SECOND INTERNATIONAL CONFERENCE ON
PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
UNIVERSITY OF ICELAND
DEPARTMENT OF ENGINEERING AND SCIENCE

A DISCUSSION OF THE WAVE RECORDS OBTAINED AT WEATHER STATION PAPA, PACIFIC OCEAN

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1. INTRODUCTION

An analysis of the action of random sea waves in the coastal zone, in the problems relating to the erosion of beaches, in the design of marine structures, and in the operation of floating craft, requires some statistical characteristics of the water surface. This paper compares some statistical characteristics of the measured deep water wave data with theoretical values.

Since December 1968, shipborne wave recorders have been maintained on the two weather ships Vancouver and Quandra which, in turn, keep position at weather station Papa in the Pacific Ocean. Ninety-five records obtained at three-hour intervals during the five most severe storms in the period December 1968 to June 1971 were selected for analysis.

A computer algorithm was developed for selecting crests, troughs, zero-crossings, etc. to give the necessary parameters for a statistical analysis of the probability distribution of wave heights and periods. The probability distribution of wave heights obtained by zero-crossing method were compared to the Rayleigh distribution.

A large number of statistical parameters such as the average of the highest $1/n^{th}$ waves, the root-mean-square values of wave height, the mean values, the standard deviations, etc. were computed from the data, and the relationships between these parameters were compared to the theoretical predictions derived from Rayleigh distributions.

A variance spectrum analysis of each record was undertaken. The characteristic wave heights deduced from the spectrum were correlated to the zero-crossing wave heights. Wave height parameters derived from Tucker's method were also compared to those from zero-crossing method.

The probability distribution of wave period was investigated. The distribution of zero-crossing period was compared to the Bretschneider's distribution function, in which it is assumed that the squares of periods follow the Rayleigh distribution. Relationships between other wave period parameters were investigated and discussed.

NOTATION

The following symbols have been adopted for the use in this paper:

H = wave height

H_z = zero-crossing wave height

$\bar{H}_{z,1/n}$ = average of the highest of $1/n^{th}$ zero-crossing wave heights

$\bar{H}_{z,1}$ = average zero-crossing wave height

$\bar{H}_{z,rms}$ = root mean square zero-crossing wave height $\left(=\sqrt{\overline{H_z^2}}\right)$

T = wave period

T_z = zero-crossing wave period

$T_{z,1/n}$ = average of the highest of $1/n^{th}$ zero-crossing wave periods

$\bar{T}_{z,1}$ = average zero-crossing wave period

$\bar{T}_{(H_{z,1/3})}$ = significant zero-crossing wave period (average of the zero-crossing periods of the highest one-third wave heights)

$\bar{T}_{z,rms}$ = root mean square zero-crossing wave period $\left(=\sqrt{\overline{T_z^2}}\right)$

σ = average squared displacement of the wave surface from the mean record level

H_{m_o} = characteristic wave height $(= 4 * \sigma)$

2. WAVE DATA

Since December 1968, shipborne wave recorders have been maintained on the two weather ships Vancouver and Quandra which, in turn, keep position at weather station Papa (lat. 50° N, long. 145° W) in the

Pacific Ocean, (Fig. 1). These instruments were manufactured by National Institute of Oceanography (N.I.O.), England (Ref. 1). The measuring system consists of two pressure-recorder-accelerometer units, one at each side of the ship. The pressure recorder indicates the heights of the wave surface above the instrument, and accelerometer records the vertical motion of the system by measuring the vertical acceleration and integrating twice electronically. The sum of these readings gives the height of the water surface above an imaginary level, and is independent of the motion of the ship (Ref. 2).

The water surface elevations were recorded on the 3.5-inch wide paper charts as a function of time at 2 inches per minute during 20-minute periods at 3-hour intervals.

Ninety-five records obtained during the five most severe storms in the period of December 1968 to June 1971 were selected for the analysis. The range of wave heights are from 7 to 56 feet.

In order to make the data compatable to the high speed computer facilities in the data-reduction process, the records were digitized at 1 mm intervals on the chart (0.0847 seconds) by D-MAC-pencil-follower digitizer, and punched on IBM cards.

The "raw data" was corrected for instrumental and hydrodynamic response factors. The depth of pressure units below the water line was 13.0 feet. The attenuation factor of the pressure units to waves of different periods was computed. This attenuation factor was multiplied by the attenuation due to the integrators to give the overall response curve, (Ref. 1 and 2).

