POAC'13
Espoo, Finland

Proceedings of the 22nd International Conference on
**Port and Ocean Engineering under Arctic Conditions**
**June 9-13, 2013**
**Espoo, Finland**

# VIBRATION OF FIXED OFFSHORE STRUCTURES UNDER ICE ACTION

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## ABSTRACT
Ice-induced vibration of fixed offshore structures has been observed in different seas, for example: Beaufort Sea, Cook Inlet, Gulf of Bothnia, Bohai Bay, Sea of Okhotsk. It can have a significant impact, as it can lead to failure due to structural fatigue (Bohai Bay) or softening of foundations, or cause problems with the serviceability of platforms. Possible future arctic developments will be in deeper water, where the phenomenon of ice-induced vibration may be one factor limiting the structure use. A 2D solution for the numerical study of ice-induced vibration of vertical-sided fixed offshore structures based upon common assumptions and mathematical models of ice properties has been developed by Saint Petersburg State Polytechnical University. The solution considers the whole process of vibration as it develops in time. The solution was compared with some model and full-scale (lighthouse and Molikpaq) results. The model has been applied in order to understand the environmental conditions and structural characteristics that are important to the incidence of ice-induced vibrations. It was shown that many factors influence this process. The relative importance of different factors to the vibration formation is analysed.

## KEY WORDS
Ice-induced vibration, frequency, ice velocity, structure response, ice load, ice action, numerical modelling, discrete element method.

## INTRODUCTION
The vibration experienced by structures has a random character and is dependent on many factors, specifically: on the structure compliance and mass, the foundation compliance, the ice modulus of elasticity and strength, the velocity of the moving ice sheet, and the type of ice failure (crushing or buckling).

Major contributions to the understanding of ice-induced vibrations on fixed offshore structures have been made by Kärnä (1992), Kärnä et al (1989, 1990, 2003, 2008), Määttänen (1978, 1998, 2001), Sodhi (1988), and Eranti (1992) who have studied this phenomenon for more than 20 years. On the basis of extensive theoretical, laboratory and field research they developed a classification of different types of vibration, investigated conditions leading to the onset of vibration, and proposed analytical solutions. However, even in laboratory experiments it is difficult to consider all of the possible factors involved in such a complex process as ice-induced vibration and to evaluate their relative significance. Therefore, a more efficient numerical solution is required comprising a wide range of conditions that may influence the ice-structure interactions.

Finite element and finite difference methodologies have previously been used to study the vibration problem; however the cyclic nature of the ice loading and failure can make application of these methodologies problematic. Each cycle of this process really consists of a sequence of processes, namely ice compression, its failure, extrusion of the pulverized ice particles, movement of the structure, and its probable loss of contact and subsequent collision with the ice. Developing descriptions for all these processes provides great challenges. A more promising methodology seems to be the use of the discrete element method reported for example by Gurtner et al (2010) Haase et al (2010), Hopkins (2006), Kolary et al (2009), Konuk et al (2009), Paaviaainen et al (2006, 2009), Polari and Tukhuri (2009), and others. This is discussed in the present paper.

**NOMENCLATURE**

$d$ maximum amplitude of structure displacement, cm;

$D$ structure diameter, m;

$F_{aver}$ average structure reaction response, MN; = $\frac{1}{T}\int_0^T F_s(t)dt$ ;

$F_i$ maximum amplitude of ice action (mean of 5 subsequent maxima), MN;

$F_m$ first amplitude of structure response (induced by action of the intact ice), MN;

$F_s$ maximum amplitude of structure response (mean of 5 subsequent maxima), MN;

$G$ structure weight, MN;

$h$ ice thickness (reference thickness = 1 m);

$K_s$ structure stiffness, MN/m;

$M$ structure mass, kt;

$R_c$ unconfined ice strength, MPa;

$t$ time, s;

$\tau$ (tau) - structure's natural period of oscillation, s;

$V_i$ ice sheet velocity, m/s

$V_s$ maximum amplitude of structural velocity of oscillation (mean of 5 subsequent maxima), m/s

$E$ modulus of elasticity of ice.

**NUMERICAL MODELLING**

The objective is to determine ice loads and structural responses arising when a moving ice field interacts with a flexible structure. Calculations are intended to establish what physical environmental conditions and structural parameters can lead to the occurrence of structure vibrations and how the main parameters of the process influence the result.

