SECOND INTERNATIONAL CONFERENCE ON
PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
UNIVERSITY OF ICELAND
DEPARTMENT OF ENGINEERING AND SCIENCE

ON THE BOUNDARY CONDITIONS AT THE BAY ENTRANCE IN THE ANALYSES OF BAY WATER OSCILLATIONS

Hitoshi Nishimura, Lecturer Department of Civil Engineering Tokyo
Kiyoshi Horikawa, Professor University of Tokyo Japan

INTRODUCTION

The Japan Islands are located in the circum-Pacific seismic zone, therefore the tsunamis generated by submarine earthquakes have attacked fairly frequently the coasts of Japan facing the Pacific Ocean and occasionally the coasts facing the Sea of Japan. The Chilean earthquake tsunami in 1960, one of the most disastrous distant tsunamis in Japan, induced the tremendous damages especially on the north-eastern coast of the Japanese main-land. The above fact was the trigger to awaken the Japanese Government to lay the tsunami disaster prevention again its keen interest. Since then the various measures for tsunami prevention have been established in the nation wide scale. One of those projects was the construction of tsunami breakwaters at the mouth of Ofunato Bay, Iwate Prefecture. The execution of breakwater construction was proceeded during the period of 1963 through 1967 by the Ministry of Transport in Japan at the cost of about ¥ 1,912 million.

The arrangement of breakwaters is shown in Figure 1. The maximum depth of water at the site of breakwater construction was about 38 m below the Low Water Level, hence a large amount of rubble was mounded up to the elevation of 16.3 m below L. W. L. in order to make the breakwater foundation. Two tide gauges have been installed since the beginning of breakwater construction; one is located at the bay head and the other is at the mouth of bay just outside of the tsunami breakwaters as shown In Figure 1.

In 1968 the Tokachi-Oki earthquake occurred and generated the tsunamis with relatively good size as shown in Figures 2 and 3. In Figure 2 are shown the data on the fluctuation of water surface record-

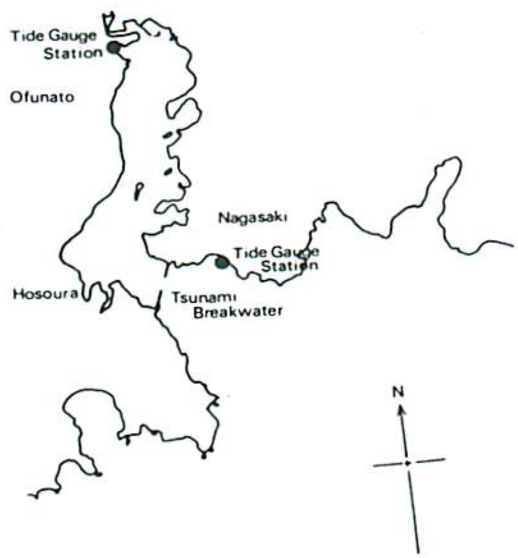

Fig. 1.

Location of tsunami breakwaters and of tide gauges in Ofunato Bay, Iwate Prefecture, Japan.

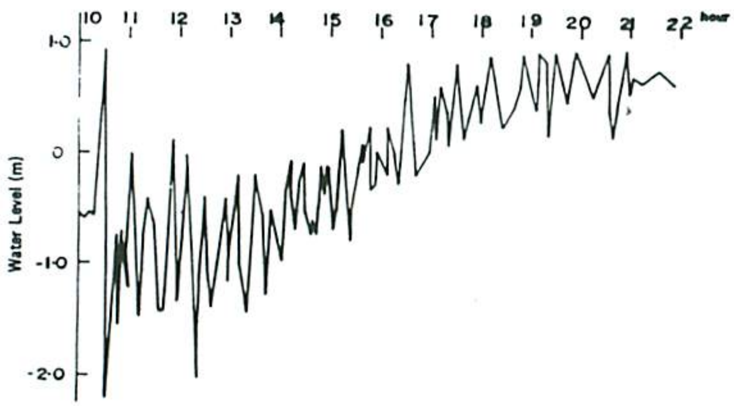

Fig. 2.

