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NUMERICAL CALCULATIONS OF LONG-PERIODIC OSCILLATIONS AT TWO NORWEGIAN HARBOURS

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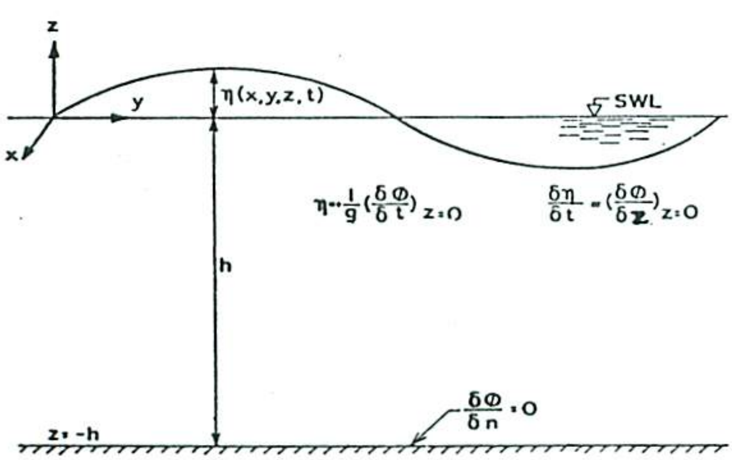
INTRODUCTION

Long periodic oscillations are experienced in many ports of the world. In future planning of harbours, more knowledge about this phenomenon is of great interest. Most harbours have irregular geometrical shapes and variations in depth. Hence simple calculation models covering regular geometry are inadequate. Today a few numerical calculation models exist which consider irregular shapes. Expensive and time consuming model tests might be avoided by further development and testing of these numerical models.

This paper describes a model, developed by J.J. Lee and F. Raichlen, used on two Norwegian fishing ports, the port of Sørvær in Finmark, and the port of Sirevåg in Rogaland. Calculated respons curves are compared to prototype and model measurements.



Definition sketch of the coordinate system.

Fig. 1

Waves in the harbour area comprise:

1) Incoming waves
2) Reflected waves } (f_2)

A potential of the form

$$\phi(x,y,z,t) = -\frac{1}{i\sigma} f(x,y)\ Z(z)\ e^{-i\sigma t} \tag{1}$$

is assumed.

Combining this potential with the Laplace equation and the assumption of constant depth, gives the following differential equations:

$$\frac{\partial^2 Z}{\partial z^2} - k^2 Z = 0 \tag{2}$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + k^2 f = 0 \tag{3}$$

Equation (3) is called the Helmholz equation after a German mathematician. Boundary conditions for solving equations (2) and (3) are:

a) The linearized surface condition

$$\eta = -\frac{1}{g}\left(\frac{\partial \phi}{\partial t}\right)_{z=0}$$

b) The linearized kinematic surface condition

$$\frac{\partial \eta}{\partial t} = \left(\frac{\partial \phi}{\partial t}\right)_{z=0}$$

c) The solid boundary condition

$$\frac{\partial \phi}{\partial n} = 0$$

d) The radiation condition (the radiated wave)

$$\lim_{r\to\infty} f_r = 0$$

e) The boundary condition at the bottom

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{for } z = -h$$

The solution of equation (2) is:

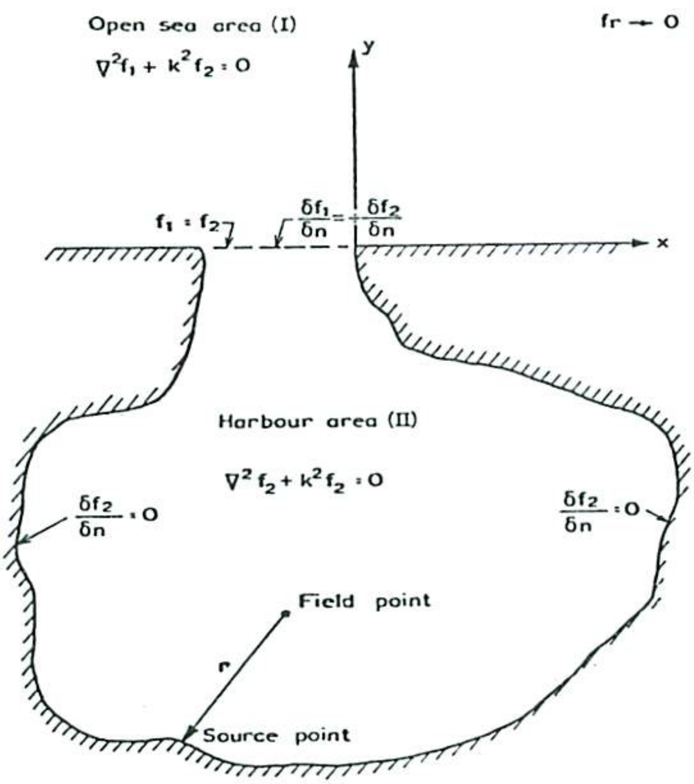
$$Z(z) = -ag\,\frac{\cosh k(z+h)}{\cosh(kh)}$$

where a is amplitude of incoming wave
g is the acceleration of gravity

THEORY

The theoretical simplifications involved in the model are shown in the following short résumé of the mathematical equations and the boundary conditions for solving these equations.

