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UNIVERSITY OF ICELAND
DEPARTMENT OF ENGINEERING AND SCIENCE

NUMERICAL WAVE FORECASTING AROUND ICELAND

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INTRODUCTION

This paper describes a numerical ocean wave forecasting program which has been developed at the Science Institute of the University of Iceland. The first part of the paper includes a brief description of the basic equations and the numerical method. This part is naturally universal in nature and applies no more to the ocean around Iceland than to other parts of the world´s ocean. In the second part the application of the model to the ocean area around Iceland is discussed and some numerical examples presented.

OCEAN WAVE GROWTH

The irregular sea surface is represented by its directional spectral density function $S(\bar{x},\theta,\sigma;t)$, where $\bar{x}$ denotes coordinates for a point on the surface, θ is direction of wave component, σ its circular frequency and t is time. The governing equation for the spectral density function is (see Refs. 1 and 2)

$$\partial S/\partial t + \nabla\cdot(S\bar{v}) - Q = 0 \qquad (1)$$

where $\bar{v}$ is a four dimensional velocity vector given by

$$\begin{Bmatrix} v_x \\ v_y \\ v_\sigma \\ v_\theta \end{Bmatrix} = \begin{Bmatrix} c_g \cos\theta \\ c_g \sin\theta \\ \partial\sigma/\partial t \\ \frac{c_g}{c}\left(\frac{\partial c}{\partial x}\sin\theta - \frac{\partial c}{\partial y}\cos\theta\right) \end{Bmatrix} \qquad (2)$$

and the vector operator ∇ has the components

$$\nabla = \{\partial/\partial x, \partial/\partial y, \partial/\partial\sigma, \partial/\partial\theta\} \qquad (3)$$

It will be assumed that no energy flow takes place between different frequency components which makes $v_\sigma = 0$. Assuming further that deep water conditions apply the component v_θ also vanishes resulting in the simple equation

$$\partial S/\partial t + c_g \cos\theta \, \partial S/\partial x + c_g \sin\theta \, \partial S/\partial y - Q = 0 \qquad (4)$$

which is the wave forecasting equation used for example by the French Meteorological Service in their socalled DSA-method (see Refs. 3 and 4). The term Q represents the net rate of energy inflow to the spectral density function, i.e. it represents the basis for wave growth.

Theoretical studies of wind wave growth have been made by O.M. Phillips (Refs. 5 and 6) and by J.W. Miles (Refs. 7-11). Phillips' analysis, the socalled resonance theory, gives initial wave growth linear with time, whereas Miles' shear flow theory results in exponential wave growth with time. Both the resonance mechanism and the shear flow mechanism are probably at work in building up the wind sea, but it appears that both theories, although presumably qualitatively correct, result in wave growth that is approximately an order of magnitude less than that observed. Investigators in this field have therefore used a semiempirical approach where the basic form of the Phillips' and Miles' equations, i.e. the linear and exponential growth terms respectively, is retained with constants chosen so as to agree reasonably well with observed values. The term Q is then expressed in the form

$$Q = P(\bar{k},\sigma) + M(\bar{k},\sigma)S(\bar{x},\theta,\sigma,t) \qquad (5)$$

where P and M stand for Phillips and Miles. The factor P is given by the equation

$$P = \frac{\alpha \, \sigma^3 \, U^6 \, K^{2.23}}{[\gamma^2 + (k_x - K)^2]\,[\delta^2 + k_y^2]}, \; m^2 \qquad (6)$$

where $\alpha = 1.357\cdot10^{-16}$

$k_x = k/\cos\theta/$, m^{-1}

$k_y = k/\sin\theta/$, m^{-1}

$k = \sigma^2/g$, m^{-1}

θ = angle measured from wind direction

$\gamma = 0.33\cdot K^{1.28}$

$\delta = 0.52\cdot K^{0.95}$

$K = \sigma/U$, m^{-1}

σ = circular frequency of wave component

U = wind velocity, m/s

In the above equation for P, Barnett's representation (Ref. 12) with U^6 has been adopted.

The factor M is taken from Inoue (Ref. 13) and has the form:

$$M = \sigma \{2.22\cdot 10^{-4} \exp[-70000(u_*/c - 0.031)^2] + 0.119(u_*/c)^2 \exp[-0.0004(c/u_*)^2]\} \quad (7)$$

where $u_* = (\tau_0/\rho_a)^{1/2}$ = friction velocity

τ_0 = wind shear stress at sea surface

ρ_a = density of air

c = phase velocity of wave

Barnett (Ref. 12) uses a somewhat simpler expression for M and a comparison of the two forms is shown in Fig. 1, where the theoretical curve based on a combined Miles-Phillips mechanism is also shown. Inoue's expression seems to be based on a broader foundation than Barnett's which determined its choice.