An algorithm was developed to compute the mean record level, to determine wave crests and troughs and to determine the period associated with zero-crossings. In this study, a zero-crossing wave height is defined as the vertical distance from a maximum elevation between an upward and downward going zero-crossing, and the following minimum elevation between a downward and upward going zero-crossing. The definition sketch is given in Fig. 2.

The statistical parameters considered to be of interest are the root mean square wave height, $\bar{H}_{z,rms}$; the average of the highest 1/n waves, $\bar{H}_{z,1/n}$ in particular, the average of the highest one-third waves (significant wave height), $\bar{H}_{z,1/3}$, and similar parameters for the wave periods were computed. All wave heights and periods are defined in terms of zero-crossings.

In addition, a variance spectrum analysis of each record was undertaken using the techniques of Fourie analysis. Each record was broken into eight blocks, each block consisting of 1024 sample points. The time step between samples was chosen to give about 60 estimates of spectral density in the frequency band corresponding to wave periods of 2 to 20 seconds. The spectral density was computed as the mean of the 8 values at each frequency.

3. STATISTICAL DISTRIBUTION OF WAVE HEIGHTS

The zero-crossing wave heights, H_z, in each record were normalized by dividing the H_z value by $\bar{H}_{z,1}$. The frequency distribution of these ratios were then formed and compared with the Rayleigh distribution, (Ref. 3 and 4). Fig. 3 shows an unbiased selection of distributions, in this diagram the bottom scale has been chosen so that the Rayleigh distribution function gives a straight line. In general, the data shows no significant deviation from the Rayleigh distribution.

Fig. 4 shows a plot of the standard deviation of the H_z values, S_H, for each record against the mean of H_z values, $\bar{H}_{z,1}$. Fig. 5 shows the relation between the average of the highest one-third of the H_z values, $\bar{H}_{z,1/3}$, and the mean of the H_z values, $H_{z,1}$. These results show a high degree of agreement with the theory. A similar degree of agreement was also found with the other wave height parameters. Fig. 6 summarizes the comparison of the ratios of the average of the highest $1/n^{th}$ H_z values in a record to the root mean square value of H_z, $\bar{H}_{z,1/n}/\bar{H}_{z,rms}$, with the theoretical values; the solid line in the graph is the theoretical curve derived from the Rayleigh distribution (Ref. 3). The agreement is particularly good for larger values of 1/n. For the very low values of 1/n, the calculated ratios are slightly lower than the theory predicts.

Fig. 7 gives a plot of $\bar{H}_{z,1/3}$ against $\bar{H}_{1/3}$ as calculated using Tucker's method (Ref. 5). It shows a good agreement.

The $\bar{H}_{z,1/3}$ values were also compared with the Characteristic wave height values in Fig. 8. The characteristic wave height, H_{m_0} $(= 4 \times \sigma)$ is computed from the spectrum where σ is the

variance of the spectrum integrated between frequencies corresponding to wave periods of 2 to 20 seconds. The equation of the least square line is $H_{m_0} = 1.03\bar{H}_{z,1/3} + 0.20$.
This also shows that the statistics of zero-crossing wave heights are Rayleigh distributed regardless of the spectral width of the data.

4. STATISTICAL DISTRIBUTION OF WAVE PERIODS

As in the case of wave height, the empirical distribution function for wave period was tested. The zero-crossing wave periods, T_z, in the set of records were normalized by dividing the T_z values by the mean value of T_z, $\bar{T}_{z,1}$. The frequency distribution of $(T_z/\bar{T}_{z,1})^2$ was plotted on graph paper with a scale so that the Rayleigh distribution function would yield a straight line as was used for wave height in the previous section. It was found that the results are in a good agreement with theory. Typical example is given in Fig. 9. The frequency distribution of the ratios of $T_z/\bar{T}_{z,1}$ was also plotted on the probability paper and compared with Bretschneider's distribution function in Fig. 10. The data appears to support Bretschneider's theory, (Ref. 6).

Fig. 11 gives a plot of the standard deviation, S_T, against the mean of the T_z values, $\bar{T}_{z,1}$. The solid line is derived from the Rayleigh distribution following Bretschneider's assumption.

The average of the highest $1/n^{th}$ wave periods were also computed. The ratios of $\bar{T}_{z,1/n}/\bar{T}_{z,rms}$ were plotted against the $1/n$ values in Fig. 12. For a given value of $\bar{T}_{z,rms}$, the distribution function of period may be determined from Fig. 12.