The coordinate system is as indicated in Fig. 1. Fig. 2 is a schematic sketch of an arbitrary shaped harbour described in the model. A straight shoreline with constant depth outside as well as inside the harbour is assumed.



Definition sketch of an arbitrary shaped harbour

Fig. 2

The area of interest is divided in two parts:

I) The open sea
II) The harbour

Waves in the open sea comprise:

1) Incoming waves (f_i)
2) Reflected waves (f_r)
3) Waves radiated from the harbour opening (f_{rad})

where f denoted the corresponding wave function.

One method of solving equation (3) is application of the theory of sources and drains. That is: Sources and drains of power satisfying the above mentioned boundary conditions are spread all over the boundary. Every point on the boundary will then act as a wave generator, generating waves which combined give the unknown wave conditions inside the harbour.

One solution which satisfies the boundary conditions is:

$$f_2 = -\frac{i}{4} \int_s \left| f_{2b} \frac{\partial A}{\partial n} - A \frac{\partial f_{2b}}{\partial n} \right| ds \tag{4}$$

where

$A = H_0^{(1)}$ (kr) is a Hankel function of the first kind and zeroth order.

f = wave function

f_b= wave function on the boundary

s = denotes integration around the boundary and subscript 2 denotes region II

Equation (4) shows that f_2 is a function of the still unknown source strengths f_{2b} and $\frac{\partial f_{2b}}{\partial n}$. To find these quanteties a matching procedure was developed:

a) In the open sea area the wave function is composed of

$$f_i + f_r + f_{rad}$$

Only incident waves at right angles to the straightlined coast are considered. This gives

$$f_i = f_r = \frac{1}{2} e^{iky}$$

where the factor $\frac{1}{2}$ is chosen for reasons of simplicity.

b) Equation (4) is developed numerically for both regions. For the open sea s denotes integration along the straightlined coast (x-axis).

c) The continuity conditions

$$f_1 = f_2 \qquad \text{and} \qquad \frac{\partial f_1}{\partial n} = -\frac{\partial f_2}{\partial n}$$

are valid at the harbour opening. It is then possible to calculate the source power f_{2b} and $\frac{\partial f_{2b}}{\partial n}$ around the boundary. The wave function f_2 is then found by equation (4), and finally the potential is determined by equation (1).

The wave amplitude in the harbour is:

$$\eta_{II} = a\ f_2\ e^{-i\sigma t}$$

The wave amplitude of the standing wave in the open sea area is:

$$\eta_I = a(f_i + f_r)e^{-i\sigma t} = a\ 1\ e^{-i\sigma t}$$

The amplification factor is defined as η_{II} divided by η_I:

$$F = \frac{|\eta_{II}|}{|\eta_I|} = \frac{|a\ f_2\ e^{-i\sigma t}|}{|a\ 1\ e^{-i\sigma t}|} = |f_2|$$

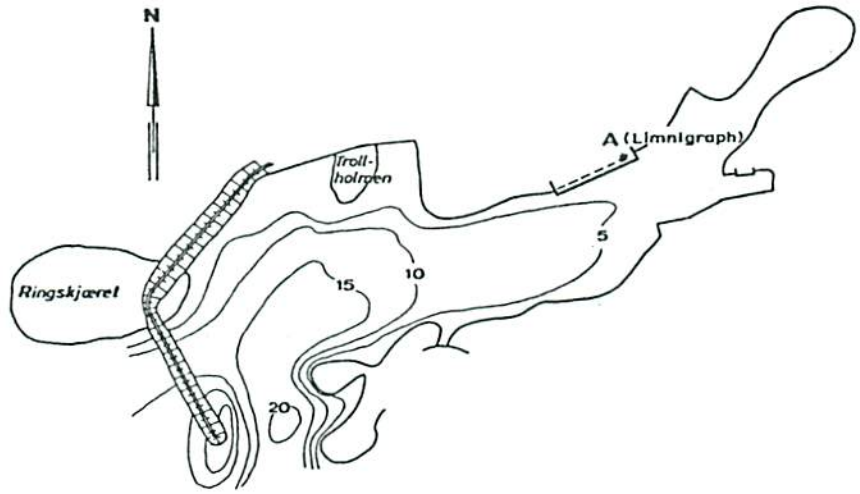
where f_2 is a complex number.

DESCRIPTION OF SØRVÆR HARBOUR

Fig. 3

The physical environment at Sørvær Harbour is shown in Fig. 3. The harbour is located on the south-west side of a large island called

Sørøya in Finnmark, of which only a small part is shown in the figure. The harbour itself is sited inside some small islets in the bay Markaila. The coastline surrounding the harbour has a very irregular geometrical shape, with rocky and rather steep slopes.