With the values for Q as given by (5) the wave spectrum will grow indefinitely. It is known, however, that dissipation of energy is always present, so that the waves with given wind conditions will reach a maximum height where the energy received from the wind is just enough to offset the dissipated energy. This is the stage called a fully developed sea, and although there is some doubt whether the fully developed sea really exists, many observed data seem to confirm the existence of such a stage. Phillips (Ref. 14) has proposed the form of the fully developed spectrum given by

$$S_\infty(\sigma,\theta) = (\alpha g^2/\sigma^5)H(\theta) \quad (8)$$

where α is a universal constant, H is the angular spreading function and g is the acceleration of gravity. A modified form of this expression based on a great number of actual ocean wave spectra was proposed by Pierson and Moskowitz (Ref. 15)

$$S_\infty(\sigma,\theta) = (\alpha g^2/\sigma^5) \exp[-\beta(\sigma_0/\sigma)^4]H(\theta) \quad (9)$$

with $\alpha = 8.1\cdot 10^{-3}$

$\beta = 0.74$

$\sigma_0 = g/U$

Inoue (Ref. 13) obtained limited wave growth by expressing the wave growth function as follows:

$$Q = \{ P(\bar{k},\sigma) [1-(S/S_\infty)^2]^{1/2} + M(\bar{k},\sigma)S \} [1-(S/S_\infty)^2] \quad (10)$$

where equation (9) is used for the fully developed spectrum. This expression includes no dissipation for example if the wind stops. Inoue added a dissipation term for those components of the spectrum propagating against the wind.

Barnett (Ref. 12) includes wave dissipation on the form

$$\partial S/\partial t = G - \tau S \qquad (11)$$

where G and τ are integral functions of S based on Hasselmann's theory for nonlinear wave-wave interactions.

The French DSA wave forecasting model (Refs. 3 and 4) takes turbulence as the main factor causing dissipation where the following expression, based on Lamb's (Ref. 16, page 624) theory for the dissipation of wave energy, is used

$$(\partial S/\partial t)/S = -7.5 \cdot 10^{-6}\ H_s^2 \cdot \sigma^4 \qquad (12)$$

In the present model this same equation was used but with a slightly different numerical constant:

$$(\partial S/\partial t)/S = -\ 6.0 \cdot 10^{-6}\ H_s^2 \sigma^4 \qquad (13)$$

This completes the description of the wave forecasting model.

NUMERICAL MODEL

The equations of wave growth and attenuation were solved using finite difference techniques. A square coordinate grid was drawn on a polar stereographic projection of the northern hemisphere as shown in Fig. 2. The grid spacing chosen is around 150 km. Three of the north-Atlantic weather ships are located in the forecast area:

Ship	Geogr. Location	Grid Point
Alpha	62°N, 33°W	6,5
India	59°N, 19°W	11,2
Metro	66°N, 02°E	17, 8-9

The spectra are calculated for all possible combinations of 15 different wave frequencies or periods ranging from 4.5 sec to 25.7 sec and 12 different wave directions. The time step was set at 2 hours which is the maximum allowed to maintain computational stability.

The model provides for variable northern boundary of the computational area to allow for changes in the drifting sea ice edge.

The numerical model was tested by hindcasting wave conditions during a storm which occurred on December 17-18, 1970. At that time a wave measuring buoy was recording wave conditions near Dyrhólaey just south of Iceland (close to grid point 11.05) which offers a check of the numerical results.

A series of weather maps from the storm period is presented in Fig. 3. A deep low pressure area approached Iceland from southwest on December 17 to 18 resulting in strong winds and high waves south of the country.

Computations were carried out for the period from 12:00 GMT on December 17, 1970 until 02:00 GMT on December 23, 1970. The initial conditions were determined from a wave analysis facsimile chart received at the Iceland Meteorological Office from the British Meteorological Office in Bracknell. From this chart (see Fig. 4) significant wave heights were estimated and assigned to each grid point. The spectrum was obtained by assuming that the waves were the result of a fully developed Pierson-Moskowitz spectrum. The directional distribution was based on a $\cos^2\theta$-distribution with the central direction coinciding with that of the wind.

Comparison of numerical results with wave height reports from the three weather ships Alpha, India and Metro is presented in Figs. 5,6 and 7. The results obtained for Weather Ship Alpha (Fig. 5) show a reasonably good agreement between observed and computed wave heights. The general trend of the records is similar but due to the inherent uncertainty of the visually observed and reported wave heights the accuracy of the computed values cannot be estimated.

Wave heights interpolated from the facsimile charts of wave height analysis received from Bracknell by the Icelandic Meteorological Office are also plotted on Figure 5. These heights are naturally not of a very high accuracy and the discrepancy between these and the wave heights observed visually onboard the Alpha is obvious. In general it can be said, however, that the observed, computed and analysed wave heights show a common trend which is about all that can be said with the limited data available from the region.

The computed wave heights of Weather Ship India do not show a very good agreement with the observed or analysed wave heights (Figure 6). This is to be expected, however, since this station is at the boundary of the computational model, where the artificial boundary conditions are applied and those are bound to introduce some errors. With better information from adjacent areas it is to be expected that improved results will be obtained at the boundary points.