Tide gauge record of Tokachi-Oki earthquake tsunami in 1968, obtaine at the entrance of Ofuna Bay.

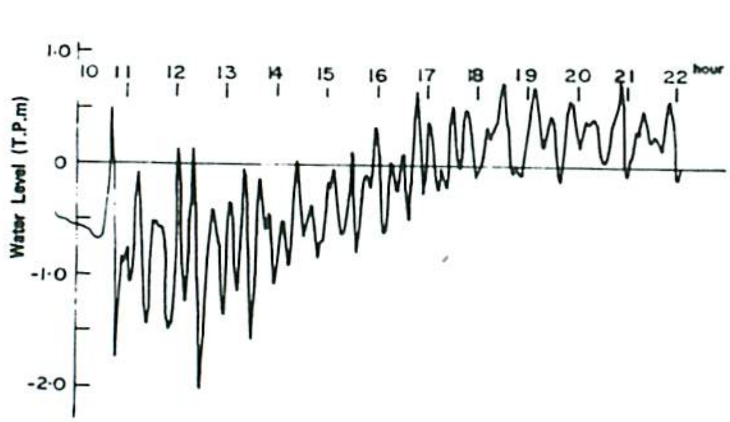

Fig. 3.

Tide gauge record of Tokachi-Oki earthquake tsunami in 1968, obtaine at the head of Ofunato Bay.

ed by the tide gauge at the entrance of bay, and in Figure 3 the data at the head of bay. Eliminating the initial part of these data where the fluctuation of water surface was abnormally large, and using the rest of data, the authors calculated the power spectra of water surface fluctuations at two sites stated above. The result of calculations are given in Figure 4, where the dotted line is the power spectrum at the bay entrance, while the solid line is that at the bay head. From these power spectra the amplification factor at the bay head is calculated on the basis of linear assumption. In Figure 5 the present curve is plotted by dotted line and is compared with the previous result obtained by R. Takahasi, I. Aida, and Y. Nagata[1] prior to the completion of breakwater in 1964. From this diagram it is easily recognized that the response characteristics of Ofunato Bay have awfully been modified by the construction of breakwaters, and that the breakwaters seem to be really effective in cutting the rise of water elevation at the bay head induced by the wave components with period longer than 30 min.

On the other hand the authors have conducted the investigations on the characteristics of bay water oscillation induced by disturbances in the open sea through both laboratory experiments and analytical treatments.[2),3),4),5)] The aim of the present investigation is to find out the validity and the limitation of the simple analytical methods such as the characteristic method[6] in the treatment of the stated problems. Figure 6 gives an example of previous results. The curves shown in this diagram are obtained analytically under the particular conditions of a laboratory model on the basis of the characteristic method. Here d is the opening of breakwaters and b is the width of bay. These curves can be compared with the corresponding curves in Figure 4. The agreement in the qualitative sense seems to be quite satisfactory.

BASIC CONCEPT OF PRESENT TREATMENT

From the fact stated above it may be realized that the one-dimensional treatment of long waves is sometimes powerful enough and is awfully simpler to analyze the bay water oscillations in comparison with the two-dimensional treatment. In such treatment, however, there exists a very important and difficult problem to be solved, namely how to give the boundary conditions at the bay entrance. Although some hypotheses have been suggested, no one of them is appropriate to be applied especially for the resonant conditions.

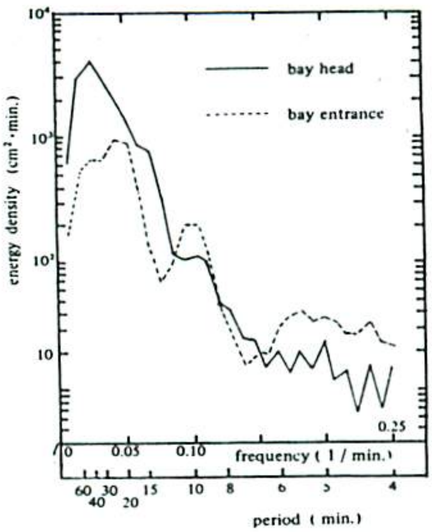

Fig. 4.