***Ice model***

The ice model makes it possible to describe in sequence the process of ice failure and the extrusion of pulverized ice particles, as well as the motion of the structure.

The modelling of the ice presented in this paper is based on that described in the program suite PFC2D-PARTICLE FLOW CODE IN 2 DIMENSIONS which is widely used in geomechanics. In this model, the medium is represented as a cluster of elements (discs in 2D or spheres in 3D) packed in a special way to form a medium with the required properties.

The basic requirement is that the field describes the real properties of ice and satisfies the failure criteria. Specifically, the medium should reflect the shear, compressive and tensile stresses of ice during its elastic deformation, as well as the absolute limits for defining the failure process. An additional requirement is that prior to starting interaction calculations the elements fill all the space of the selected field area without voids, without overlapping and without any prestressing. This can be achieved by using a procedure recommended by Cundall and Strack, 1979. As a result of this procedure the initial positions of bonds between each element and its neighbours are determined. Bonds with specified stiffness and strength model the normal and shear forces arising from the normal and transverse relative displacements of adjacent element centres. Initially all forces are zero. A change in the distance between the centres of adjacent elements causes compression or tension. Relative displacement in the transverse direction causes shear force. The displacement of each element (absolute and relative to neighbours) is calculated during the development of the interaction process. If compressive, tensile or shear forces associated with the relative displacement of elements exceed the local crushing, tensile or shear limit respectively, then the corresponding connection fails. During subsequent movement the element can make contact with other elements but the tensile strength of this element is never recovered. As shown in Cundall and Strack, 1979 the failure criteria for the contacts in the media are similar to the Mohr-Coulomb law, written for forces instead of stresses.

It is important to calibrate the internal (bonds) stiffness and strength with the global ice properties. Therefore numerical experiments with ice “samples” were conducted. Compressive, tensile or shear forces were applied to the “samples” surfaces, and increased to the sample’s failure. The relationship between the global and the local elastic and failure parameters was established after a large number of experiments.

***The structure***

A wide vertical fixed (bottom-founded) structure with one degree of freedom is considered using a 2D out-of-plane solution. The mathematical model examines the mass (added mass is included) located on a spring with a defined stiffness (which combines soil reaction and stiffness of the structure body).

***The calculation***

Level ice with thickness *h*, elastic modulus *E* and specified strength properties is given a far-field boundary velocity $V_i$ which causes the ice to be pushed against the structure. The velocities of ice particles near the structure will differ from $V_i$ as a result of the ice /structure interaction.

The ice load on the structure develops as the result of ice/ structure interaction. The normal force is calculated from the ice bond stiffness and the element’s “intrusion” into the structure. The shear force is calculated as the sum of the shear force increments from the ice bond shear stiffness and the incremental transverse displacement at each time step.

***The main input parameters in the calculations***

The vibration depends upon many parameters:

Structural system:

- Mass *M* (including added mass)
- Stiffness $K_s$
- Width *D* of the front wall (assumed to be no less than ten times the ice thickness)
- Damping $\varsigma$

Ice properties and parameters:

- Elastic modulus *E*
- Ice velocity $V_i$
- Unconfined strength $R_c$
- Thickness *h*

The following combinations may determine the development of the interaction process:

- Structural natural period $\tau = 2\pi\sqrt{M / K_s}$
- Relative weight $p_w = Mg/DhRc$
- Mass $p_m = M/Dh$

- Relative stiffness $p_s = K_s/Eh$

The study of vibration involves the analysis of many contributing influencing factors. Combinations of these factors are very important and deserve particular attention, but their detailed investigation and discussion needs a huge volume of calculations, which is beyond the scope of this paper. Therefore, the influence of only some of these combinations of factors will be examined in most of the experiments discussed below.

Some parameters, as listed below, are assumed to be constant:

- Width of the structure (the experimental loads and reactions and displacements are with reference to the structure width used in the experiments);
- Ice thickness ($h$=1m);
- Ice strength (1 MPa).

The data in Figures 2 - 8 refers to a constant relative mass $M/Dh$= 6.7 kt/m$^2$. The influence of the structure's mass was studied in a small series of experiments (Figure 9). Other parameters used in the calculation are specified in the figures.