Map of SØRVÆR harbour

Fig. 4

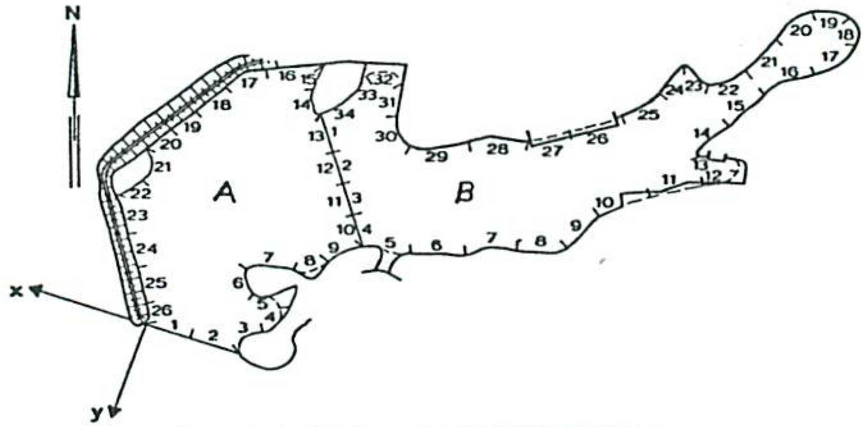
Fig. 4 shows the layout of the harbour. It is protected by a rubble mound breakwater, built in 1970-71. A wave-recorder was installed at the wharf (point A in the figure) in August 1970. The total length of the harbour is approximately 860 meters, and the numbers shown are the water depth in meters.

NUMERICAL CALCULATIONS

Based on the above mentioned mathematical equations, J.J. Lee and F. Raichlen developed a computer program where the total harbour area was subdivided into smaller parts. To use this program directly, Sørvær harbour was divided in two basins, A and B as shown in Fig. 5.

The harbour boundary, approximated by straightlined segments is also shown in the figure. The average depth of basin A is about 18 m, and of basin B about 6 m.

The result is a function of the water depth used in the calculations. For Sørvær harbour the average depth, 12 m, is chosen, but for comparison d = 20 is also used. Fig. 6 and 7 show the response curves for d = 12 m and d = 20 m respectively. Both show three typical resonance periods. As it could be expected, the only effect of decreasing