Power spectra of water surface fluctuations.

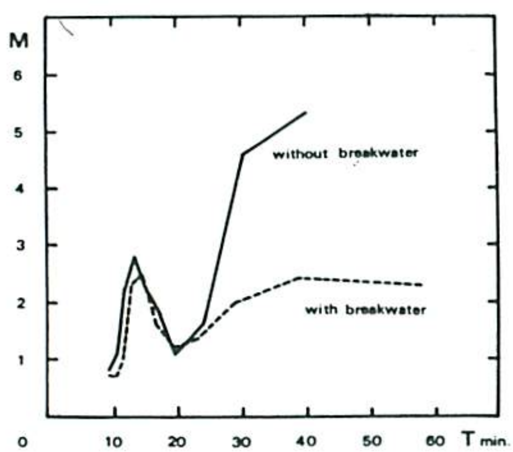

Fig. 5.

Response characteristics of Ofunato Bay before and after the completion of tsunami breakwaters.

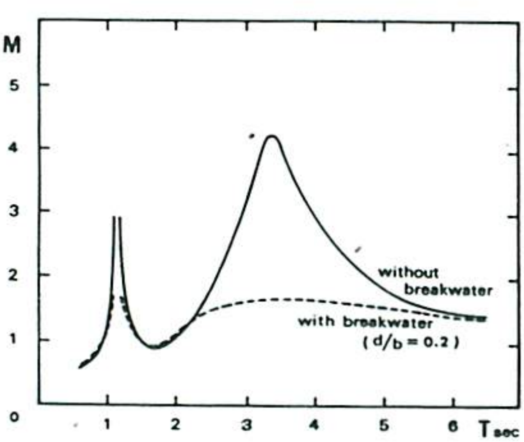

Fig. 6.

Calculated response characteristics under the specified conditions of laboratory model.

In order to simplify the complex oscillations near the bay entrance, the authors consider idealized channel with semi-infinite length which is connected to the open sea. The phenomena of water oscillations inside and outside the bay are thus separated into the following two parts:

1) The energy of waves propagating in the open sea towards the shoreline with normal incident is partly transmitted into the channel, and the rest is reflected towards offshore at the entrance.

2) The energy of waves propagating through the channel towards the entrance is partly dispersed into the open sea, while the rest is reflected at the entrance.

In each case, both the reflection and transmission coefficients are obtained as functions of the ratio of channel width to wave length. It should be noted that these coefficients have complex values, because the reflection and transmission of waves are generally accompanied by the apparent discontinuities of phases which appear due to the application of one-dimensional treatment.

Oscillatory characteristics of bay water are easily investigated for incident waves with various periods by taking into account the reflection of waves at the bay head and superposing the two parts mentioned above. Therefore, the rates of transmission and of reflection determine the amplification factor of bay water oscillations, and the phase lag of reflected waves proves the necessity of "entrance correction".

It should be mentioned here that the similar approaches have been taken by M. Hayakawa[7], and by H. Yamada and S. Ishida[8] in experimental and analytical treatments respectively.

LONG WAVES ADVANCING INTO A SEMI-INFINITE CHANNEL

The coordinate axes of x and y are taken in a horizontal plane as shown in Figure 7, and that of z is taken vertically upward above still water. The water depth h is assumed to be uniform in the whole region.

As it is assumed also that the fluid is inviscid and incompressible, and its motion is irrotational, there exists a velocity potential ϕ which satisfies the following Laplace equation:

$$\nabla^2 \phi = 0 \qquad (1)$$

The velocity potential may be expressed as

$$\phi = i\sigma \cdot \eta(x,y,t) \cdot Z(z) \qquad (2)$$

where σ is an angular frequency. The functions of $Z(z)$ and $\eta(x,y,t)$ should be the solutions of the following equations respectively:

$$\frac{d^2 Z}{dz^2} = k^2 Z \qquad (3)$$

$$\frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} + k^2 \eta = 0 \qquad (4)$$

where k is a wave number.

