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DEPARTMENT OF ENGINEERING AND SCIENCE

NUMERICAL SIMULATION OF CURRENTS RECORDED IN A NORWEGIAN FIORD

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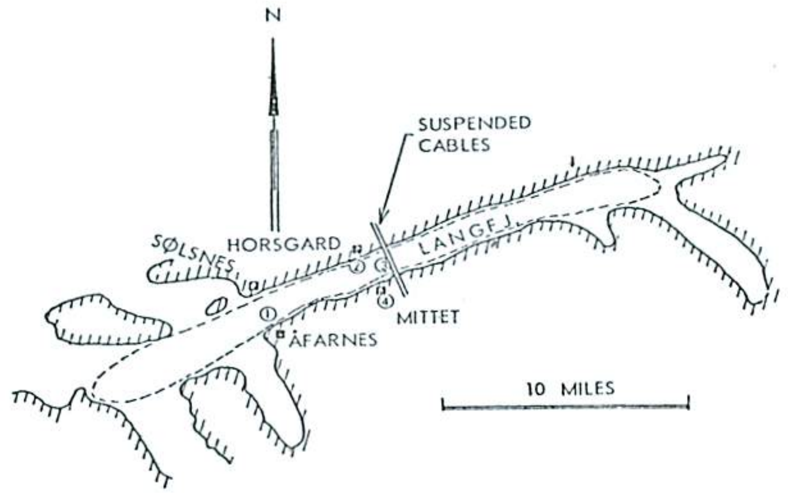
ABSTRACT

The horisontal wind velocity at elevation 11 m and the horisontal water velocity at 5, 10 and 20 m depth were recorded simultaneously during a 20 day period in a long and narrow fiord. In order to simulate the surface wind current velocity on a digital computer, the diffusion equation was solved numerically with the observed - time dependent - wind stress as boundary condition. The water velocities computed were compared to the velocities recorded. The maximum surface current velocity was found to be about 9% of the maximum wind velocity. This number is significantly larger than the number 2 - 3% usually experienced on the open ocean. The large surface velocities are attributed to the stable stratified surface waters, which prevents the water masses from vertical mixing.

INTRODUCTION

This study is aimed at the understanding of air-sea momentum transfer processes which are of importance when floating structures like floating bridges are planned. Also, for pollution control, the influence of wind on the surface waters is of general interest.

The Norwegian fiords provide unique conditions for the study of momentum transfer from air to water because they are long and