The results for Weather Ship Metro are shown in Figure 7. Here the computed and observed wave heights are seen to be in fairly good agreement for the first two days but after that the computed values are consistently lower than the observed ones. The analysed records do not at all agree with the others in the beginning but come closer towards the end at the computational period.

Figure 8 presents computed wave heights at grid point 11.05

which is close to Dyrhólaey where a wave measuring buoy was located during this period. The measured wave heights are also plotted in Fig. 8. It is clear that the initial conditions were not at all accurate but aside from that the agreement between computed and measured values is not bad.

CONCLUSIONS

The results obtained by the numerical wave forecasting program described in the paper indicate that the numerical model gives wave heights in fair agreement with visually observed and measured wave heights in the area. Artificial boundary conditions imposed at boundary points at sea will, however, introduce errors making results there less reliable than those for interior points.

The numerical computations were made on the University Science Institute IBM-1620 computer. Each run consisting of evaluating the wave spectrum at all grid points in the computational area for one time step of two hours took about one hour. This is of course too long a time to make the model practical for the forecasting of waves. This problem, however, will be eliminated by a larger and faster computer, IBM-370/135 which recently has been installed at the State and Municipal Computing Center in Reykjavík. It is planned to adapt the model for this computer in the near future.

In order to eliminate the uncertainty at the southern edge of the computational grid, arrangements must be made for exchange of information with other countries making wave forecasts in the North Atlantic. This way the boundary conditions on the southern boundary will be more reliable. It is likely that such information can be obtained from the U.S. Fleet Weather Facility, the British Meteorological Office and from the Norwegian Meteorological Office.

When such cooperation has been started, the wave forecasting model described here could in the not too distant future be put into use. Wave forecasts for the ocean around Iceland will then be possible on a routine basis along with other environmental forecasts made by the Icelandic Meteorological Office. This will be of great importance for increased efficiency and added safety both for the Icelandic fishing and merchant fleets.

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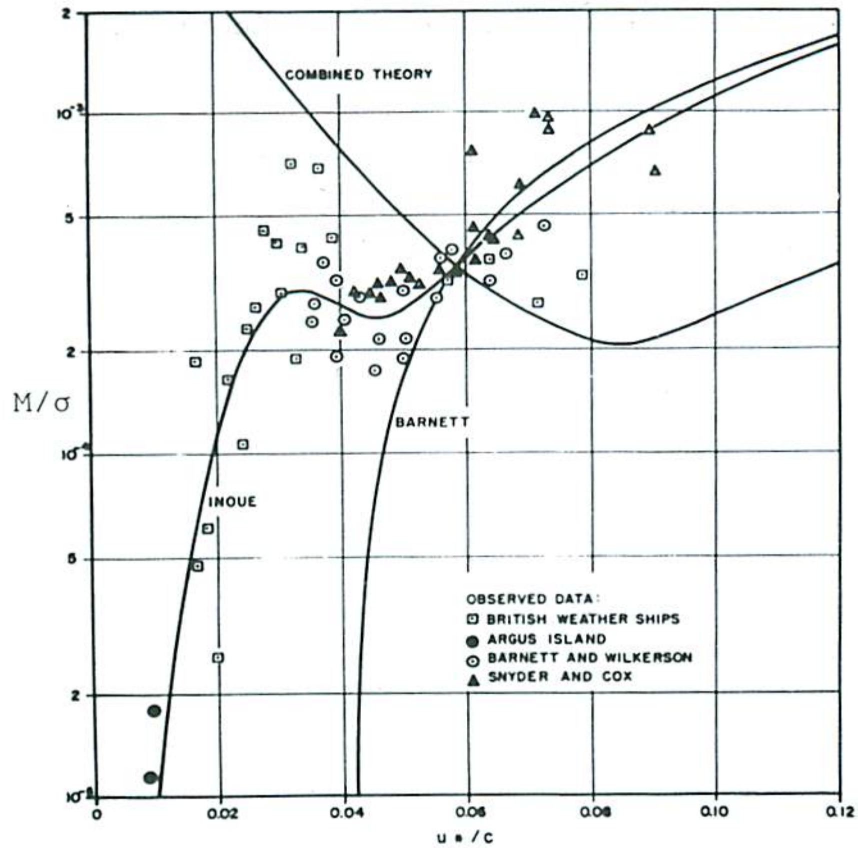