The solution of Equation (3) under the boundary condition of $(w)_{z=-h} = \left(\frac{\partial \phi}{\partial z}\right)_{z=-h} = 0$ is given by

$$Z = \frac{\cosh k(h+z)}{k \sinh kh} \qquad (5)$$

Under the consideration of $\eta \sim e^{i\sigma t}$, it is recognized that $\eta(x,y,t)$ is the water surface elevation above still water level.

The incident and transmitted waves are written respectively as $\eta_1^- = a_1^- e^{i(\sigma t+kx)}$ and $\eta_2 = \eta_2^- = \mathbb{T} a_1^- e^{i(\sigma t+kx)}$. Here $\mathbb{T}$ is a transmission coefficient. The horizontal velocity u_2 corresponding to η_2 is formulated as $u_2 = -\mathbb{T} k\sigma Z a_1^- e^{i(\sigma t+kx)}$. In the present treatment, it is assumed that the width of channel b is small in comparison with the wave length L, hence it is considered that the wave is uniform in the direction of channel width. The wave reflected perfectly from the shoreline is expressed by $\eta_1^+ = a_1^- e^{i(\sigma t-kx)}$ and the disturbed wave is η_*. Therefore the wave pattern in the open sea η_1 and the corresponding horizontal velocity u_1 are written as follows:

$$\eta_1 = 2a_1^- \cos kx \cdot e^{i\sigma t} + \eta_* \qquad (6)$$

$$u_1 = -2ik\sigma Z a_1^- \sin kx \cdot e^{i\sigma t} + i\sigma Z \frac{\partial \eta_*}{\partial x} \qquad (7)$$

Under the conditions of 1) u_1 being a constant in the region of $x = 0$ and $|y| \le b/2$ (b : width of channel) and zero in $x = 0$ and $|y| > b/2$, and 2) $\eta_* = 0$ at infinity, η_* is solved as follow

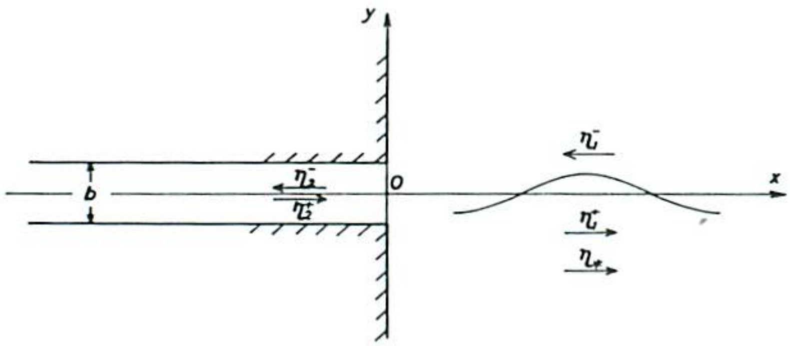

Fig. 7.

Definition sketch (I).

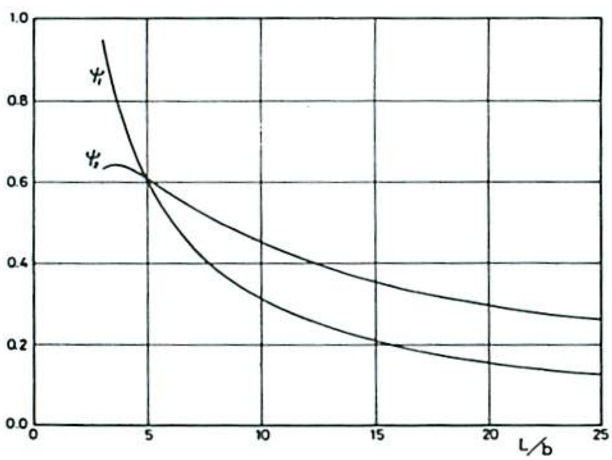

Fig. 8.