The parameters $\tau$ , $p_w, p_m, p_s$ raise an issue concerning the methodology of conducting the experiments. Usually when the influence of a parameter is investigated only this one parameter is varied while the others remain constant. But in the vibration problem it is often not possible to take this approach. For example, if the influence of $\tau$ is considered then the variation of $K_s$ or $M$ in the equation for $\tau$ causes either $p_s$ or $p_w$ to change.

***The main outputs***

The main outputs of the calculations are time-history series of the following:

- ice loads acting over the ice / structure contact area**;**
- structure reaction**;**
- structure displacement;
- structure velocity.

## RESULTS OF CALCULATION

***Description of the interaction process***

Successive phases of the ice-structure interaction in the ice nearest to the contact area zone are given in Figure 1.


Figure 1. Successive phases of the ice-structure interaction:
1 – initial phase, 2 – onset of failure near the contact border, 3 – extrusion of pulverized ice particles, 4 – formation of a wedge and its interaction with the structure,
5 – structure interaction with crushed ice.

The phases of interaction illustrated in Figure 1 are similar to those usually observed in laboratory tests. The intact ice approaches the structure and exerts pressure on it (1). As the load increases, the ice starts to fail and expands (2). This process is accompanied with failure of internal bonds between the elements, and dilatancy. Under large compressive forces the ice is crushed and partly extrudes (3). The surface of the intact ice takes the form of a wedge. Later the pressure in the wedge exceeds the failure limit and the ice fails (4). The crushed, pulverized material interacts with the structure (5).

A similar phenomenon has been described several times (e.g. Timco and Frederking, 1995, Tuhkuri,1993). The subsequent process is a repetition of phases 2, 3, 4 and 5. Two types of actions are occurring after the first phase as can be seen from the illustrations above: the significant loading by the wedge on a small part of the contact area, and action of the weak broken ice on a wide part of the area. This means that the high total load exerted on the structure during the initial interaction (the first stage) does not occur on the contact surface again and the load typical for the subsequent processes is always only a fraction of the initial load. This is confirmed by laboratory experiments where the subsequent loads usually do not exceed half of the initial load.

***Description of the load/ structure response***

Time series of load exerted on the ice-structure contact area $F_i$ divided by *(D h $R_c$)*, and the structural response $F_s$ normalised using the same parameter are illustrated in Figures 2 and 3. Figure 2 corresponds to an ice velocity $V_i$=0.03 m/s and different natural periods of the structure. Figure 3 corresponds to the same structural natural period (0.8 s) and different ice velocities. Changes in the structural response occur in all cases.

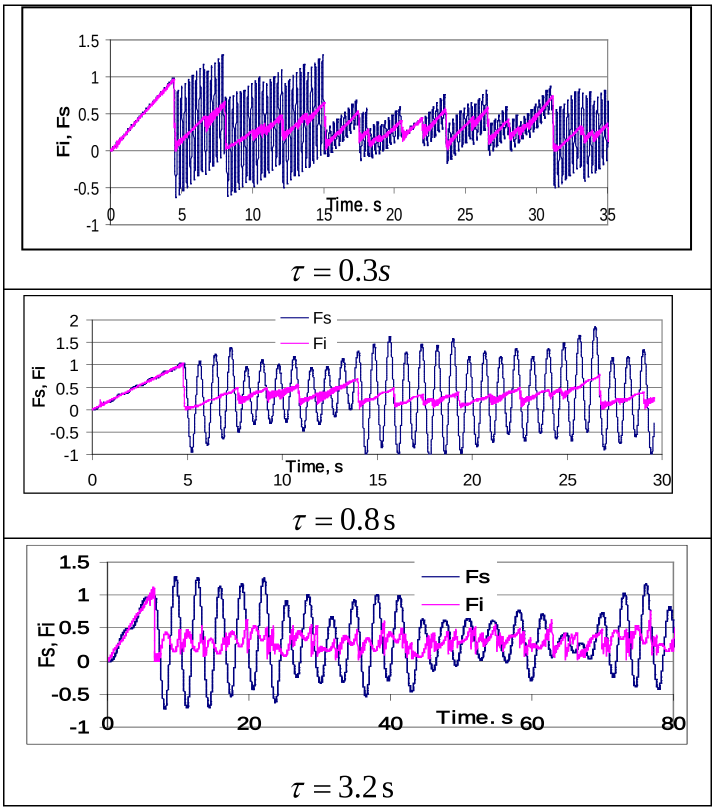


Figure 2. Relation between ice load and structural response at various natural periods of the structure, for an ice velocity $V_i$=0.03 m/s