Fig. 13 gives a plot of the significant zero-crossing period, $\bar{T}_{H_z,1/3}$ against $\bar{T}_{z,rms}$, where the significant zero-crossing period is the average of the zero-crossing periods of the highest one-third wave heights. The equation of the least square line is $\bar{T}_{H_z,1/3} = 0.983\,\bar{T}_{z,rms}$. It may be concluded that the significant zero-crossing period is practically equal to $\bar{T}_{z,rms}$.
The height and period associated with an individual wave were usually found to show little relationship to each other. However, during the storm, the wave period does increase in proportion to wave height. A typical joint probability distribution from the data was plotted in Fig. 14. It shows a moderate degree of correlation. The relationship is approximately linear, the least square line is $\bar{T}_{z,rms} = 0.205\,\bar{H}_{z,rms} + 7.35$

This may not necessarily be applicable when the wave heights is less than 7 feet.

5. CONCLUSIONS

The statistical distribution of zero-crossing wave heights and periods appear to show regularity from one wave system to another. Wave parameters derived using the zero-crossing method closely follow the theory derived for the narrow band spectrum. Rayleigh distribution may be used in practice to estimate the distribution of heights and Bretschneider's distribution of the periods of ocean waves.

6. ACKNOWLEDGEMENTS

The author is indebted to the Oceanographic Branch of the Marine Science Directorate of the Department of the Enviroment of Canada, and, in particular, to Mr. G. L. Holland for making the original data available, and to Mr. C. K. Hurst, Chief Engineer, the Department of Public Works of Canada for permission to publish this paper.

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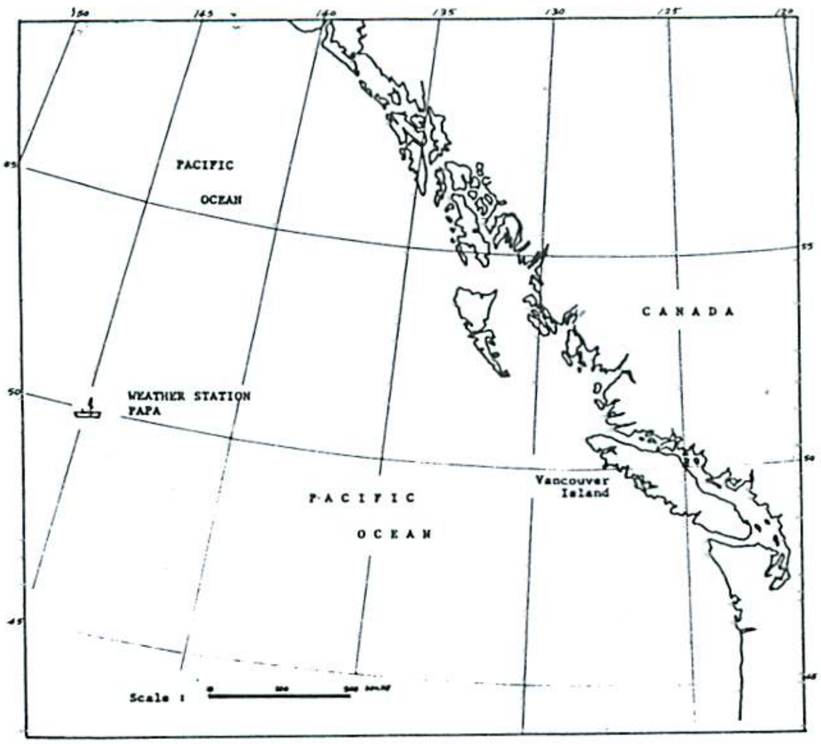