Functions of ψ_1 and ψ_2 .

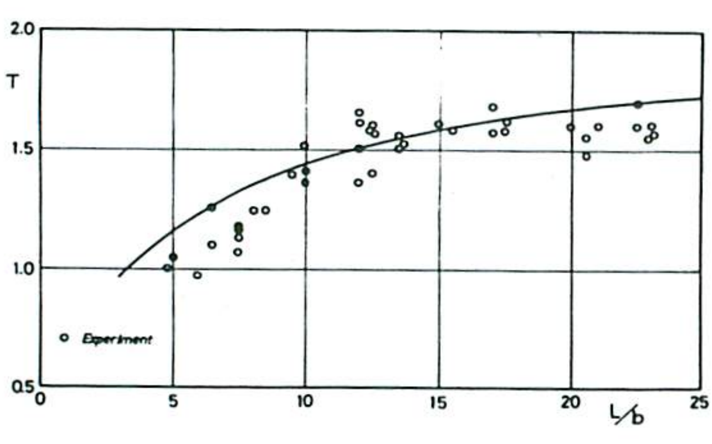

Fig. 9 (a).

Theoretical curve for the amplitude of complex transmission coefficient and experimental data.

$$\eta_*(x,y,t) = C\left[\frac{2i}{\pi}\int_0^1 \frac{\sin(\xi kb/2)\cos\xi ky}{\xi\sqrt{1-\xi^2}}\, e^{-i\sqrt{1-\xi^2}\,kx}\, d\xi \right.$$

$$\left. - \frac{2}{\pi}\int_1^\infty \frac{\sin(\xi kb/2)\cos\xi ky}{\xi\sqrt{\xi^2-1}}\, e^{-i\sqrt{\xi^2-1}\,kx}\, d\xi\right] e^{i\sigma t} \qquad (8)$$

Hence $\eta_*(0,0,t)$ and $u_1(0,0,t)$ are given by

$$\eta_*(0,0,t) = C\left[\frac{2i}{\pi}\int_0^1 \frac{\sin(\xi kb/2)}{\xi\sqrt{1-\xi^2}}\, d\xi - \frac{2}{\pi}\int_1^\infty \frac{\sin(\xi kb/2)}{\xi\sqrt{\xi^2-1}}\, d\xi\right] e^{i\sigma t}$$

$$= C(i\psi_1 - \psi_2)\, e^{i\sigma t} \qquad (9)$$

$$u_1(0,0,t) = ik\sigma CZ e^{i\sigma t} \qquad (10)$$

Here ψ_1 and ψ_2 are functions of the ratio between wave length L and the channel width b as shown in Figure 8.

By using the continuity conditions of water surface and horizontal velocity at $x = 0$, $y = 0$ such as $\eta_1 = \eta_2$ and $u_1 = u_2$, under the condition of relatively small width of channel, the complex transmission coefficient $\mathbb{T}$ is obtained as

$$\mathbb{T} = Te^{i\theta_T} = \frac{2}{\psi_1 + 1 + i\psi_2} \qquad (11)$$

The amplitude and phase of $\mathbb{T}$ are given in Figure 9(a) and Figure 9 (b) respectively. The open circles plotted in Figure 9 (a) are the experimental data, and they are in good agreement with the theoretical curve. In the laboratory experiment the water depth and width of channel were 10 cm and 20 cm respectively, and the wave height was less than 1 cm.