At very low velocities and low $\tau$ the main factor determining the phenomenon is the abrupt jumps in ice loads caused by the structure losing contact with the ice with a subsequent impact. At moderate ice velocities steady-state vibrations develop. No dynamic response amplification is seen at the high ice velocities. Similar dependences of load-time histories upon ice velocities have been reported previously, for example by Kärnä and Turunen, 1989 and 1990, Huang and Shi, 2007.

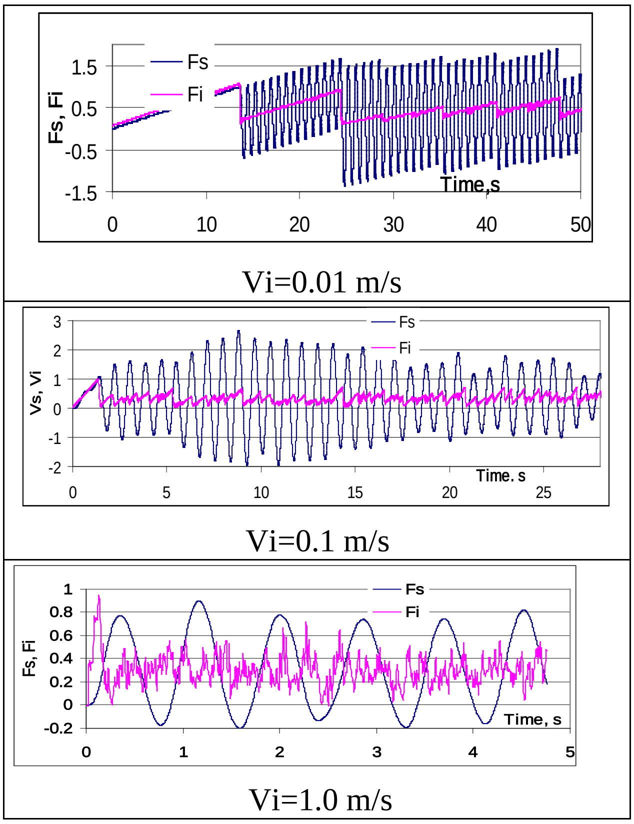


Figure 3. Relation between ice load and structural response at various ice velocities, for a structure natural period $\tau = 0.8\ s$

Fourier analysis of some time series is presented in Figure 4, (a) and (b).