MAP OVER LANGFJORD, NORWAY

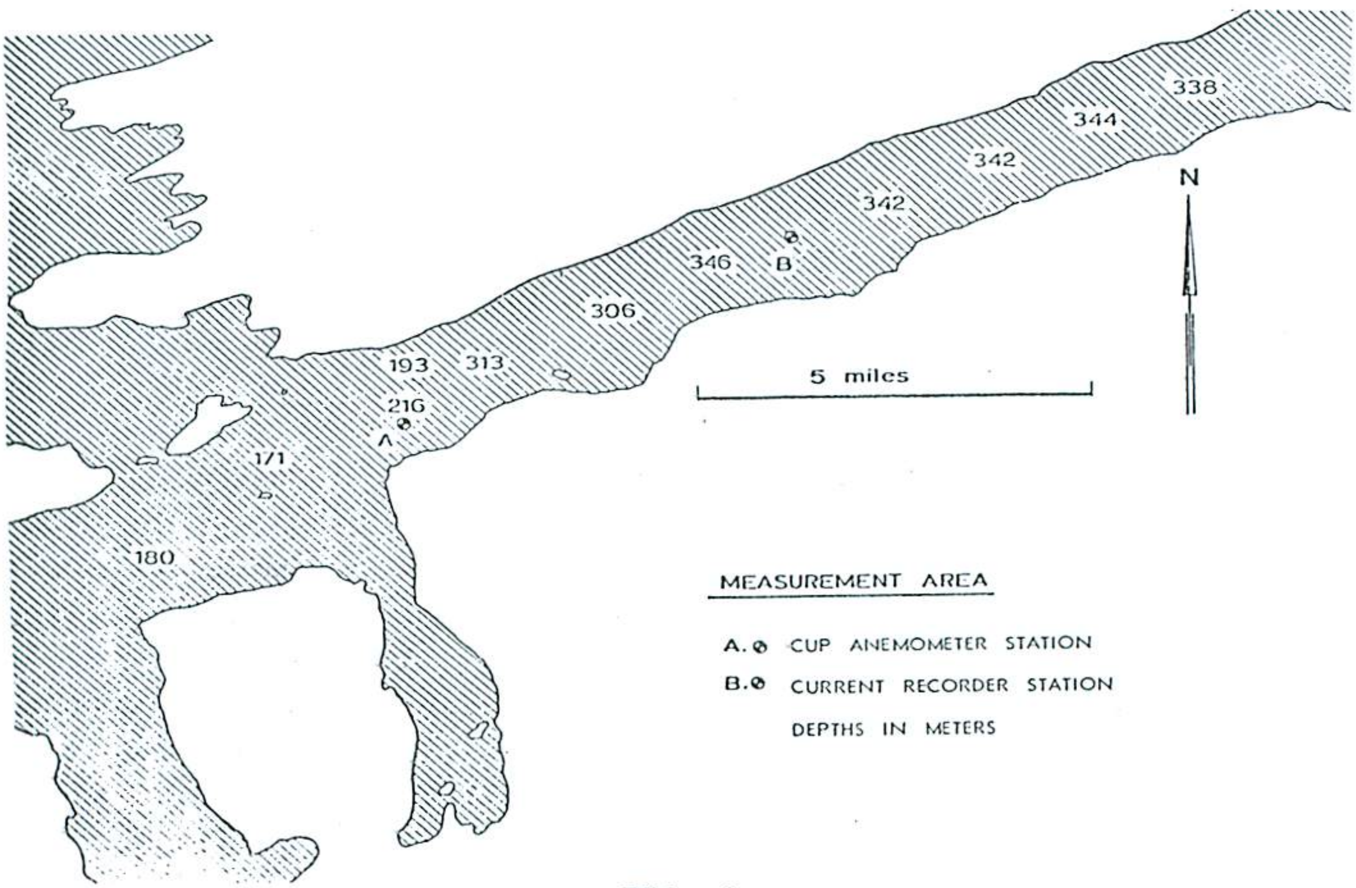
① Cup anemometer station

② Tide gage station

③ Waverider and current recorder station

④ Recording and control station

- FIG. 1 -



- FIG. 2 -

narrow and surrounded by high mountains with steep slopes which force the wind and currents to follow the fiord direction. The Langfjord at Molde, on the West Coast of Norway, is almost straight and is therefore very suitable for such a study. (Fig. 1 and 2).