REFLECTION OF LONG WAVES PROPAGATING TOWARDS OPEN SEA

The wave progressing through a channel towards open sea is $\eta_2^+ = a_2^+ e^{i(\sigma t - kx)}$ and the wave reflected at the entrance of channel is $\eta_2^- = \mathbb{R} a_2^+ e^{i(\sigma t + kx)}$, where $\mathbb{R}$ is a complex reflection coefficient (See Figure 10). Therefore the composite wave in the channel η_2 is expressed as follows:

$$\eta_2 = \eta_2^+ + \eta_2^- = a_2^+(e^{-ikx} + \mathbb{R}e^{ikx})\cdot e^{i\sigma t} \qquad (12)$$

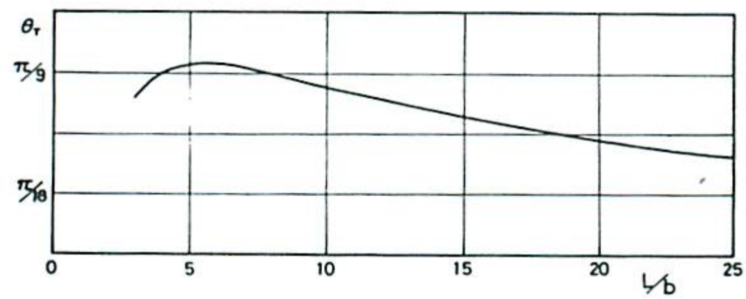

Fig. 9 (b).

Theoretical curve for the phase of complex transmission coefficient.

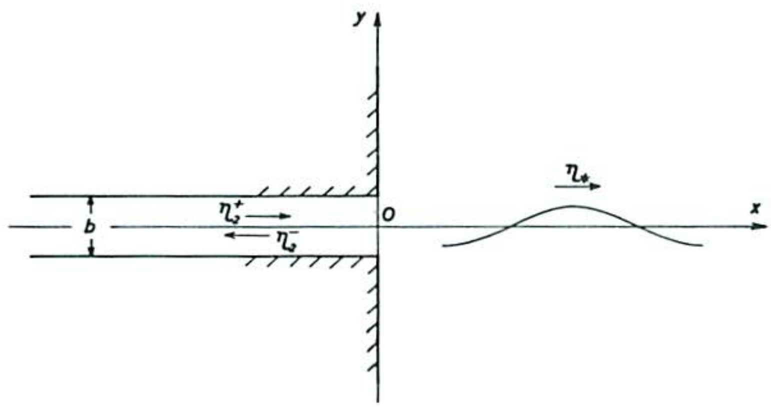

Fig. 10.

Definition sketch (II).

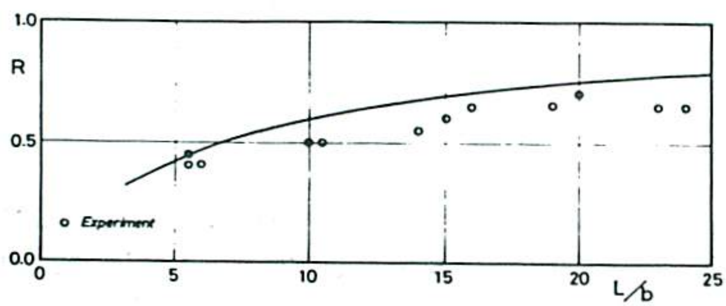

Fig. 11 (a).

Theoretical curve for the amplitude of complex reflection coefficient and experimental data.

that is a partial clapotis; while the corresponding current velocity u_2 is given by

$$u_2 = k\sigma Z a_2^+ (e^{-ikx} - \mathbb{R} e^{ikx}) \cdot e^{i\sigma t} \tag{13}$$

The disturbed wave η_* in this case is also expressed by Equation (8)

The continuity conditions of water elevation η and horizontal velocity u at the entrance of channel $x = 0$ give the complex reflection coefficient as follows:

$$\mathbb{R} = -Re^{-i\theta_R} = \frac{\psi_1 - 1 + i\psi_2}{\psi_1 + 1 + i\psi_2} . \tag{14}$$

The theoretical curves for the amplitude and the phase of $\mathbb{R}$ are compared with the experimental data in Figure 11 (a) and Figure 11 (b) respectively. The agreement between them is quite satisfactory.

