order Runge-Kutta integration. The solution gives new positions, velocities and accelerations of the ice and the floater. Then the cycle can be repeated.

SIMULATIONS

The mathematical model described in the previous section was implemented in a Matlab code which can be used as a numerical tool for studying the hydrodynamics of floating structures and an ice field. The program does not allow contacts between the objects and thus has to be upgraded in order to simulate mechanical interactions. It is possible to do so using e.g. the concept developed by Lubbad and Løset (2011), since their model also implies solving rigid-body dynamics.

A simple validation test has been performed to make sure that the program is capable of predicting sensible results. For this, the added mass coefficient of a fully submerged sphere in an unbounded fluid was calculated numerically. Since the shape of the body is symmetrical, the coefficients associated with different directions of translational motions are equal to each other, hence only one of them denoted here as a_{11} was investigated. According to the analytical solution (see e.g. Newman, 1977), for a sphere of radius R, the added mass a_{11} should be equal to $\frac{2}{3}\rho R^3$. In the numerical tests, the sphere surface was discretized by panels as it is shown in Fig. 5. Three cases with different numbers of elements in the mesh were modelled in order to evaluate the mesh independency.

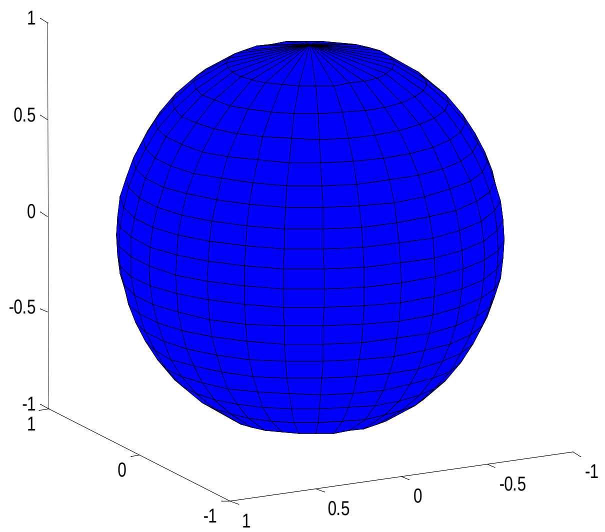

Figure 5. Discretization of the sphere.

The coefficients a_{11} calculated from the model were compared with the theoretical values in Table 1. As it can be seen, the numerical results are in good agreement with the results of the analytical solutions, and the discrepancy (error) decreases when a finer mesh is used. These ensure a high conformity of the method and its convergence to the exact value.

Table 1. Comparison of numerical and theoretical values of the added mass coefficient *a11* of the sphere.

Sphere radius R, m	Number of elements	Numerical result for a_{11}/ρ	Theoretical value of a_{11}/ρ	Error, %
1.0	400	2.0215	2.0944	3.5
1.0	800	2.0633	2.0944	1.5
1.0	2400	2.0802	2.0944	0.68

Another test has been performed to analyse a remote interaction between a floater and broken ice pieces. The purpose of this study was to show the importance of the fluid inertia force on the ice when considering its dynamics, clearing and accumulation along the hull. The floater model were reproduced based on the data with the main dimensions of the drilling vessel *Kulluk* presented by Nixon and Ettema (1987). Only double-body (hull-floe) interactions were investigated here. For this, a single fully submerged 1×5×5 m ice floe was placed underneath the hull at a distance H from the floater as shown in Fig.6. For each simulation, the distance was varied from 0.5 to 30 m but the floe's orientation and the floater's pitch and heave remained fixed.

As it can be expected, any motion of a huge marine structure will be transferred to some extent on nearby floating objects by the water. This fluid inertia effect can be quantified in terms of potential flow theory using different added mass coefficients. In our case for the considered floe-floater system, one such coefficient C_a has been identified numerically using the following definition:

$$C_a = -\frac{F_x^{floe}}{a_x^{floater} V\rho} \qquad (11)$$

where V is the volume of the floe; F_x^{floe} the force on the ice along the x-axis due to the floater motion in x direction with acceleration $a_x^{floater}$. During the simulations, different values of C_a corresponding to different distances H have been obtained and plotted (Fig.6) against the relative distance H/L where L is the representative length of the floe (here 5 m). It can be seen from the diagram that for pieces of broken ice freely floating in the vicinity of an unstable floater at the distance up to 3 of their size the vessel motion effect can be even more important than their own inertia since ice and water densities are comparable. However, the strength of this remote interaction decreases with increasing distance very fast and far away from the floater the floes will not experience its motion. It should be mentioned that the last conclusion is only applicable as long as the free surface effect can be neglected since no wave radiation has been simulated here.