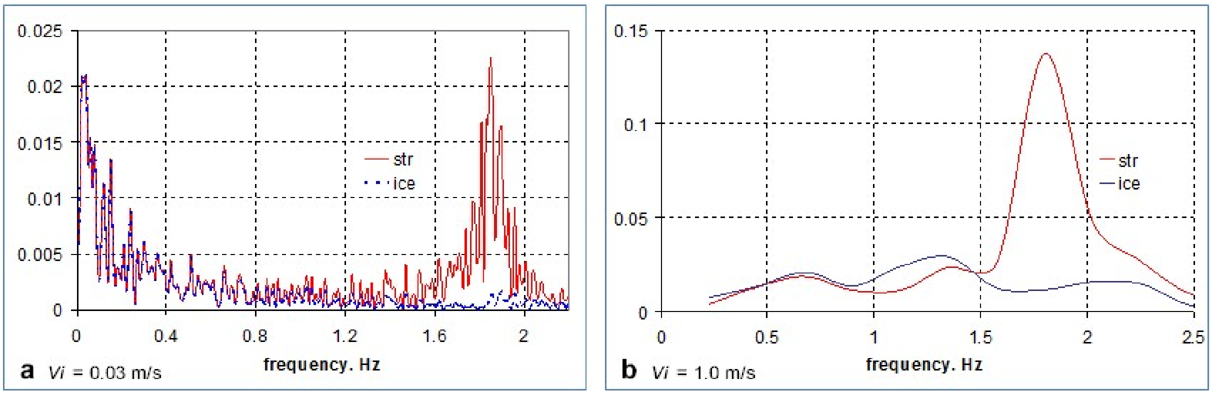


Figure 4. Fourier analysis of some time series

The left (a) part in Figure 4 corresponds to a time series similar to that shown in Figure 2 where $V_i$ =0.03 m/s and $\tau = 0.8$ s. The right (b) to $V_i$ =1.0 m/s and the same $\tau$. Dependences for both the structure's reaction (str) and the ice loads (ice) are presented. There are two maxima on the structural reaction graph in Figure 4a. The first peak reflects oscillations with great period arising due to the structure losing contact with the ice, free motion and subsequent hitting of the ice. The curves for ice load and the structural response coincide in this peak. The second peak takes place on frequencies close to the structural natural ones. Peaks for maximum structural reaction and maximum load in Figure 4b take place at different frequencies. A decrease of the first peak and an increase of the second peak is typical for the intermediate velocities.

The results shown above demonstrate that both the ice velocity and the structural natural period influence the vibration process. The results have been obtained for one natural period are not valid for other periods at the same ice velocity.

***Quantitative results: Influence of ice velocity and structural natural period***

The combined influence of the structural natural period (tau) and the ice velocity ($V_i$) on the maximum structural response is illustrated in Figure 5. The amplification of the structural response is plotted for ice velocities from 0.01 to 1.0 m/s.  The amplification factor is expressed as $F_s/F_i$, i.e. the ratio of the mean of the maximum structural response $F_s$ to the mean of the maximum of the ice load $F_i$. As can be seen, there are almost no dynamics in the interaction when the ice velocity is more than 0.5 m/s. In the experiments, the maximum dynamic response was registered in the $V_i$ range 0.03 - 0.1 m/s for all considered $\tau$ but the level of reaction depends significantly on this parameter.

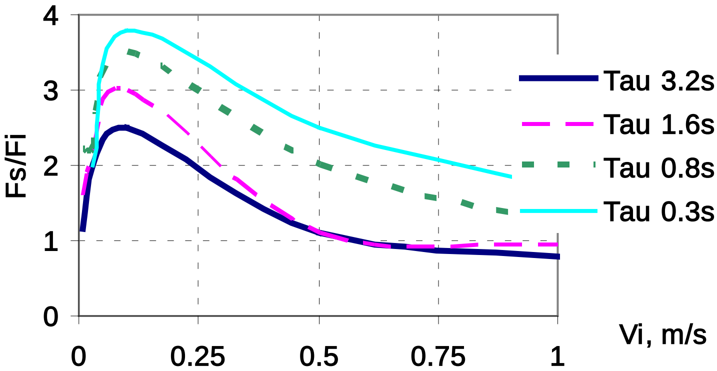


Figure 5.  Influence of ice velocity and structure's natural period on structure response

At low velocities abrupt jumps in ice loads caused by the structure breaking-off from the ice with a following impact induces maximum reaction. In contrast, at moderate ice velocities steady-state vibrations develop, and no dynamic response amplification is seen at high ice velocities.. Similar dependences of load time histories upon ice velocities have been reported earlier, for example by Kärnä and Turunen, 1989 and 1990, Huang and Shi, 2007.

***Quantitative results: Structure oscillation velocities***

Similar to ice loads, a structure's velocity depends upon its natural period, the velocity of the ice motion and other factors discussed above. As an illustration, time series of the structure's velocity is outlined in Figure 6, where, the straight line represents the velocity of the ice motion. It can be seen in the figure that the structure's velocity can be higher than the ice velocity. This phenomenon has been already described before in several studies (e.g. Kärnä and Turunen, 1989 and 1990, Huang and Shi, 2007).

A general relationship between ice velocity $V_i$ and the maximum structural velocity $V_s$ is given in Figure 7 along with the dependence for natural periods of 3.2 s, 1.6 s and 0.8 s. The dashed line in the figure indicates the situation when $V_s = V_i$. At low $V_i$ the structure velocity $V_s$ significantly exceeds the ice velocity, it then reaches the $V_i$ level and later decreases. The ice velocity which satisfies a condition $V_s=V_i$ depends on the structural period. Such behaviour has been detected in some experiments, which showed that the shorter the natural period is, the smaller the region where $V_s > V_i$ (Huang and Shi, 2007). Usually in our numerical experiments the maximum of the structure reaction was seen near the region where $V_s = V_i$.