Numerical diviation of SØRVÆR harbour

Fig. 5

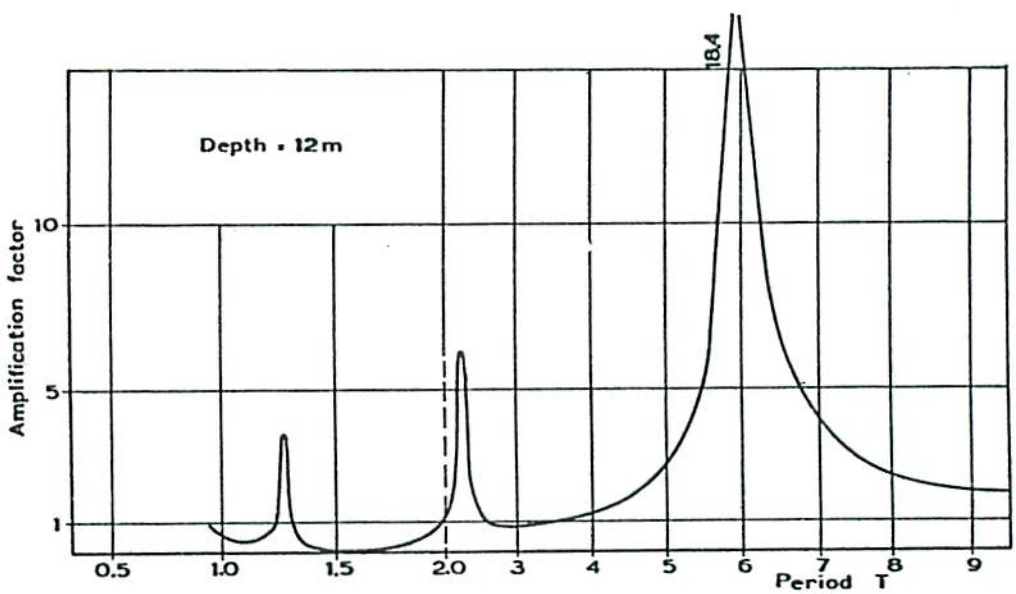


Fig. 6 Responsecurve, SØRVÆR

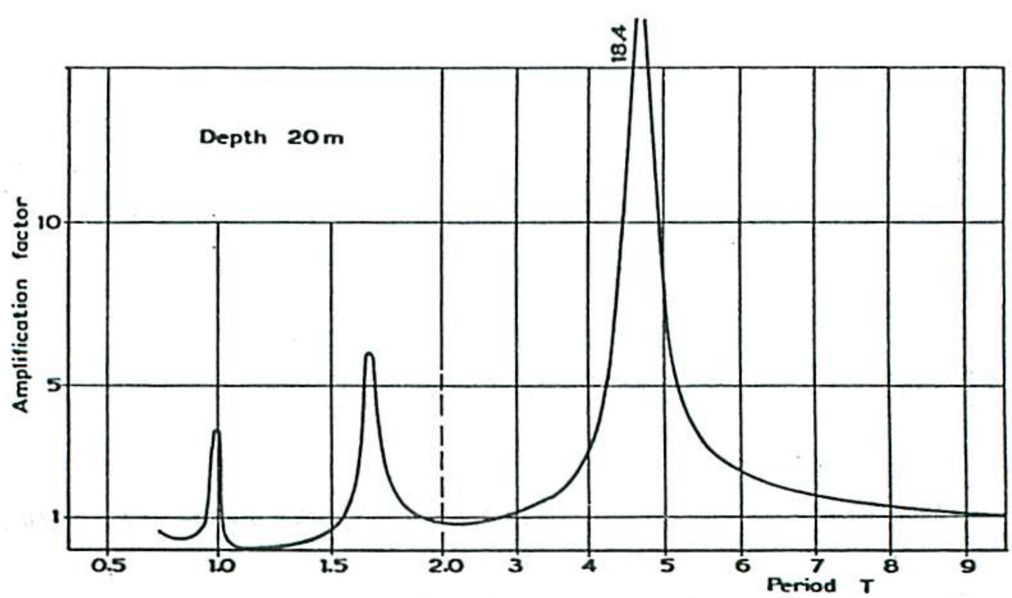
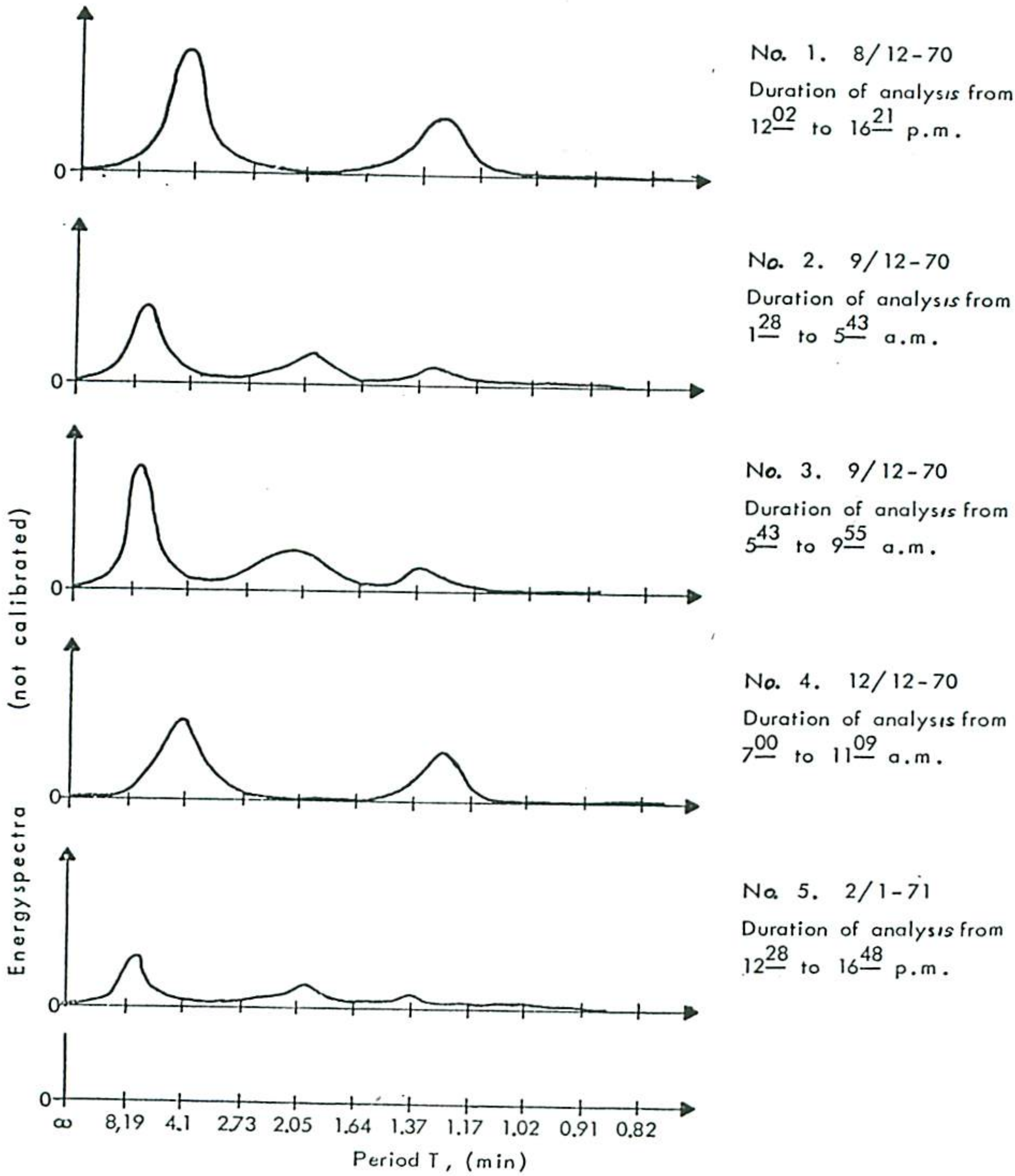