Figure 1. Theoretical and empirical growth rate curves

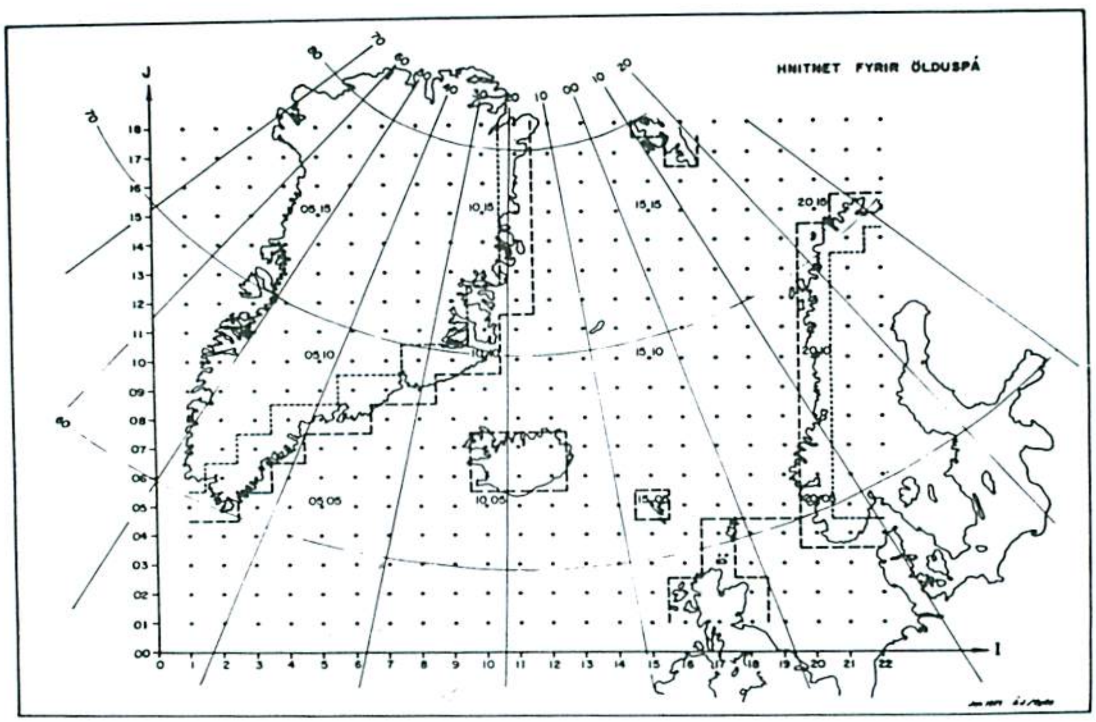


Figure 2. Coordinate grid for numerical wave forecasting model