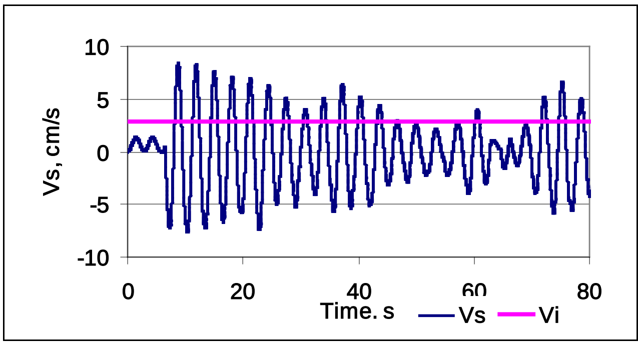


Figure 6. Vibration rates
$V_i = 0.03$ m/s, $\tau = 3.2$ s

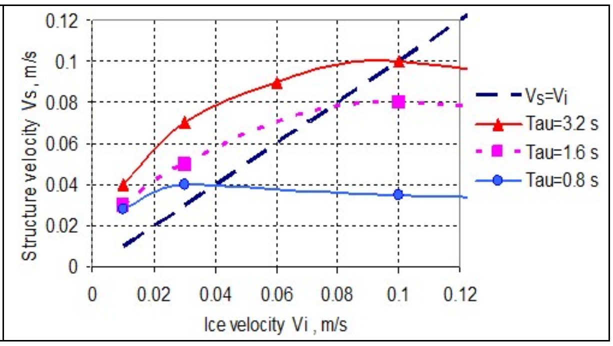


Figure 7. Maximum structural velocity $V_s$ versus ice velocity $V_i$

***Quantitative results: structure oscillation displacement***

The dependence of the maximum amplitude of oscillations displacement, normalized to a reference ice thickness of 1 m, on the ice velocity is represented in Figure 8.

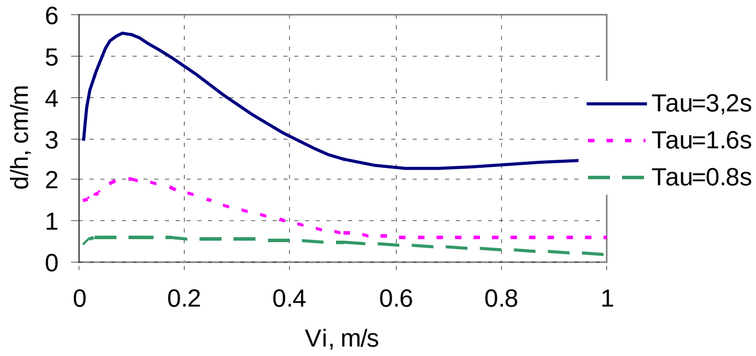


Figure 8. Influence of ice velocity and structural natural period on amplitude of oscillation

Judging by this dependence, the most dangerous conditions are those with low ice velocities and compliant structures whose amplitude of oscillation may be ten times larger than the amplitude of stiffer structures.

***Effect of the elastic modulus of ice***

The influence of this parameter on the vibration has not previously been investigated in any detail. To the best of the authors' knowledge, only one study has been published dealing with this issue (Huang and Shi, 2007). To examine the role of this parameter, numerical experiments where only the ice elastic modulus $E$ was changed while the structure's mass and stiffness, and the ice strength properties, remained the same.

The results show that reducing the value of the elastic modulus leads to a considerable increase in structural reaction loads (in particular, the mean loads) and structural velocities. For example, if ($K_s/hE$) changes from 10 to 100 then the structural reaction load increases by about 1.5 times and the structural velocity by about 3 times. These results are qualitatively consistent with the results of experiments (Huang and Shi, 2007).

A possible explanation is as follows. Figure 1, phase 4 indicates that the ice/structure contact area at some instants can be very narrow or zero. This allows the structure to move against the ice due to the structural stiffness and displacement. The lower the ice elastic modulus (e.g. due to warmer ice) the longer will be the structure path before full contact with ice develops, the greater will be the structure velocity before full contact occurs, and the higher will be both load and displacement.

### *Effect of the structural relative mass*

The relative structural mass (the structural mass *M* on the ice/structure contact area *Dh*) has not been considered before but has an important influence on the response. As an example, the amplitude of the displacement and velocity obtained for three relative masses are plotted in Figure 9. All other parameters (except *D*) are the same. The structure's natural period is 1.6 s, the ice thickness is 1 m. Figure 9a shows that the point where the velocities of ice and structure are equal depends significantly on the relative mass of the structure.