FIGURE 1 LOCATION OF WEATHER STATION PAPA

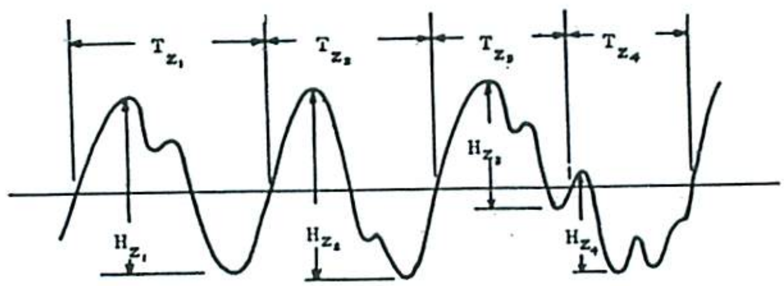


FIGURE 2 DEFINITION SKETCH OF THE ZERO-CROSSING METHOD

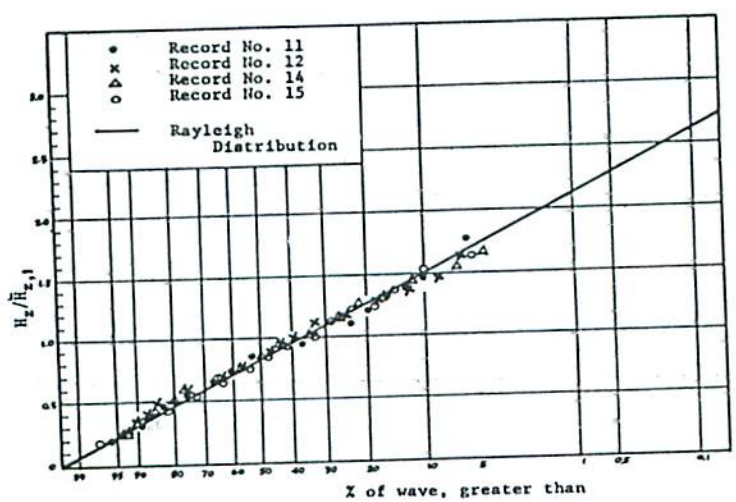


FIGURE 3