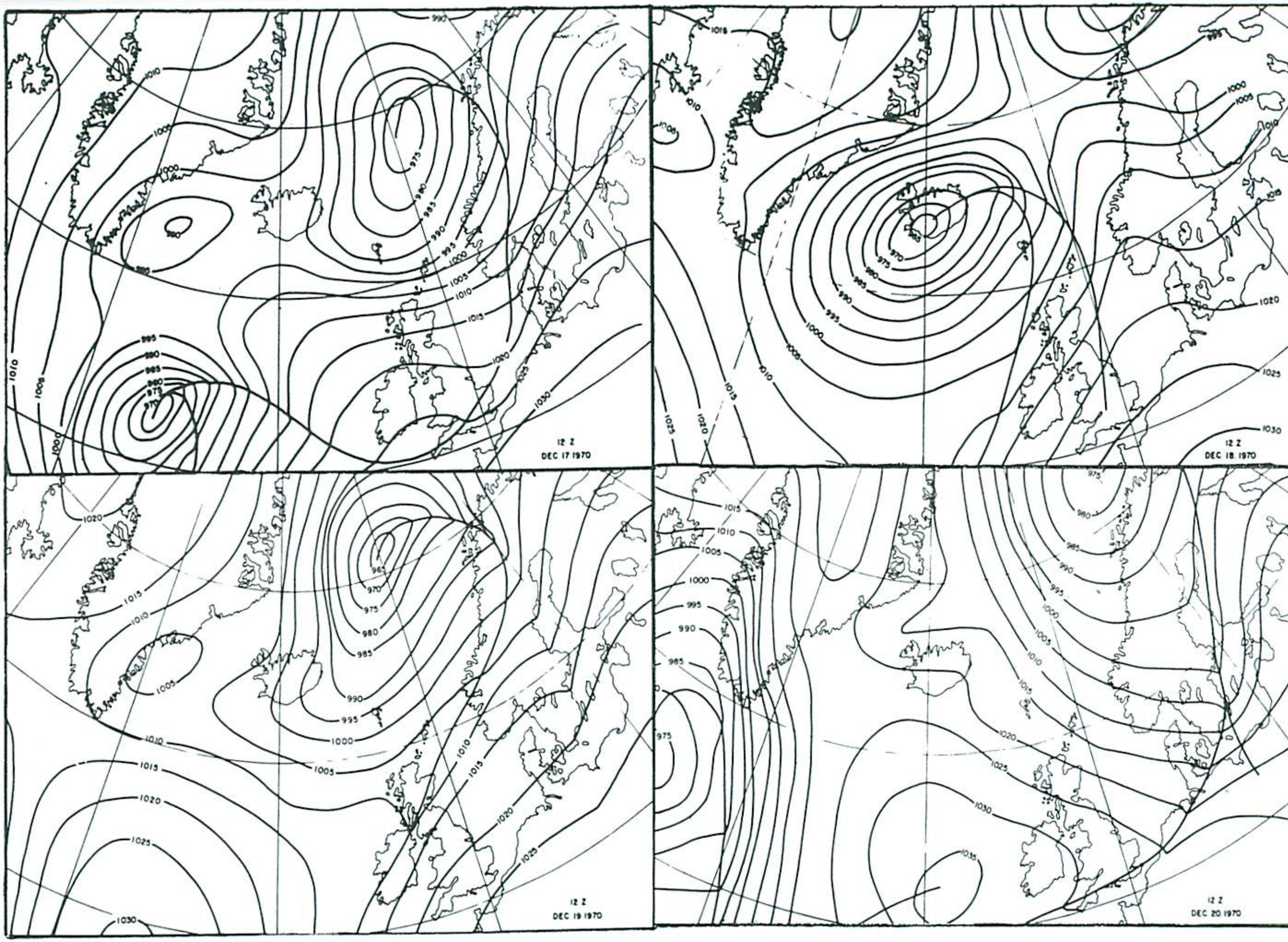


Figure 3. Weather maps from 17-20 December 1970

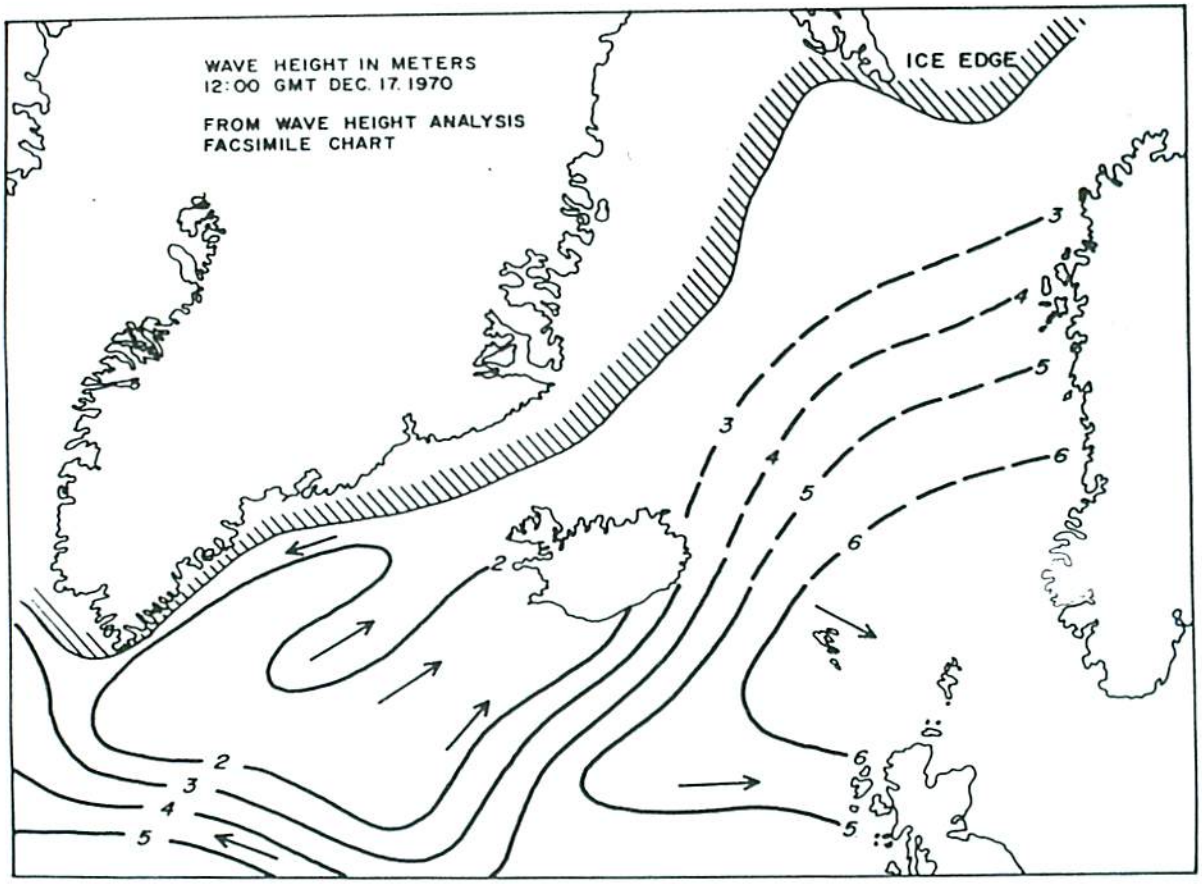


Figure 4. Wave height distribution at 12:00 GMT on Dec. 17, 1970.

