SECOND INTERNATIONAL CONFERENCE ON
PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
UNIVERSITY OF ICELAND
DEPARTMENT OF ENGINEERING AND SCIENCE

STOCHASTIC ANALYSIS OF ICE-STRUCTURE INTERACTION

C. Sundararajan
Postdoctoral Fellow

D.V. Reddy
Professor

Faculty of Engineering and Applied Science
Memorial University of Newfoundland

St. John's
Newfoundland
Canada

SYNOPSIS

The random response of an offshore structure due to forces caused by ice floes is studied. The loading, given by an actual field record obtained by Blenkarn [1] at Cook Inlet, Alaska, is assumed to be stationary and ergodic. The structure is idealised as a damped single-degree-of-freedom elastic system and the response obtained by the spectral method.

INTRODUCTION

A probabilistic dynamic analysis is presented in this paper for the response of fixed offshore structures to ice floes forced past them.

One of the most comprehensive investigations of impact forces of ice on structures is that due to Korzhavin [2], which includes studies on strength of ice under impact loading, indentation of ice and splitting of ice floes by a pier with a vertical edge and failure of ice floes on inclined piers. Peyton [3-7] made significant contributions to the study of ice properties, laboratory and field investigations of ice-structure interaction (with particular reference to Cook Inlet, Alaska) and recommendations for design. Ice force measurements made on a specially instrumented test pile and other platforms at Cook Inlet have been described by Blenkarn [1]. Neill [8,9] has carried out extensive measurements of ice forces on several bridge piers in Alberta, Canada. The work included the synchronization of force recording and movie photography at one site to compare force fluctuations with the nature of the ice failure. The use of model basins to simulate and predict full-scale ice-structure interactions has been described by Voelker and Levine [10] and Coon [11] has investigated the forces and deformations associated with small floes contiguous with each other. Nevel, Perham and Hogue [12], have described laboratory tests to define limiting ice force levels and identify different modes of ice failure.

An analytical study for the response of a cantilevered test pier to deterministic ice forces has been presented by Matlock, Dawkins and Panak [13]. Assuming that the primary response of the structure to the excitation of the ice floe would be in its fundamental mode of vibration, the test pier was idealised by a damped single-degree-of-freedom system and the response determined for a postulated "saw-tooth type" of deterministic loading. The results obtained for a numerical solution of the differential equation check well with Peyton's [6] observed values.

The high degree of randomness noticed in the ice force measurements at Cook Inlet [1] indicates a strong need for stochastic analysis.

STRUCTURAL RESPONSE TO RANDOM ICE FORCES

The structure considered by Matlock et al [13] is idealised by a damped single-degree-of-freedom elastic model (Fig. 1), in the same manner as in their investigation by choosing

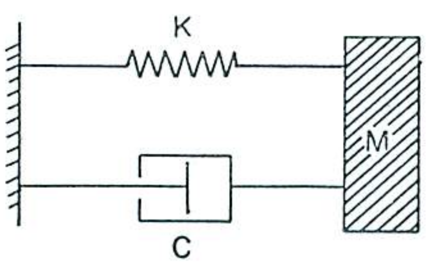

FIG. 1: THE ELASTIC SYSTEM

The stiffness K to produce the same force-displacement behaviour and the mass M, the same fundamental frequency as in the actual structure. The damping constant C of the model is assumed to be the same as that for the actual structure.

Let the force at any time to be P(t). The mean value of the forces P_{mean} is deducted from P(t) to obtain Q(t). The deflection due to P_{mean} is

$$\delta_{mean} = \frac{P_{mean}}{K} \qquad (1)$$

The forces $Q(t)$ are random in nature and assumed to be stationary and ergodic. The record of $Q(t)$ is digitized and fed into a Fourier analysis programme which gives the approximate spectral density S_{QQ} of the force deviations Q as a function of the frequency ω.