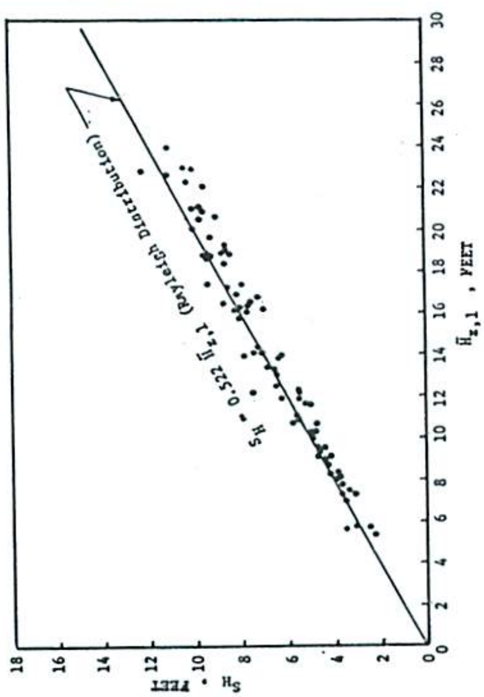


FIGURE 4

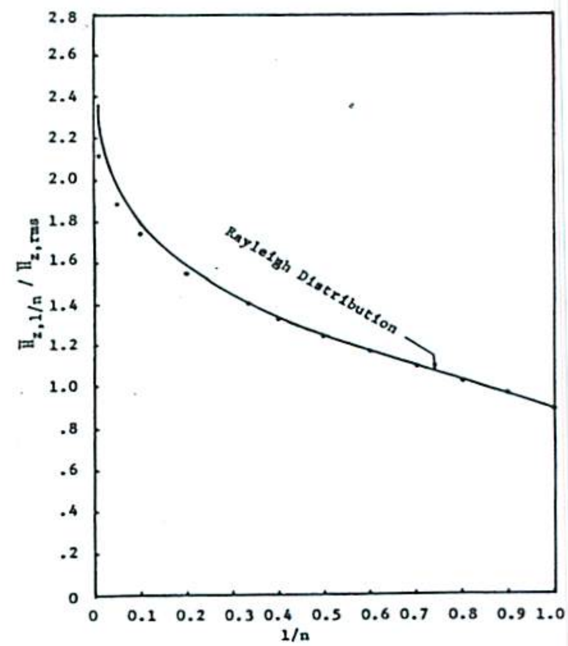


FIGURE 6

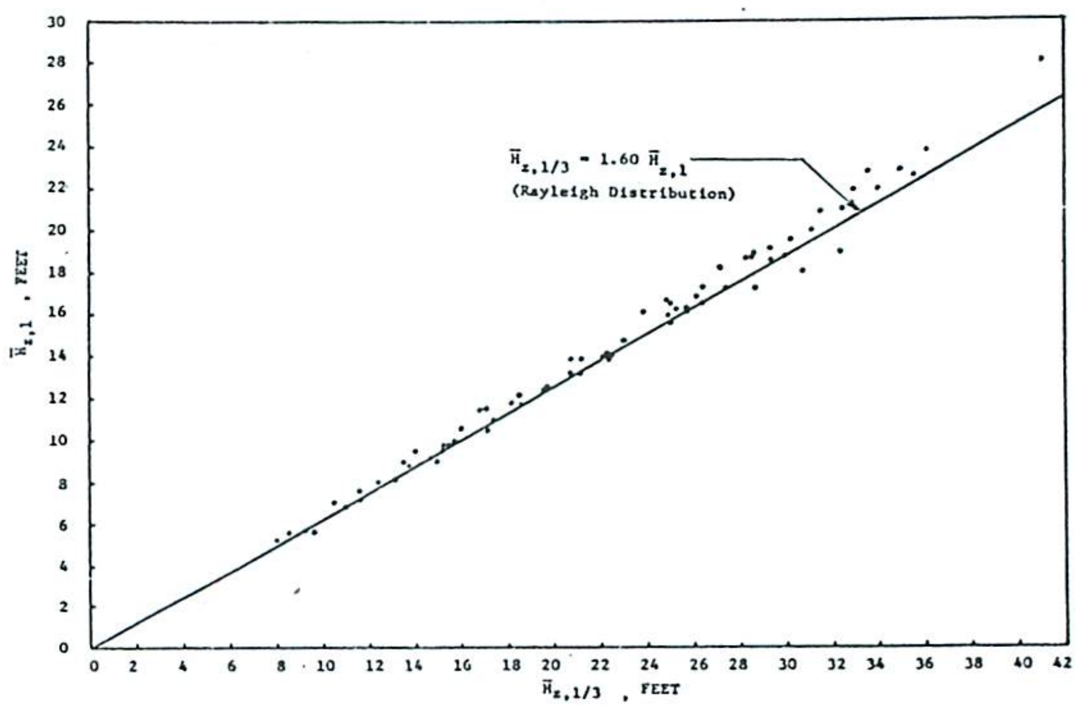
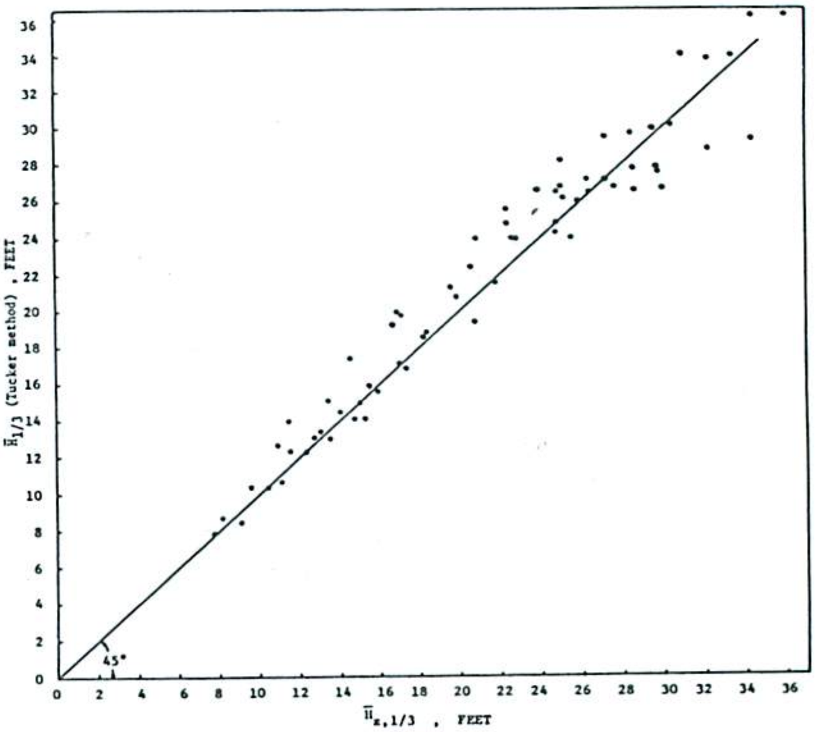