RECORDINGS

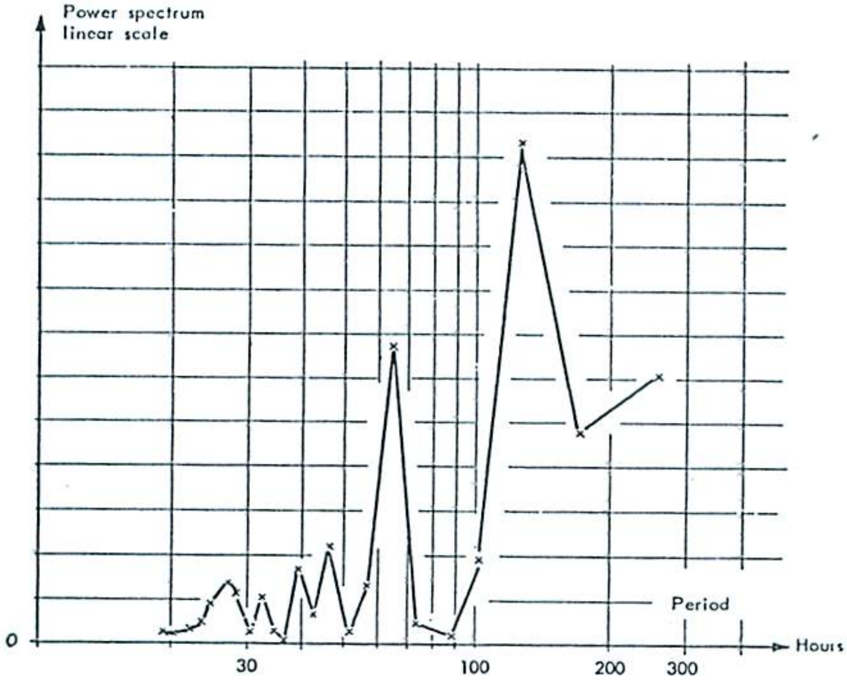
The transfer of momentum from the atmosphere to the water is provided largely by wind waves, which, in turn, transfer the momentum into currents. Only a small fraction of the momentum is advected away with the wave field. (Of order 10%, K. HASSELMANN et. al. 1973). The study of momentum transfer from the atmosphere to the water can therefore be made successfully without including wave recordings in the measurement program.

The location of the current recorders are shown on Fig. 2. Horizontal velocity and direction were sampled each 5. min. and the data stored on magnetic tape. (Aanderaa-recorder). Depths of recording instruments were 5,10 and 20 meters. The location of the wind recorder, recording horizontal wind velocity and direction instantaneously on a strip chart, are shown on Fig. 2, on a small island.

ANALYSIS

Twenty-two days of simultaneous recordings during April and May 1972 provided the basic material for analysis of the momentum transfer. Velocity components in the longitudinal fiord direction were considered only. Hourly averages were computed for all the recordings.

Autospectra were computed with a Fast Fourier Transform (FFT) routine and the results are shown on Fig. 3 and 4. The wind spectral peaks at 128 and 64 hours are due to variations of the wind caused by moving low pressure systems. The autospectrum of the 5 m current velocity shows the same peaks at 128 and 64 hours. At 10 m the peak at 64 hours is absent; it is probably filtered out by the depth. At 20 m the water velocities are apparently weak.