Fig. 7 Responsecurve, SØRVÆR.

the water depth is to increase the resonance periods. Amplification factors at resonance remain unchanged.

Prototype data have been collected since August 1970. Five different periods of severe wave-activity are chosen and analysed. The corresponding spectra are shown in Fig. 8.



ENERGYSPECTRA FOR LIMNIGRAM. SØRVÆR HARBOR.

Fig. 8

Theoretically calculated and prototype measured response periods are compared in Table 1.

Table 1 Theoretically calculated and Prototype measured response

Calculations		Measurements	
D = 12	D = 20 m	1 & 4 case	2.3. & 5. case
1.3 min 2.1 " 6 "	1.0 min 1.7 " 4.5 "	1.3 min 5 "	1.3 min 2.1 " 8 "

1) Calculations give three of the four resonance periods observed. The fourth period probably is the resonance period for the Markaila bay.

2) Three resonance periods are caused by resonance in the harbour. If a water depth d = 12 m is used, resonance periods are in good agreement with those measured.

WAVE PROFILE AT RESONANCE

For a harbour exposed to long periodic waves it is of great practical importance to determine the most critical places in the harbour. Wharfs and mooring devices should be placed where velocities are lowest.

The horizontal stream velocity can be expressed as follows:

$$u = \sqrt{u^2 + v^2} = \frac{ag}{\sigma} \sin\sigma t \sqrt{\left|\frac{\partial f_2}{\partial x}\right|^2 + \left|\frac{\partial f_2}{\partial y}\right|^2} \qquad (5)$$

where $u = \text{Real}\ (\frac{\partial\phi}{\partial x})$

$v = \text{Real}\ (\frac{\partial\phi}{\partial y})$

Eq. (5) shows that the highest current velocities occur where the gradients $\frac{\partial f_2}{\partial x}$ and $\frac{\partial f_2}{\partial y}$ are at maximum. For standing waves this means at the nodes. Figs. 9 and 10 show the wave profile at the resonance periods T = 1.3 min and T = 2.1 min respectively. The numbers given in the figures are $|f_2|/|f_2|_{max}$. Looking at the profile along the axes of the harbour direction, it may be noted that there are two nodes in the profile for the 1.3 min period and one node for the 2.1 min period. The figures show roughly the most critical places. Stream velocities may have been calculated, but any further investigation should include knowledge on the frequency of occurrence of the different resonance periods, which is not yet available.