FIGURE 5



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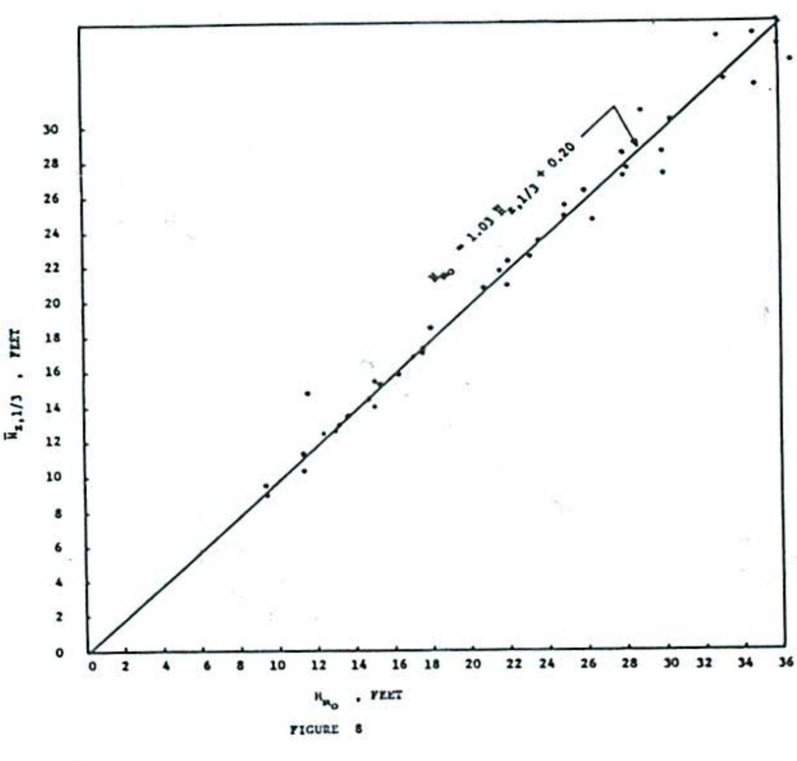

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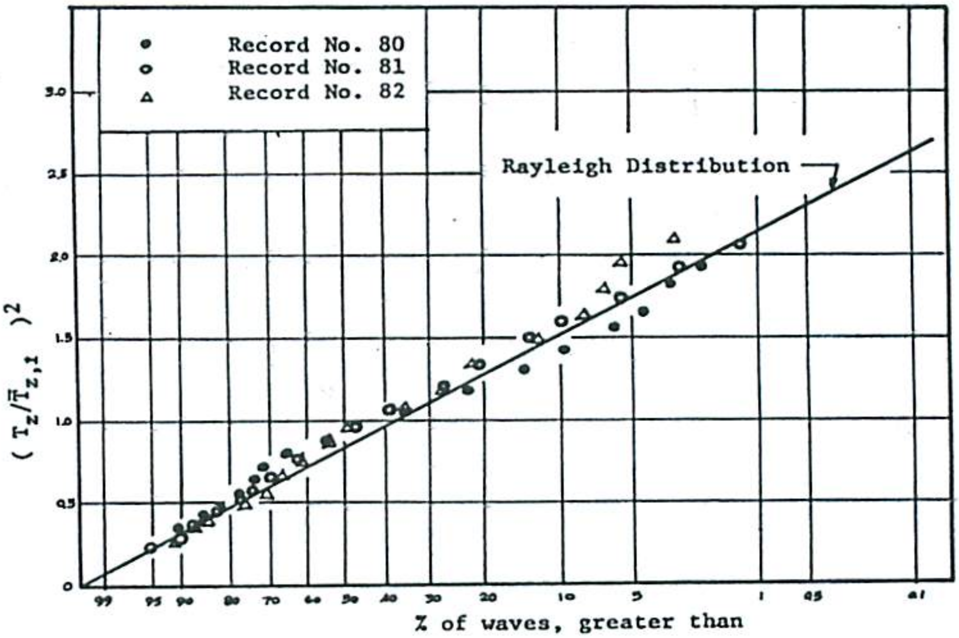