The equation of motion of the single-degree-of-freedom system is

$$M \frac{d^2X}{dt^2} + C \frac{dX}{dt} + KX = Q \tag{2}$$

where $X(t)$ = random displacement of the mass due to $Q(t)$. Equation (2) can be rewritten as

$$\frac{d^2x}{dt^2} + 2\beta\omega_o \frac{dx}{dt} + \omega_o^2 x = F \tag{3}$$

where

$$\omega_o = \sqrt{\frac{K}{M}}, \quad \beta = \frac{C}{2\sqrt{KM}} \quad \text{and } F(t) = \frac{Q(t)}{M}$$

The solution of equation (3) is well known, for example [14]. The spectral density of the displacement X is given by

$$S_{XX}(\omega) = \frac{S_{FF}(\omega)}{(\omega o^2 - \omega^2)^2 + (2\beta\omega\omega_o)^2} \tag{4}$$

where

$$S_{FF}(\omega) = \frac{1}{M^2} S_{QQ}(\omega) \text{ is the spectral density of } F(t) \tag{5}$$

The mean square of the displacement X is obtained by

$$\overline{X}^2 = \int_o^\infty S_{XX}(\omega)\, d\omega \tag{6}$$

ILLUSTRATIVE EXAMPLE

The system parameters of the single-degree-of-freedom system of Matlock et al [13] are

K = 50 kips/inch
M = 0.14 kip-sec^2/inch
C = 6% of the critical value (i.e. β = 0.06)
ω_o = 3 cps

The force acting on the system is represented by three segments, of 4 seconds each, taken from Blenkarn's [1] force record of a much longer duration (Fig. 2). A check of tide and current tables, corrected to the location, suggests that the flow velocity generally exceeded 3 ft/sec.

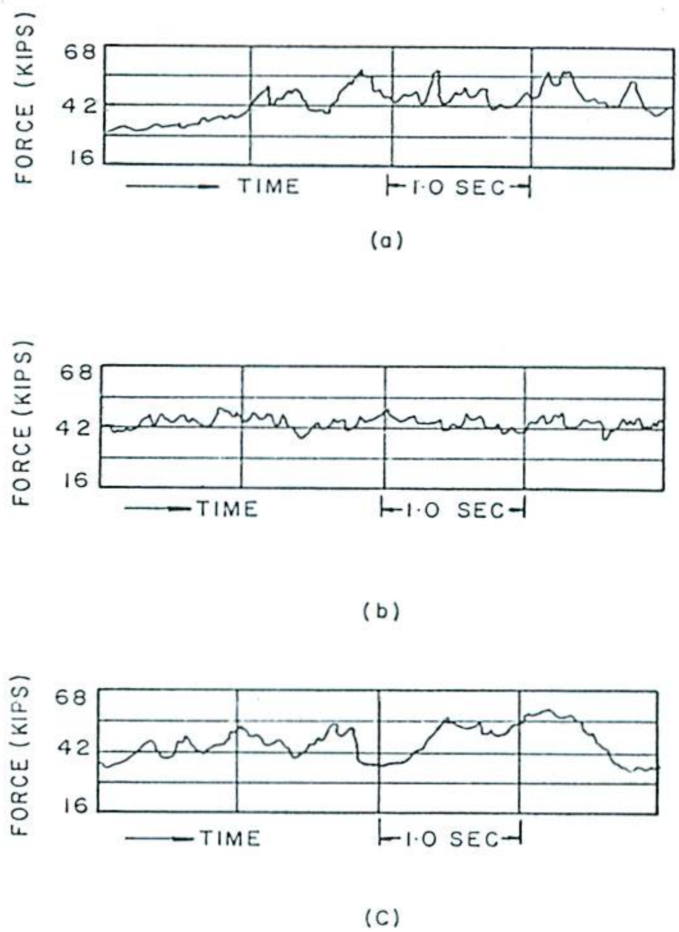

FIG. 2: ICE FORCE RECORD

The responses obtained by the Spectral Method are summarised in Table I.

TABLE I

P(t)	Fig 2(a)	Fig 2(b)	Fig 2(c)
P_{mean}	44.9 kips	43.8	46.6
δ_{mean}	0.898 in.	0.876	0.932
$S_{QQ}(\omega)$	Fig 3(a)	Fig 3(b)	Fig 3(c)
$S_{XX}(\omega)$	Fig 4(a)	Fig 4(b)	Fig 4(c)
$\overline{X}^2$	0.01724 in^2	0.003587	0.01892