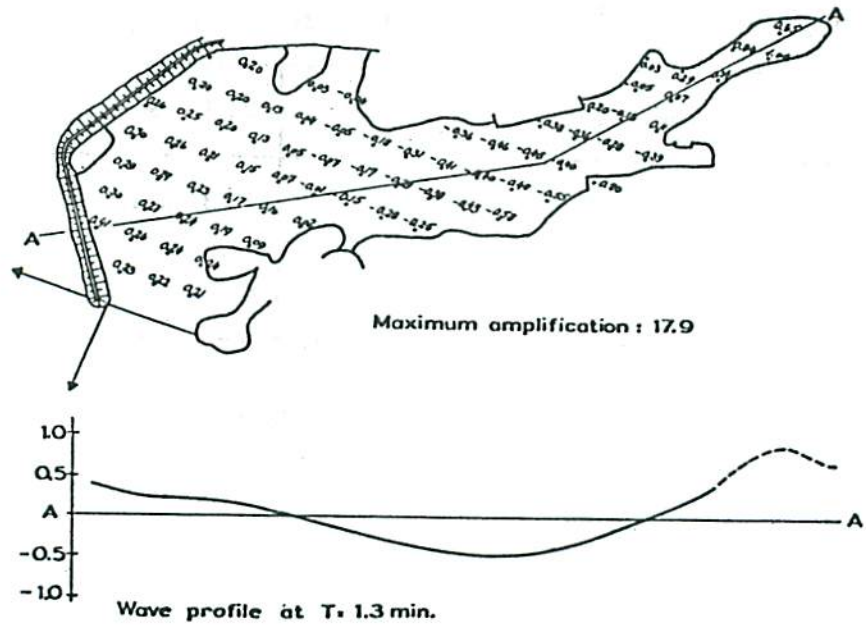


Fig. 9

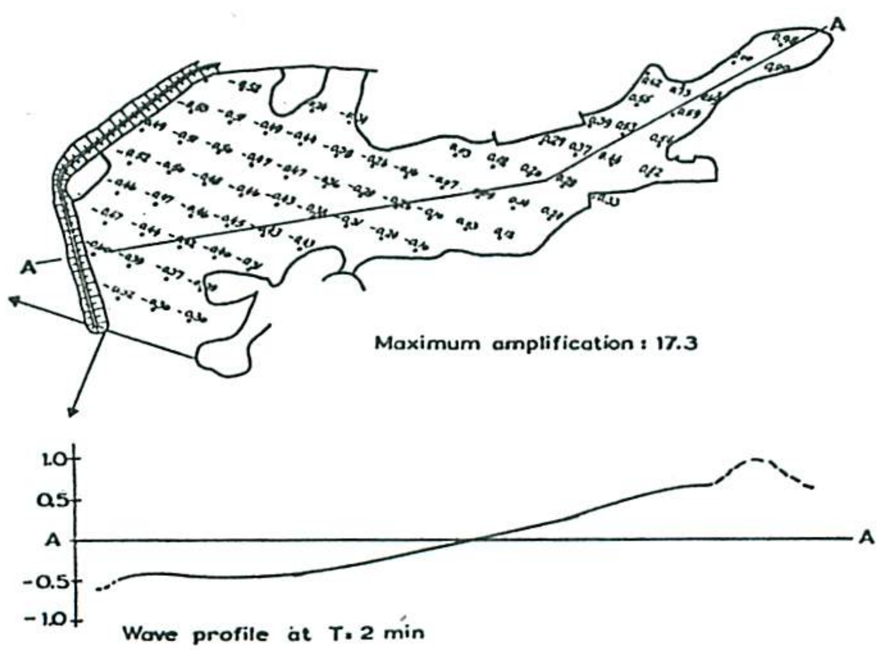
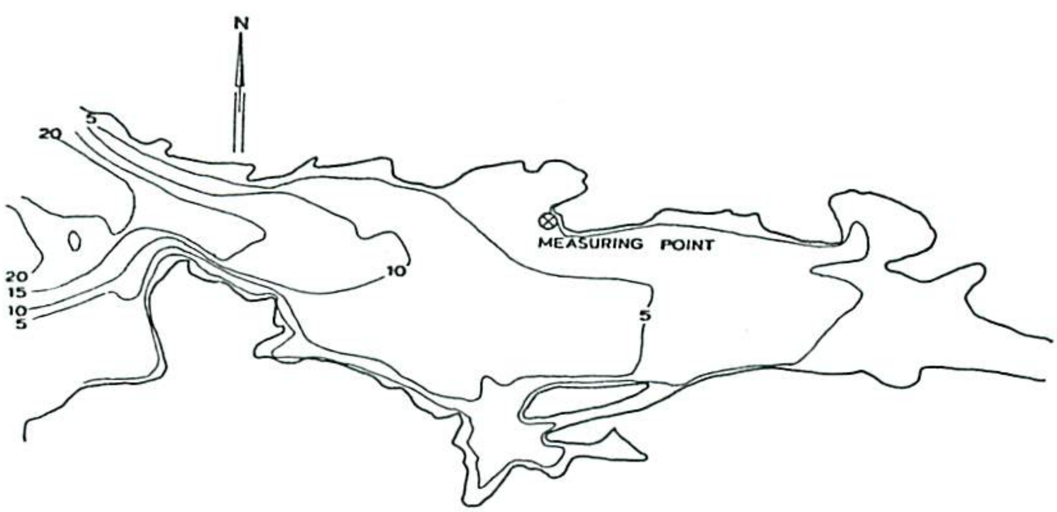


Fig. 10

COMPARISON BETWEEN CALCULATIONS AND MODEL TESTS

Sirevåg harbour Fig. 11, is located on the southwest coast of Norway. Like Sørvær it is exposed to severe long periodic wave action. Actually there are some geometrical similarities between Sirevåg and Sørvær. Both harbours are long, with irregular geometry. Shorelines are irregular, rocky with rather steep slopes. Both harbours may be classified as typical fishing ports on the Norwegian coast.

In 1962 River and Harbour Laboratory at the Norwegian Institute of Technology carried out some model tests on long periodic oscillations in Sirevåg harbour. 14 different breakwater design and positioning were tested, but with little effect on the harbour oscillations.



SIREVÅG Harbour without breakwater

Fig. 11

Several reasons make comparison with these model tests less valuable than in the case of Sørvær.