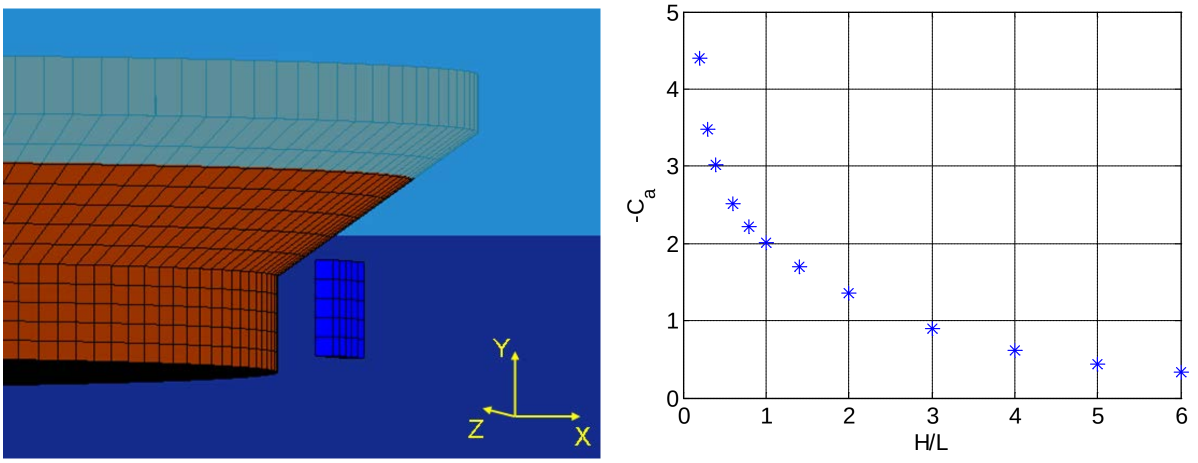

Figure 6. The model of the floater and an ice floe (left); and the added mass coefficient ($-C_a$) of the floe versus its relative distance *H/L* to the hull (right).

POSSIBLE APPLICATIONS AND FURTHER DEVELOPMENT

The developed software program is based on the mathematical model described in the paper and can be used as a stand-alone hydrodynamic tool for computation of the added mass coefficients of a multi-body system floating in a fluid. The model also can be employed for numerical studies of the fluid effect on the interaction processes between an Arctic vessel and broken ice as it was done in the previous section.

The most promising potential application of the proposed model is to connect it in parallel with an efficient simulator of the mechanical hydrostatic ice-structure interaction processes (e.g. one developed by Lubbad and Løset in 2011). It is possible to do so if all the objects in the simulator are governed by the ordinary differential equations of rigid body motion. In this case, the total force on a body can be expressed as the superposition of the contact forces and buoyancy computed by the simulator and the hydrodynamic forces obtained from the presented model. The boundary element subroutine can be run on any integration time step using only current positions of the objects and their velocities. Once discretizing the computational domain (the bodies' surfaces), there will be no need to update or regenerate the mesh during simulation. This is a big advantage in comparison with Arbitrary Lagrangian-Eulerian (ALE) methods often used for FSI modelling. The unified concept of the proposed hydrodynamic rigid-body model of a floater in ice is believed to give a new approach for simulations of the complex ice-structure interaction processes in a wide range of practical problems.

CONCLUSIONS

In this paper an analysis of the main forces which contribute to the global dynamic behaviour of a broken ice field around and underneath the hull of a floater has been performed. The major conclusive remarks are the following:

- The results of this study were used to develop a numerical hydrodynamic ice-floater model which was further utilized for the investigation of the remote interaction between the floater and an ice floe.

- The model was implemented as a Matlab code that was validated in a simple test. The program also has shown its capability to be used as a stand-alone tool for calculation of the added mass coefficients of different bodies and their systems.

- An innovative approach to simulate the hydrodynamic ice-structure interaction processes has been proposed in connection with the future development of the concept.

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