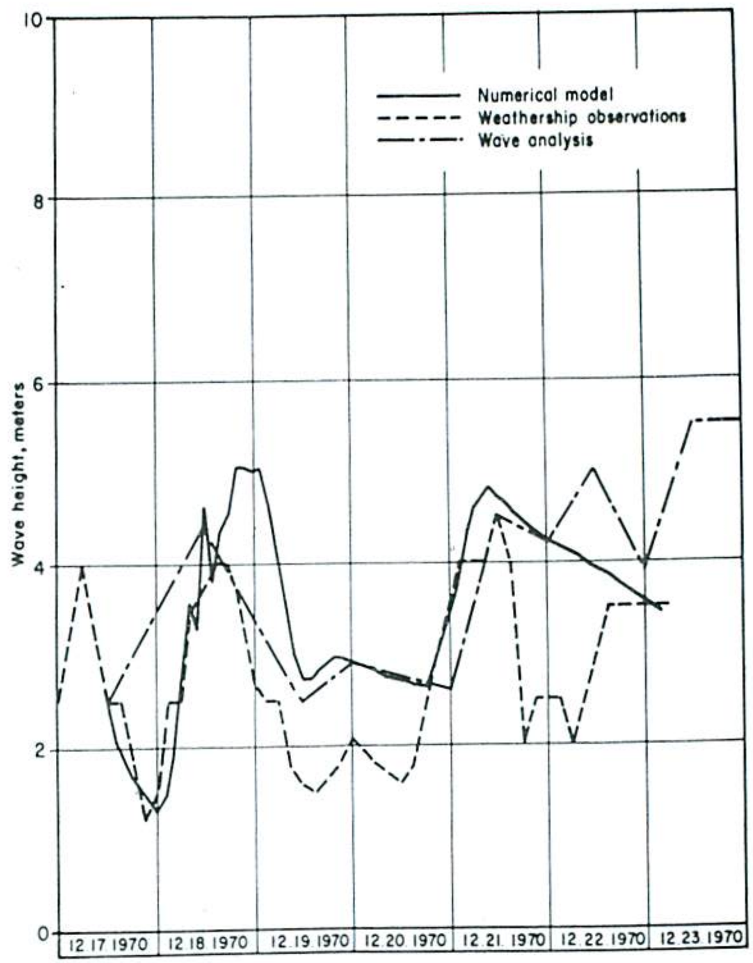


Figure 5. Comparison of numerically hind-casted and observed wave heights Weathership Alpha

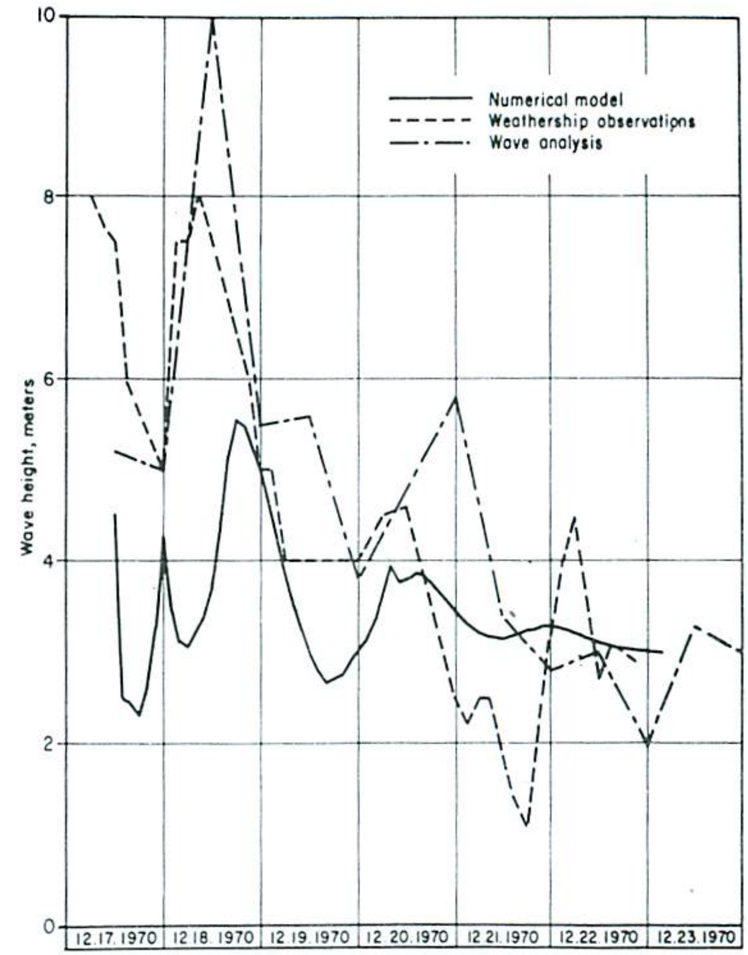


Figure 6. Comparison of numerically hind-casted and observed wave heights Weathership India