FIGURE 9

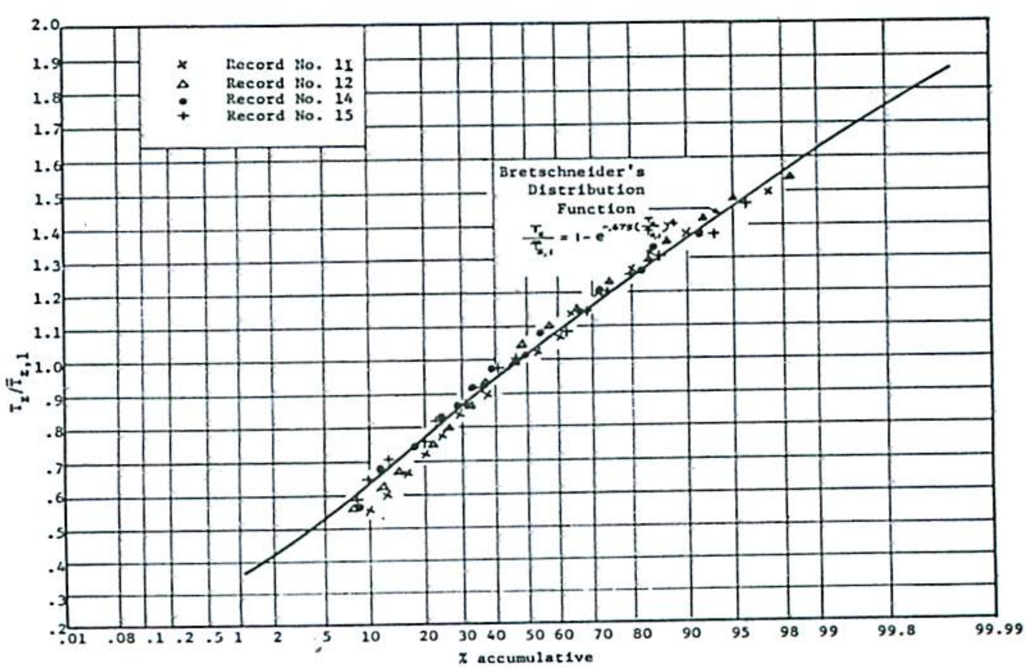

FIGURE 10

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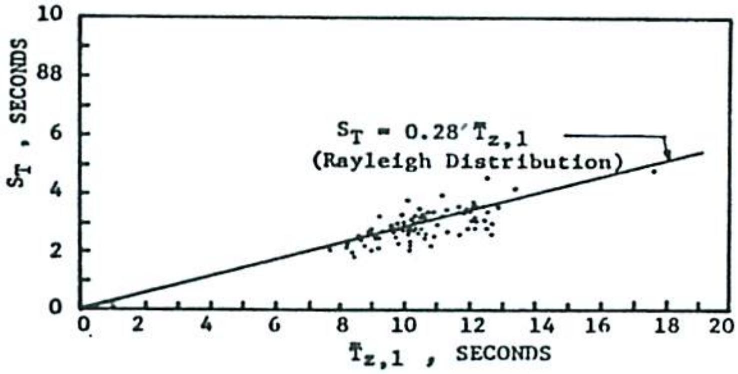