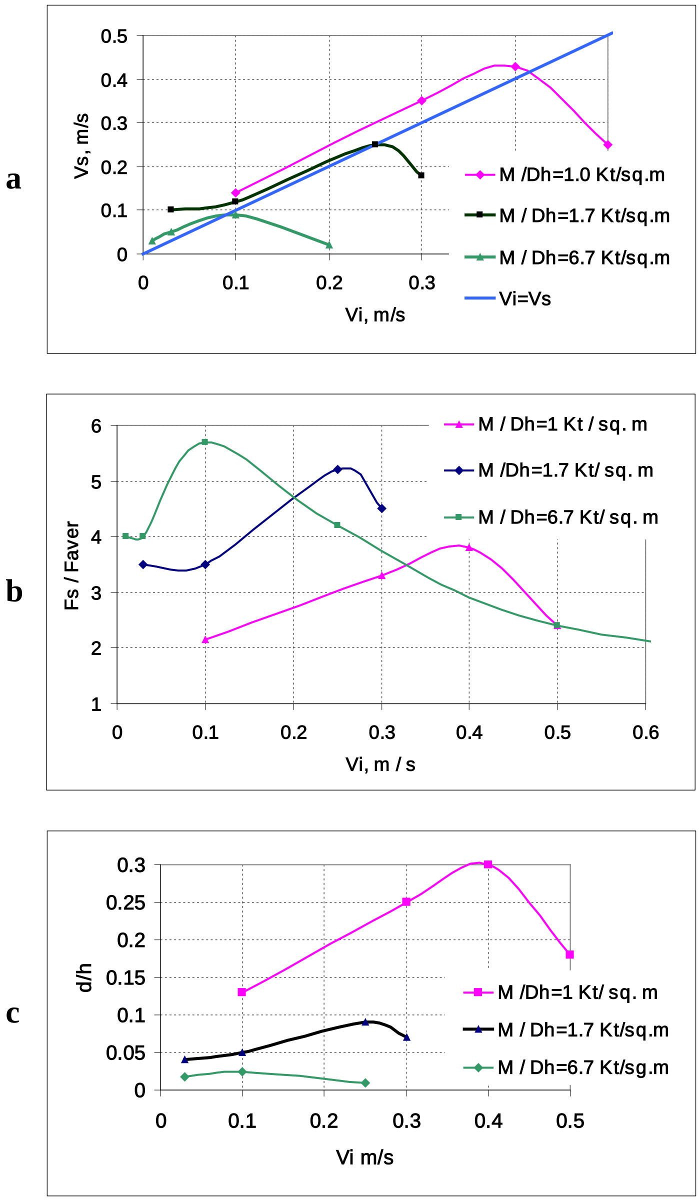


Figure 9. Influence of the structure's relative mass on the structural velocities, amplitudes and response

## COMPARISON OF CALCULATED DATA WITH LABORATORY EXPERIMENTS AND FIELD OBSERVATIONS

As mentioned above the general character of the dependences represented in Figures 4, 7, and 8 qualitatively coincides with the results described by Kärnä and Turunen, 1989 and 1990, Yue and Bi, 2000, Huang and Shi, 2007, and Kärnä et al, 2008; and other sources.

***Loads***

Figure 10 (from Kärnä et al, 2008) is analogous to Figure 5. On the whole, the maximum load amplification factor noted in both studies is more than 3.

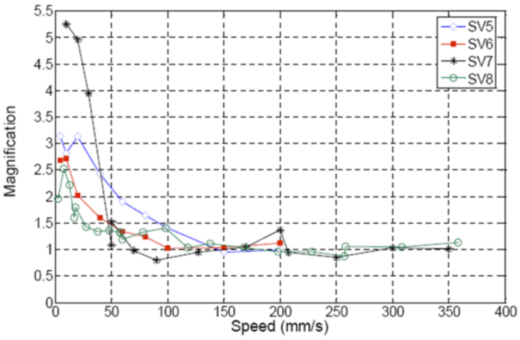


Figure 10. Dependence of load amplification factor on ice velocity (Kärnä et al, 2003)

However, as shown in (Kärnä et al, 2008) the dynamics of the process stops, on average, at ice velocities in the range 0.1 - 0.15 m/s, while in our experiments the absence of dynamics was seen if the ice velocity exceeds 0.5 m/s. Probably this difference is the result in differences of relative structural masses (*M/Dh*, see Figure 9).