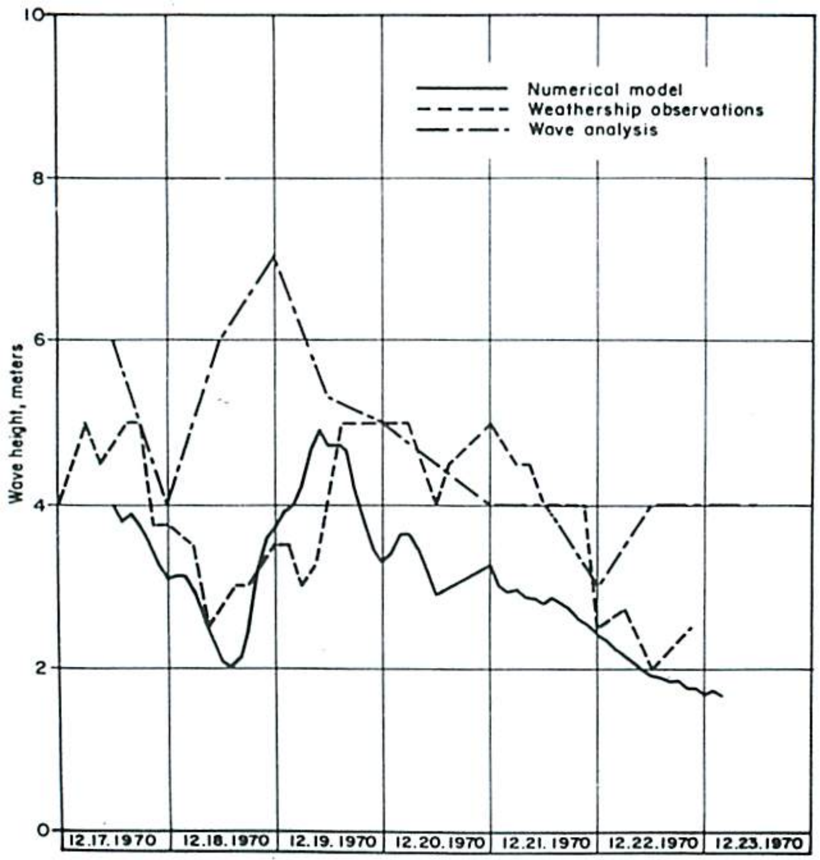


Figure 7. Comparison of numerically hindcasted and observed wave heights Weathership Metro

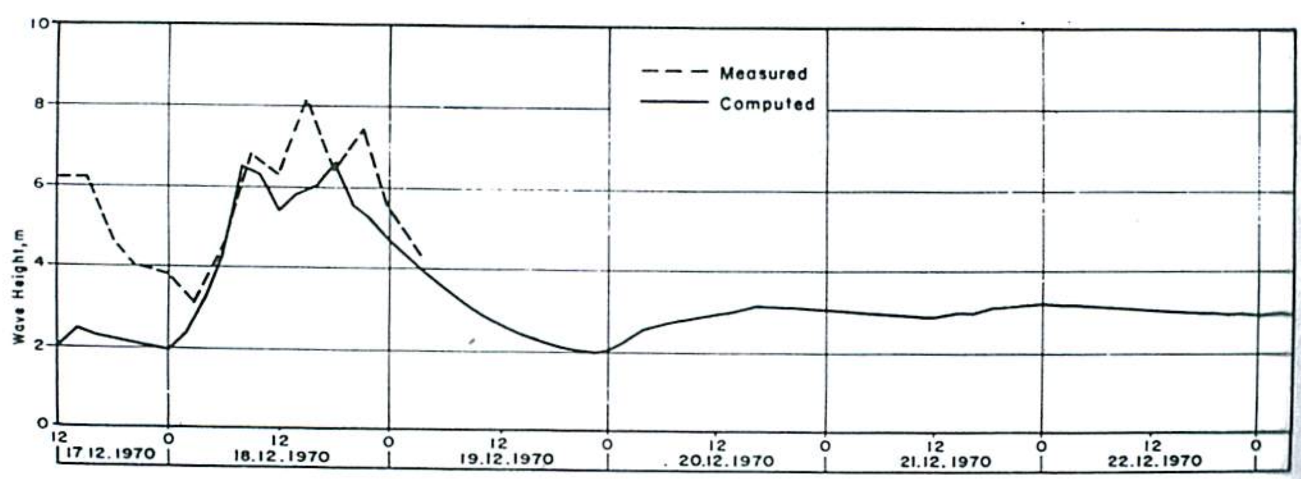


Figure 8. Comparison of numerically hindcasted and measured wave heights near Dyrhólaey, Iceland

DISCUSSION

Mr. H. Rye, River and Harbour Laboratory, Technical University of Norway, Trondheim, Norway.

You are using a parameterization based on the wave growth theories by Miles and Phillips. In order to fit the observations, you modify the constants only.