K. Kajiura[6] introduced the effective width b_e in the open sea in the one-dimensional analyses of the bay water oscillation, and expressed the transmission coefficient T and the reflection coefficient R as functions of the ratio between the bay width b and the effective width b_e as follows:

$$T = \frac{2}{1 + (b/b_e)} \tag{15}$$

$$R = \frac{1 - (b/b_e)}{1 + (b/b_e)} \tag{16}$$

By using Equations (11) and (15) or Equations (14) and (16), the ratio b_e/b is expressed as a function of L/b. The relationship derived from the transmission coefficient is not the same as that derived from the reflection coefficient as shown in Figure 12. From this fact it may be recognized that the effective width introduced by K. Kajiura is only an expedient index.

RESONANCE CHARACTERISTICS OF BAY WATER OSCILLATION

The incident long wave in open sea is expressed as $a_1^- e^{i(\sigma t + kx)}$, and the wave invading into a bay is written as $\mathbb{T} a_1^- e^{i(\sigma t + kx)}$. The latter wave is reflected perfectly at the head of bay. Then the reflected wave progresses towards the open sea and is re-reflected partly at the entrance of bay. Such phenomena are repeated infinitely in this region,

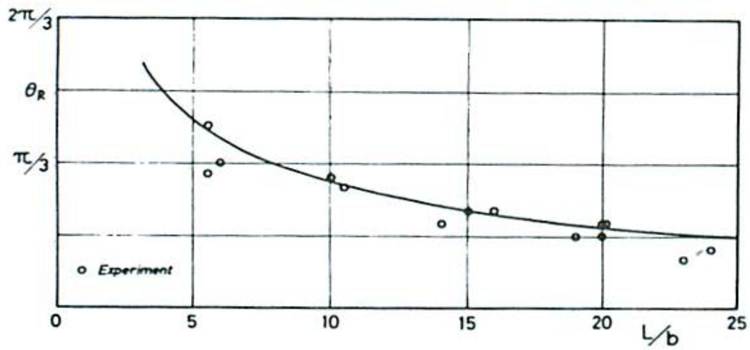

Fig. 11 (b).

Theoretical curve for the phase of complex reflection coefficient and experimental data.

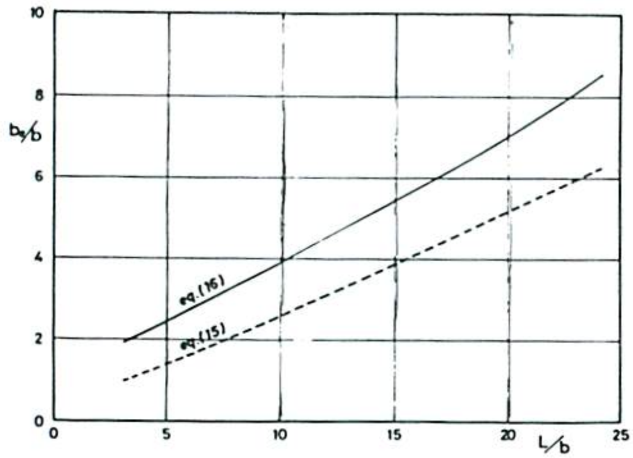

Fig. 12.

The ratios between the effective width b_e and the channel width b determined theoretically on the basis of transmission and reflection coefficients respectively.

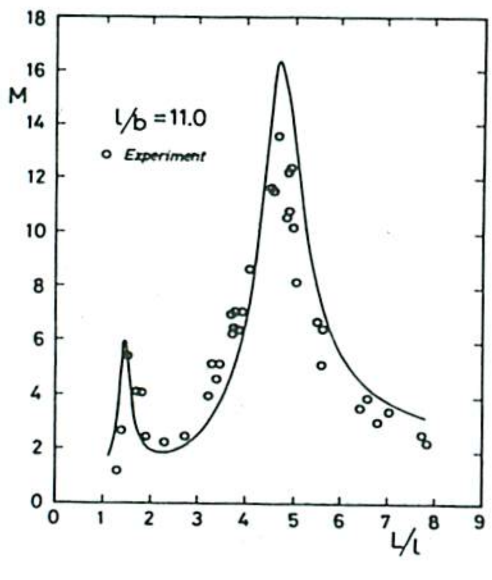

Fig. 13.