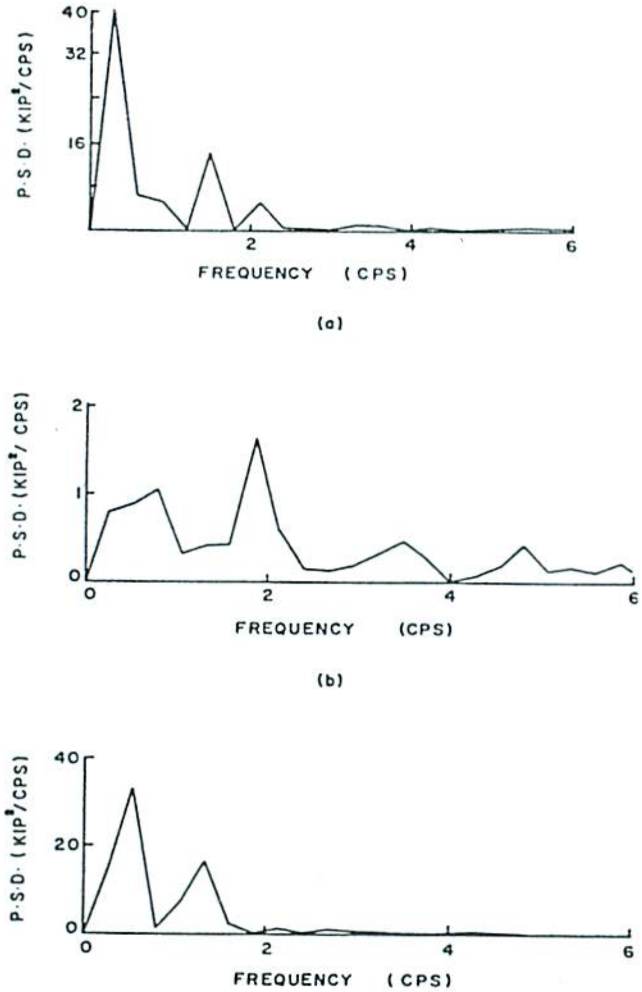

FIG. 3: POWER SPECTRAL DENSITY OF FORCE

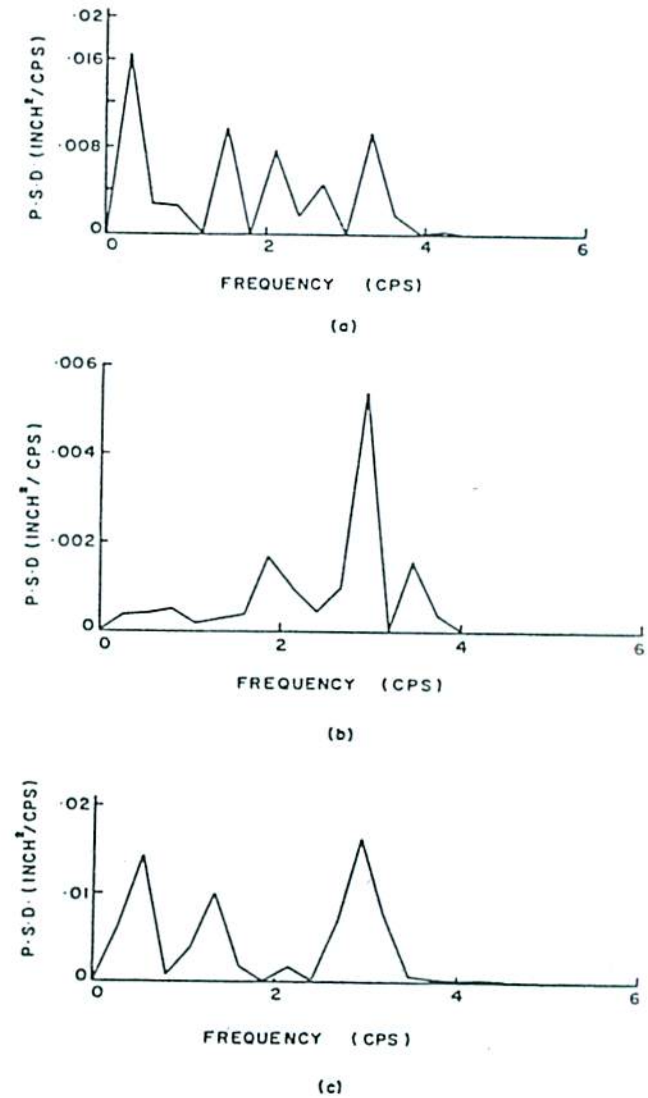

FIG. 4: POWER SPECTRAL DENSITY OF DEFLECTION

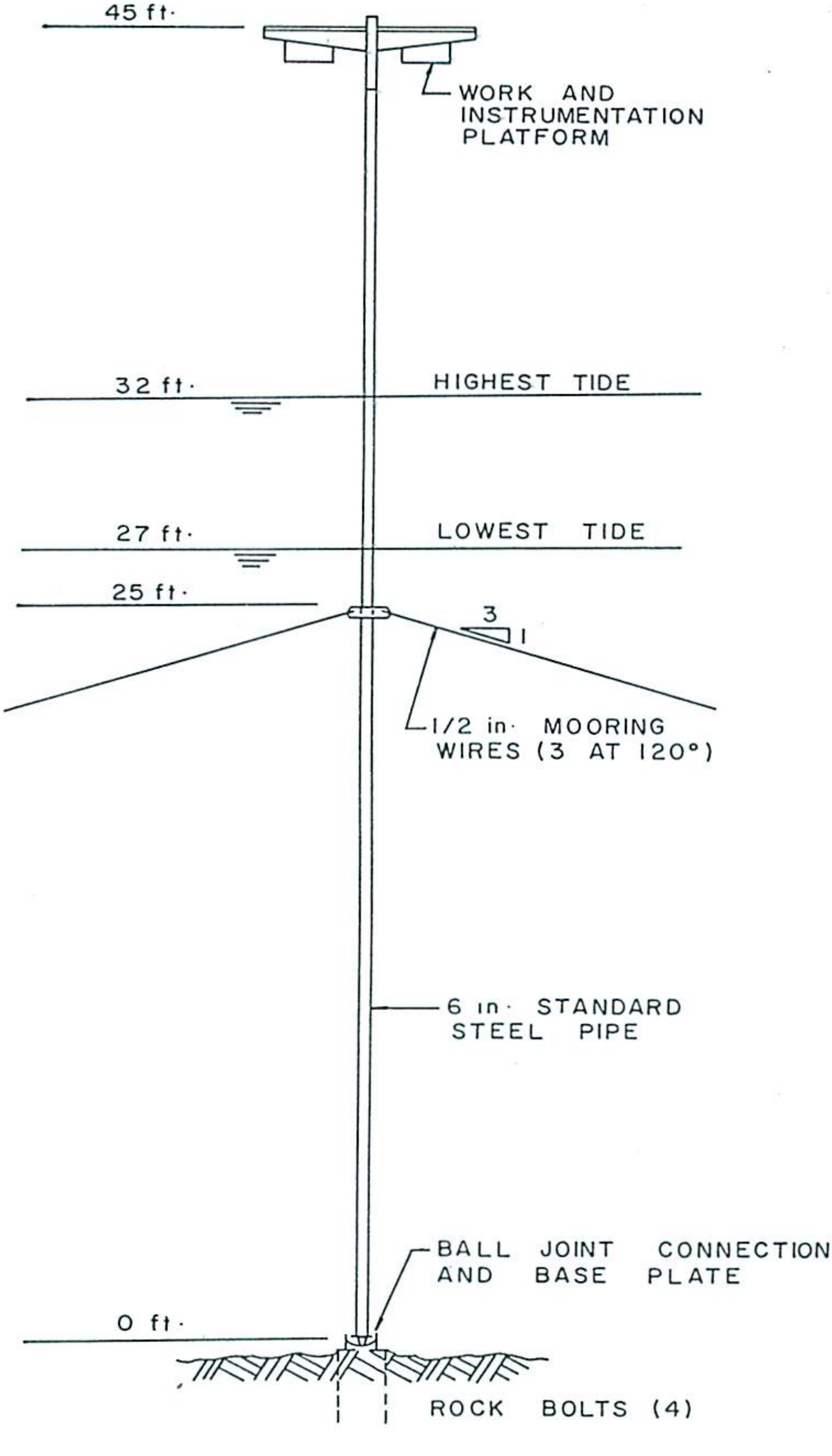

FIG. 5: CONFIGURATION OF TOWER OFF ST. PHILLIPS [15]

DISCUSSION

The power density spectra enable the distribution of frequency-related vibrational energy to be visualised. As one would expect from the familiar resonance curve for a linear system, the power spectral density plots of displacement (Fig. 4) indicate that the system responds to excitation for $\omega < \omega_o$ and $\omega = \omega_o$ ('peaking' of resonance) but filters out excitations for $\omega >> \omega_o$. While the first and third spectral density plots indicate that the assumption of stationarity and ergodicity is warranted, the discrepancy in the second record suggests a strong need for the analysis of more segments of the force record. For reliability of measurement, the product of the record length and the band width must be sufficiently large.