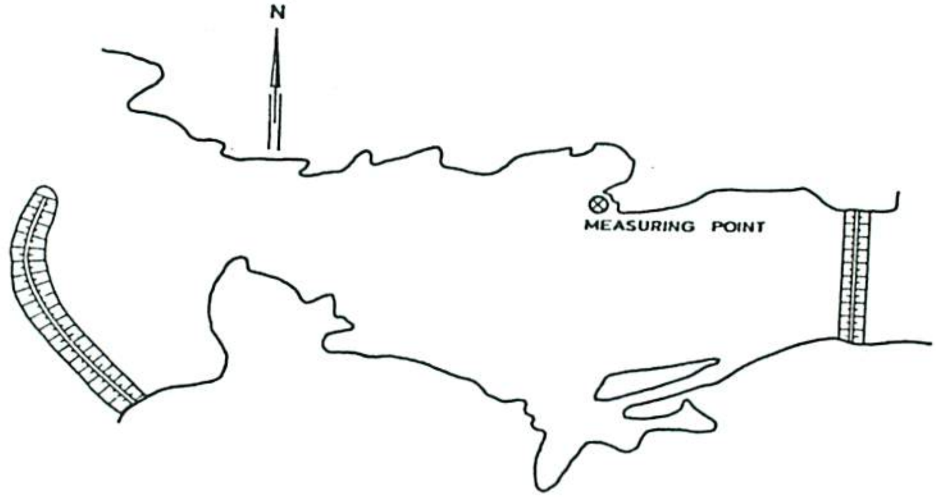
1) The laboratory model of Sirevåg harbour were built for short-periodic wave measurements. The tests with long periodic waves were run afterwords without changing the model.

2) Model scale effects.

3) The amplification factor is defined in different ways for the numerical model and the model tests.

4) Only a few points on the response curve were measured by the tests. It is then possible that maximum amplification may have been higher than measured.

In spite of these uncertainties, two of the alternatives were chosen for comparison with the calculated response curves, one without breakwater the other with the breakwater alternativ shown in Fig. 12. The corresponding response curves are shown in Figs. 13 and 14. Most interesting here is the comparison between the amplification factors at resonance. Having in mind point 3 and 4 in the before mentioned review of possible uncertainties the difference undoubtedly is mostly caused by friction. The comparison is shown in table 2.