FIGURE 11

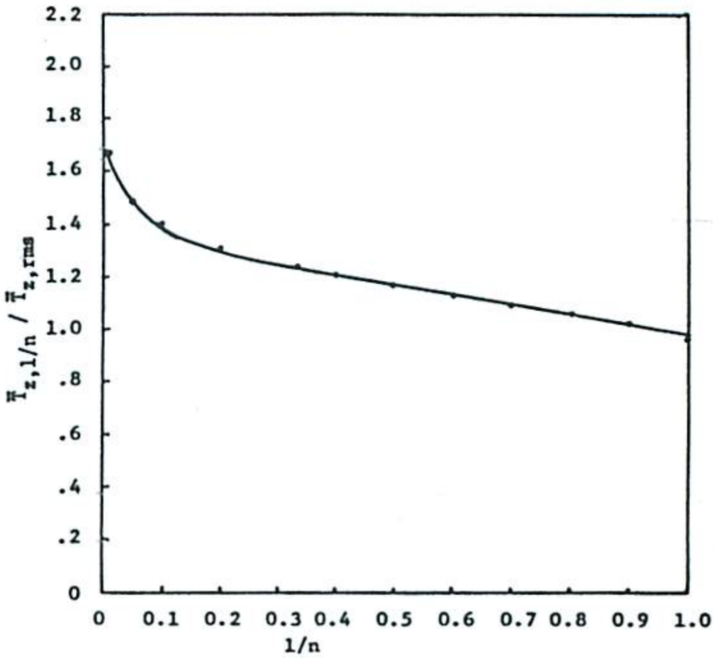

FIGURE 12

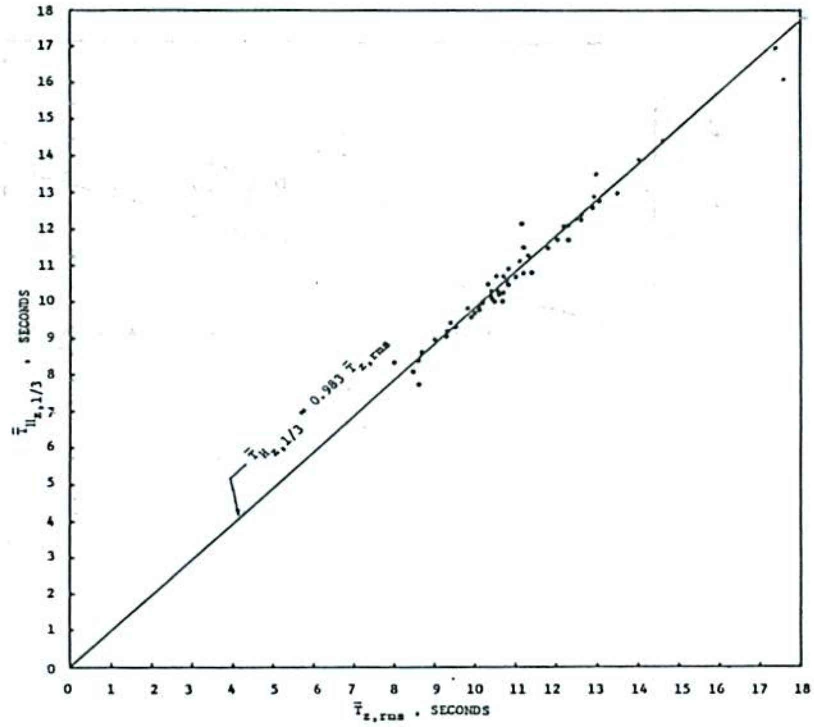

FIGURE 13

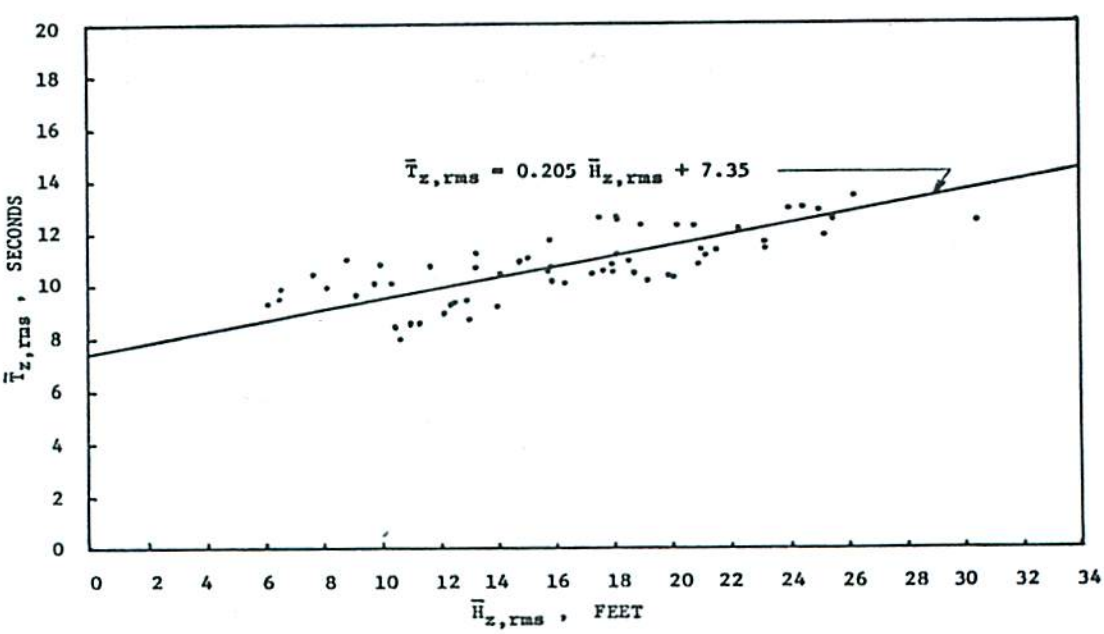

FIGURE 14