The procedure described in this paper can be easily extended to a structure idealised as a multi-degree-of-freedom system. In the winter of 1973-74, it is proposed to measure ice forces on an offshore tower (Figure 5) erected at St. Phillips by the Ocean Engineering group of the Memorial University of Newfoundland [15]. The response of the tower to measured random ice forces will be calculated by the spectral density method and compared with the observed response. It is hoped that the results will have some useful applications in establishing code requirements for sea ice pressures on offshore structures. From their force measurements on bridge piers, Sanden and Neill [16] have indicated that "the saving that would ensue from a reduction in specified ice forces justifies considerable effort by many organisations". It might be noted that Blenkarn's [1] suggested value of an upper bound of steady pressure (150 psi) and his peak forces per unit width (60,000 to 70,000 lb/ft), due to sea ice, are comparable with the observations of Neill, Saunders and Schultz [17] on the Pembina River in Alberta.

ACKNOWLEDGEMENT

The authors are grateful to Dr. A.A. Bruneau, Dean, Faculty of Engineering and Applied Science, Memorial University of Newfoundland for his keen interest and encouragement. Thanks are due to Professor M.O. Toll for helping with the digitizing table and the PDP-12 programme, CALPLOT, for determining the spectral densities. The valuable discussions with Professors C.R. Neill, D.S. Sodhi and D.B. Muggeridge;

and the computing assistance of Mr. Brian Gamberg are gratefully acknowledged. The first author (C.S.) was partly supported by N.R.C. grant No. A-8119.

REFERENCES

1) Blenkarn, K.A., "Measurement and Analysis of Ice Forces on Cook Inlet Structures", Preprint, Offshore Technology Conference, Vol. II, pp. 365-378 (1970).

2) Korzhavin, K.N., "Action of Ice on Engineering Structures", Novosibirsk, Akad. Nauk., SSSR, 1962, English Translation, Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire, U.S.A., 1971.

3) Peyton, H.R., "Sea Ice Strength", Final Report to Office of Naval Research, Geophysical Institute, University of Alaska, College, Alaska, 1966 .

4) Peyton, H.R., "Sea Ice Forces", Proceedings of Conference on Ice Pressures Against Structures, Laval University, Quebec, Nov. 1966, National Research Council of Canada, Ottawa, N.R.C. No. 9851 (March 1968).

5) Peyton, H.R., "Ice and Marine Structures, Part I - The Magnitude of Ice Forces Involved in Design", Ocean Industry, Vol. 3, No. 3, pp. 40-44 (1968).

6) Peyton, H.R., "Ice and Marine Structures, Part II - Sea Ice Properties", Ocean Industry, Vol. 3, No. 9, pp. 59-65 (1968).

7) Peyton, H.R., "Ice and Marine Structures, Part III - The Importance of Design Alternatives", Ocean Industry, Vol. 3, No. 12, pp. 51-58 (1968).

8) Neill, C.R., "Ice Pressure on Bridge Piers in Alberta", IAHR Symposium,"Ice and its Action on Hydraulic Structures", Reykjavik, Iceland, 1970.

9) Neill, C.R., "Force Fluctuations during Ice-Floe Impact on Piers", Ice Symposium, Leningrad, U.S.S.R., 1972.

10) Voelker, R.P. and Levine, G.H., "Use of Ice Model Basins to Simulate and Predict Full Scale Ice-Structure Interactions", Preprint, Offshore Technology Conference, Vol. II, pp. 583-592 (1972).

11) Coon, M.D., "Mechanical Behaviour of Compacted Arctic Ice Floes", Preprint, Offshore Technology Conference, Vol. II, pp. 603-608 (1972).

12) Nevel, D.E., Pertram, R.E. and Hogue, G.B., "Ice Forces on Vertical Piles", Conference Preprint, U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire, 1972.

13) Matlock, H., Dawkins, W.P. and Panak, J.J., "Analytical Model for Ice-Structure Interaction", Proc. ASCE, EMD, Vol. 97, pp. 1083-1092 (1971).

14) Bolotin, V.V., "Statistical Methods in Structural Mechanics", Holden-Day Inc., San Francisco, pp. 115-117 (1969).

15) Cammaert, A.B., "The Design and Construction of an Offshore Tower", 1972 Annual Technical Conference sponsored by The Association of Professional Engineers of Newfoundland and The Engineering Institute of Canada (Region V), October, 1972.

16) Sanden, E.J. and Neill, C.R., "Determination of Actual Forces on Bridge Piers due to Moving Ice", Preprint, Proc. 1968 Convention, Canadian Good Roads Association.

17) Neill, C.R., Saunders, H.D. and Schultz, H, "Measurements of Ice Forces on Bridge Piers", 1970 and 1971", Research Council of Alberta, Edmonton, Alberta, 1972.