Response curve determined theoretically under the specified condition of laboratory model and the corresponding experimental data.

hence the water surface elevation at the bay head reaches finally to such a stationary state as expressed by

$$\eta_2 = 2T a_1^- e^{i(\sigma t - kl)} \sum_{n=0}^{\infty} R^n e^{-i \cdot 2nkl}$$

$$= 2T a_1^- e^{i(\sigma t - kl)} / (1 - R e^{-i \cdot 2nkl}) \tag{17}$$

where l is the distance between the entrance and the head of the present bay. Therefore the amplification factor M is given as follows:

$$M = \left| \frac{\eta_2}{a_1^-} \right| = \frac{2T}{\sqrt{1 + R^2 + 2R \cos(2kl + \theta_R)}} \tag{18}$$

From the above equation the correction for the bay length Δl is expressed approximately as

$$\Delta l = \frac{\theta_R}{2k} \tag{19}$$

In order to investigate the applicability of the above treatment, a sample calculation for the response curve of bay water oscillation was made under a specified condition, and was compared with the laboratory data which were obtained previously at the Coastal Engineering Laboratory, University of Tokyo. The agreement between the theoretical curve and the laboratory data is quite satisfactory as shown in Figure 13.

CONCLUSIONS

The purpose of present investigation is mainly to analyze the complex phenomena of bay water oscillation by using the rather simple method such as the one-dimensional analyses. In this case the establishment of boundary coditions at the bay entrance is the main difficulty to be overcome. In order to solve the stated problem the authors treat the following two cases analytically and verify their appropriateness by comparing them with laboratory data. The first case is that the wave in the open sea is coming towards the entrance of semi-infinite channel with normal incidence, and the second case is that the wave propagating through the channel is coming towards the open sea.

In the treatment of the above two cases, the authors introduce a complex transmission coefficient and a complex reflection coefficient respectively. By using the results of present treatment, the response characteristics of bay water oscillation are easily calculated. The theoretical curve thus obtained under the specified conditions is quit satisfactory to represent the tendency of laboratory data.

ACKNOWLEDGEMENT

This study was supported in part by the grant of the Ministry of Education, Japan. Appreciation is extended to the valuable assistance given to the authors by Mr. Y. Goda, Port and Harbour Research Institute, Ministry of Transport.

REFERENCES

1) Takahasi, R., I. Aida and Y. Nagata : The submerged long-wave recorders and the obsevation of the seiches in Ofunato Bay, Jour. Oceanographical Soc. of Japan, Vol. 22, No. 1, Feb. 1966. (In Japanese)
2) Horikawa, K. and H. Nishimura : Characteristic oscillation in a basin with a branch, Coastal Engineering in Japan, Vol. 11, 1968.
3) Horikawa, K., N. Shuto and H. Nishimura : Characteristic oscillation of water in an L-shaped bay, Coastal Engineering in Japan, Vol. 12, 1969.
4) Horikawa, K. and H. Nishimura : On the function of tsunami breakwaters, Coastal Engineering in Japan, Vol. 13, 1970.
5) Nishimura, H., K. Horikawa and N. Shuto : On the function of tsuna breakwaters (2), Coastal Engineering in Japan, Vol. 14, 1971.
6) Kajiura, K. : Effects of a breakwater on the oscillations of bay water, Bulletin, Earthquake Research Institute, University of Toky Vol. 41, 1963. (In Japanese)
7) Hayakawa, M. : Studies on tsunamis, Report Nos. 1-3, Umi To Sora (Sea and Sky), Vol. 21, Nos. 7, 9 and 12, 1941. (In Japanese)
8) Yamada, H. and S. Ishida : Boundary condition and energy dissipation at the bay entrance in harbour hydraulics, Report No. 26, Institute of Applied Mechanics, Kyushu University, 1967. (In Japanese)
9) Ippen, A. T. and Y. Goda : Wave induced oscillations in harbors, Report No. 59, Hydrodynamics Laboratory, M. I. T. , 1963.