SIREVÅG Harbour, alt. with breakwater

Fig. 12

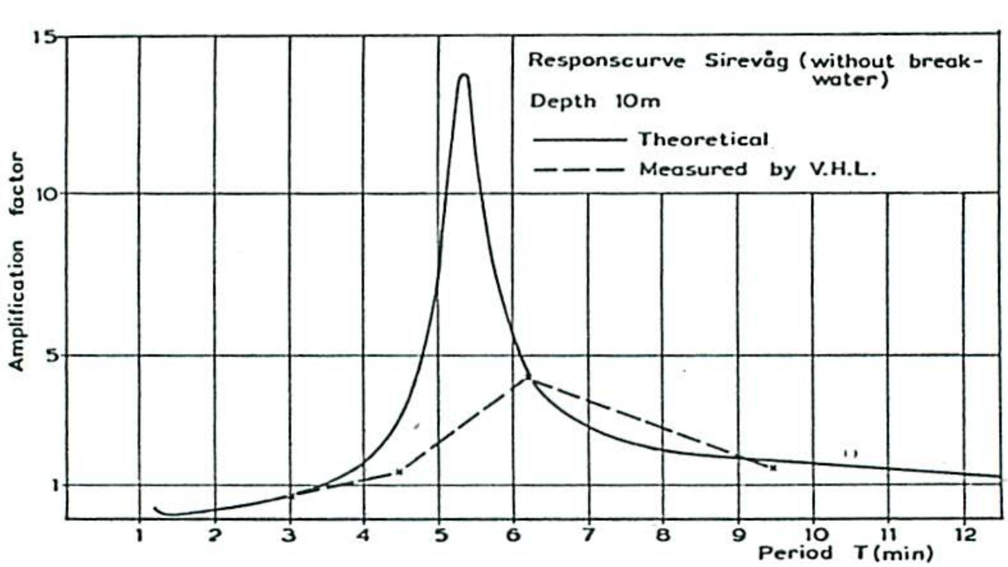


Fig. 13

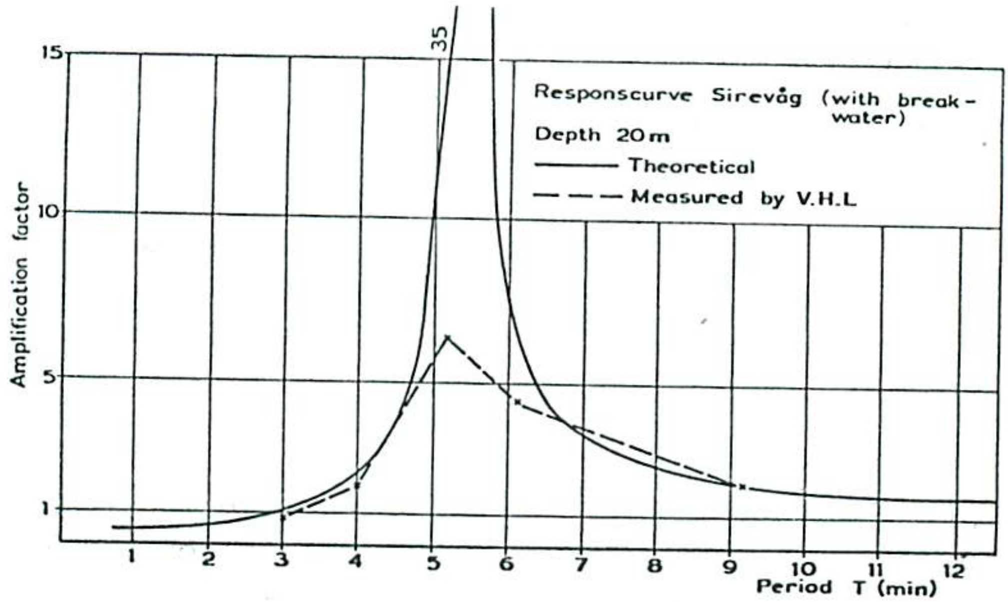


Fig. 14

		Resonance period		Amplification factor	
		Calculated	Measured	Calculated	Measured
With breakw.	d = 20 m	5.1	5.2	35.4	6.3
Without breakw.	d = 10 m	5.5	6	13.6	4.3

Table 2

The calculated amplification factor turned out to be about 3 to 6 times larger than for the model tests.

For comparison: An ordinary springmass-dashpot system has an amplification factor which is 6 times larger for the damping coefficient 0.3 than for the damping coefficient 0.05, at resonance.

DISCUSSION AND CONCLUSIONS

The numerical model contains some simplifications the most important assumptions being:

1) Friction neglected

2) Constant depth, both outside and inside the harbour

3) A straight and vertical coast

Re 1

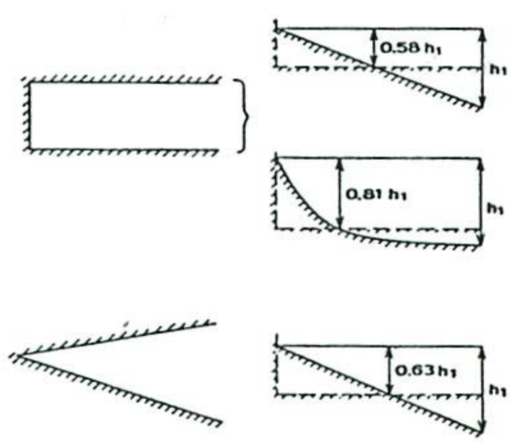
Lee and Raichlen (1) have shown that friction has little effect on the resonance period. There is only slight increase in the resonanc period when friction is considered. The amplification factor is influenced much more, (compare the springmass-dashpot system) consequently a reduction of the calculated value at resonance of about 3 - 6 times is not surprising. Of course this value depends on the particular harbour geometry, but is an indication of what we can expect for other similar harbours.

Re 2

The assumption of constant depth influences the resonance period, and as shown in Figs. 5 and 6 the calculated resonance periods depen strongly upon the depth chosen. Fig. 15 shows some harbours of simple geometry. The equivalent constant depth harbour, a harbour giving the same resonance period as for the harbour of variable dept is shown in the same figure. A depth of 0.6 to 0.8 times the depth at the harbour opening will in these cases give the correct resonanc period. For Sørvær the calculated resonance period was in good agreement with the prototype measurements for d = 12 m, which is about the average depth for the harbour.

Re 3

Straightlined and vertical coast is a very rough simplification, which is most important for the shorter wave periods. For these waves it is difficult to attain the regular standing wave system in the open sea area. It is hard to say how much this simplification influences the results. From an engineering point of view, however, the results must be classified as relatively satisfactory.



Constant depth for harbours of simple geometry.

Fig. 15

REFERENCES

1. J.J. Lee "Wave inducted oscillations in Harbours of arbitrary shape". Report KH-R-20, W.M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

2. J.J. Lee og F. Reichlen "Wave inducted oscillations in harbors with connevted Basins". Report KH-R-26, W.M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

3. Bernard Lé Méhauté og Li-San Hwang "Harbor design: Scale model or Computer?" Tetra, Tech. Institute Pasadena California

4. River and Harbour Laboratory, Norwegian Institute of Technology "Wave measure mento and Model tests, Sirevåg Harbour". Report No F-924, 1960-62 (In Norwegian)

5. Wilson, Hendrickson og Kilmer "Feasibility study for a surge-action model of Montery harbor, California". Science Engineering Associates. San Marino, California.

6. H. Rye og G. Viggosson "Long periodic oscillations at the port of Sørvær, The Lopp sea, Norway". POAC-conference, Trondheim, 1971.

7. Lee-San-Hwang og Kenneth Olsen "Oscillations in a Bay of Arbitrary Shape and Variable Depht". Vol. 76, No. 21 Journal of Geophysical research, July 1971.