The load amplification factor of about 3 – 3.5 was also identified in experiments (Huang and Shi, 2007) but unfortunately, the velocities at which dynamic effects disappeared were not determined.

***Structure velocities***

Comparison of structural velocities ($V_s$) obtained from calculations and from full scale measurements at a lighthouse (Engelbrektson, 1997) are shown in Figure 11.

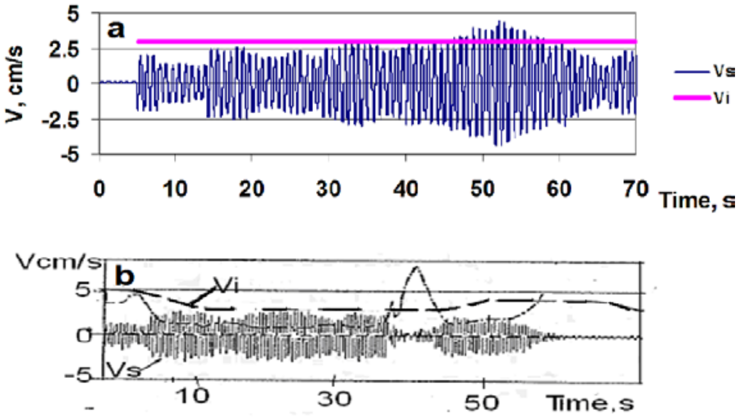


Figure 11. Comparison of structural velocity obtained (a) from calculations, and (b) measured during lighthouse vibrations (Engelbrektson 1997)

In calculations, the ice moved at a velocity of 0.03 m/s (bold straight line in Figure 11a). The vibration in the full scale lighthouse measurements started when the ice velocity, for some reasons, dropped from 0.05 to 0.03 m/s (dashed line in Figure 11b).

Dependence similar to that represented in Figures 7 and 9a can be obtained from other work (Kärnä and Turunen, 1989 and 1990, Yue and Bi, 2000, and Huang and Shi, 2007).

## CONCLUSION

The study comprises an extensive series of numerical experiments to investigate relationships between structural characteristics, ice properties and vibration parameters for ice interactions with fixed flexible vertical structures. The 2D in-plane solution describes multiple failures of ice, extrusion of pulverized particles from the zone of interaction and includes the effect of structure compliance on the interaction process.

The following conclusions can be made from the results obtained:
1. All the results are qualitatively and, in some cases, quantitatively consistent with data from model tests and field measurements. However it should be noted that the quantitative results presented in this paper are valid only for the input data used in these calculations.
2. It is concluded from our work that it is practically impossible to vary in a series of experiments only one of the parameters (other than the ice field velocity) that influence the oscillation phenomenon. Variation of one parameter induces simultaneous change of some others. This conclusion should be taken into account in experimental work.
3. The vibration process is stochastic, e.g. see Figure 11b. A small decrease in ice velocity from 0.05 to 0.03 m/s induced vibration and a subsequent increase in ice velocity eliminated it. The onset of vibration and its level depend upon a combination of many factors, which may vary by region and structure. Currently, only approximate estimations of vibration formation and level can be achieved, and a universal prediction scheme remains a future challenge.
4. Considerable vibration occurs at the critical ice velocity, i.e. when the structure's velocity is close to that of the ice velocity. Within this velocity range, the structural response can be much higher (up to 3 - 3.5 times) than the static reaction, and the amplitude of the oscillations reach a maximum. At ice velocities less than the critical velocity, the structure's velocity may exceed the ice velocity.
5. Some new factors influencing the oscillation were found:

- Vibration is markedly affected by the correlation between the stiffness of the structure/foundation system and the elastic modulus of ice. An increase of the parameter $K_s/hE$ results in growth of both the average load and the maximum reaction. The structure velocities rise in this situation.
- The relative structural mass parameter $M/Dh$ is important and should be taken into consideration when planning laboratory model tests

## ACKNOWLEDGEMENTS

The authors are grateful to BP Exploration Operating Company for financial support of these studies.

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