However, recent research results show that the exponential growth stage of the wave is due to wave-wave interactions and not due to any Miles-Phillips mechanism.

Wouldn't this fact change the way you do the parameterization?

Thorbjörn Karlsson, Science Institute, University of Iceland, Reykjavík, Iceland.

I am not familiar with the JONSWAP report to which you refer. A different wave generating mechanism would certainly change the wave forecasting program. However, the wave generating routine is but a small portion of the overall package and the program is very easily adapted to any wave generating mechanism.

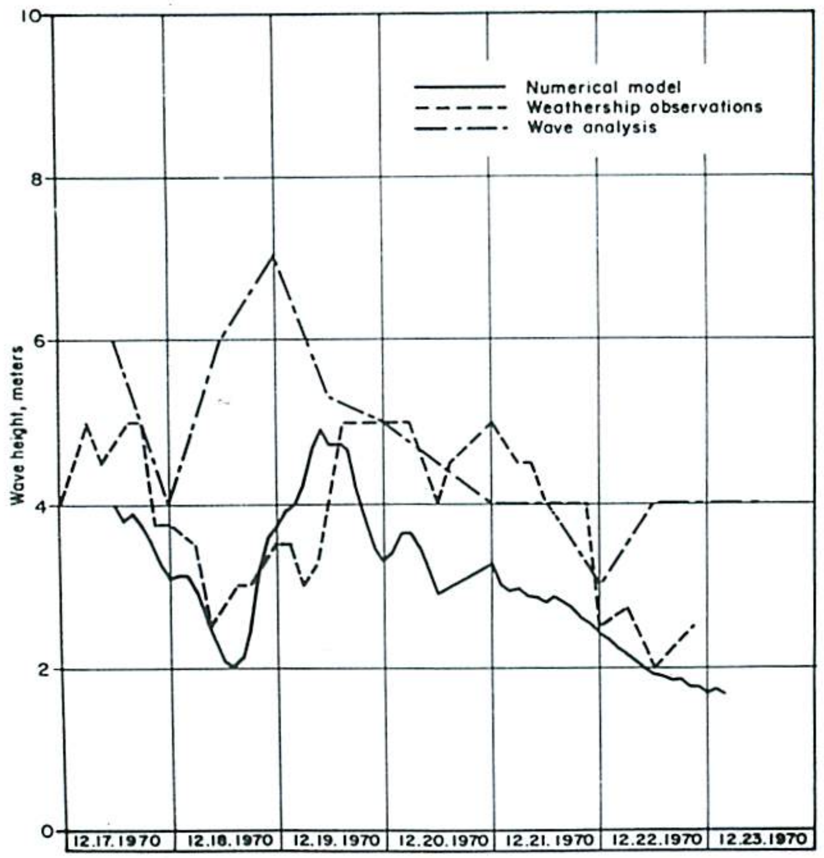
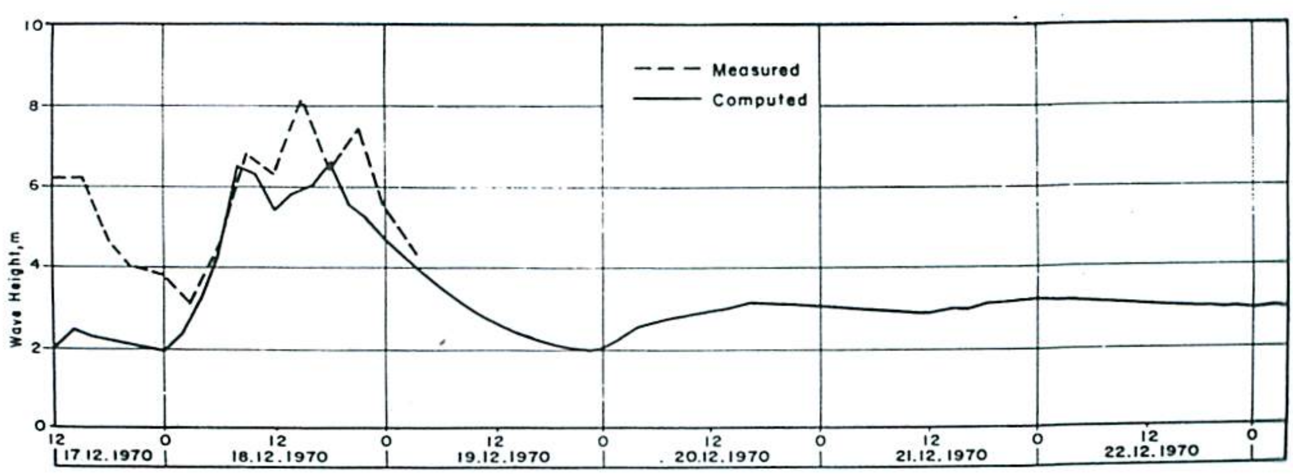


Figure 7. Comparison of numerically hindcasted and observed wave heights Weathership Metro



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