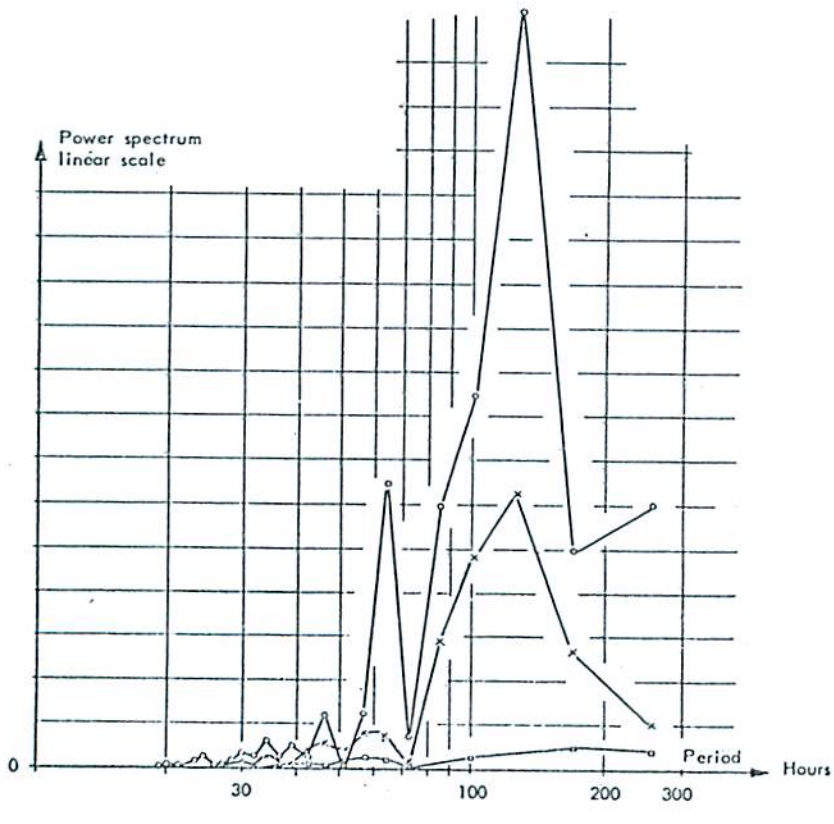
LEGEND

×——× Wind velocity spectrum

POWER SPECTRUM OF THE WIND VELOCITY RECORDED IN LANGFJORD.

15.4. - 6.5. 1972

- FIG. 3 -



LEGEND

o——o Current spectrum 5 M depth

×——× 10 M depth

□——□ 20 M depth

POWER SPECTRUM OF THE CURRENTS RECORDED IN LANGFJORD.

15.4. - 6.5. 1972

- FIG. 4 -

No tidal current component could be detected from the spectra. This is probably due to the large depth of the fiord; estimates of the tidal currents show them to be of order 2 - 3 cm/sec. By comparing Fig. 3 and 4, it was concluded that the currents recorded are mainly caused by the wind.

Fig. 5 shows the cross-correlation functions between the wind stress and the currents at 5,10 and 20 m depths respectively. While the cross-correlation at 5 and 10 m depths are positive, the cross-correlation at 20 m depth is negative. This indicates that undercurrent may be present at 20 m depth.

Fig. 6 shows the phase spectra between the wind stress and the currents. When the tidal currents and horizontal pressure gradients are assumed absent, it will be possible to evaluate an eddy viscosity coefficient from cross-spectral computation by a method developed by D.M. FARMER (Farmer, 1972). Assuming the diffusion equation with a constant diffusion coefficient to be valid, it can be shown that an analytical solution for the diffusion equation exist with a time-dependent, harmonic oscillating surface stress as a boundary condition provided that

$$\nu_{\varepsilon} = const. \cdot \frac{w}{\phi^2} \tag{1}$$

where ν_{ε} is the diffusion coefficient, w is the frequency of the time-varying boundary condition, and ϕ is the phase lag between the surface stress and the current. The constant equals to:

$$const. = \frac{z^2}{2} \tag{2}$$

where z is the depth of the current with the phase lag ϕ.

In nature, the time variations of the surface boundary stress are made up by many different frequencies. These can be resolved by the spectrum technique. A diffusion coefficient is therefore obtained for each frequency w and each measurement depth z by means of the phase angle ϕ in the phase spectrum. The results from the computations are shown in the following table:

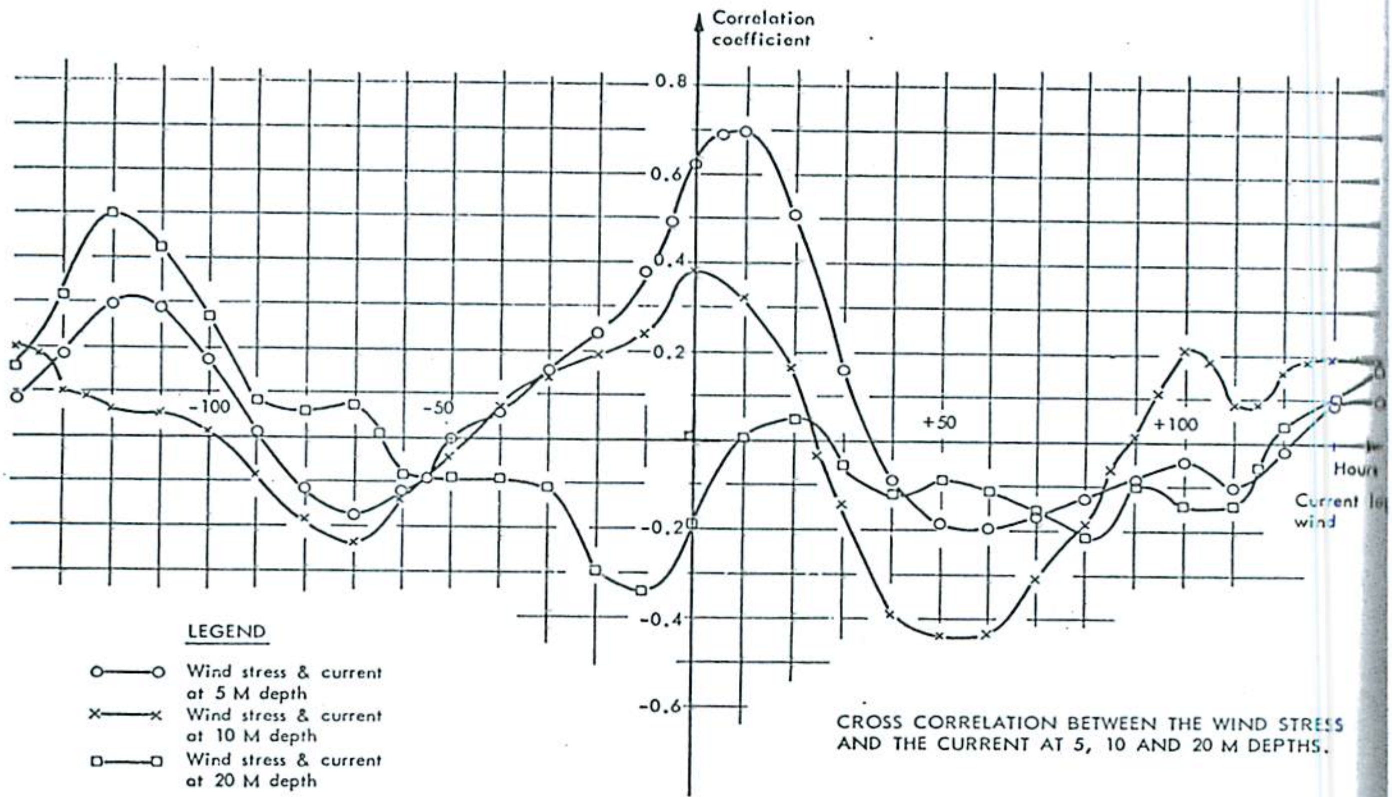
PERIOD HOURS	EDDY VISCOSITY ν_ϵ 5 M DEPTH	EDDY VISCOSITY ν_ϵ 10 M DEPTH	EDDY VISCOSITY ν_ϵ 20 M DEPTH
128	208	35.80	16.21
102	460	322	17.08
85.3	276	129	6.85
73.1	3.74	12.06	12.28
64.0	2.21	5.09	19.72
56.9	2.14	4.96	21.85
51.2	2.94	16.93	10.08
46.5	4.32	18.64	14.58
42.7	2.43	7.41	26.84
39.4	1.51	3.65	35.12
36.6	1.48	3.88	30.10

The numbers are diffusion coefficients in square cm per sec. The values at the lowest frequencies (85.3 hours and more) are possibly influenced by the presence of horizontal pressure gradients and should therefore be neglected. The values for the 20 m depth recording are also doubtful due to the possible existence of undercurrents. (Remember the negative cross-correlation shown on Fig. 5). For the upper 5 meters, an eddy viscosity of about 2 - 5 cm^2/sec. was estimated. Between 5 and 10 meters, the viscosity is apparently larger.

NUMERICAL SIMULATION OF THE SURFACE CURRENTS

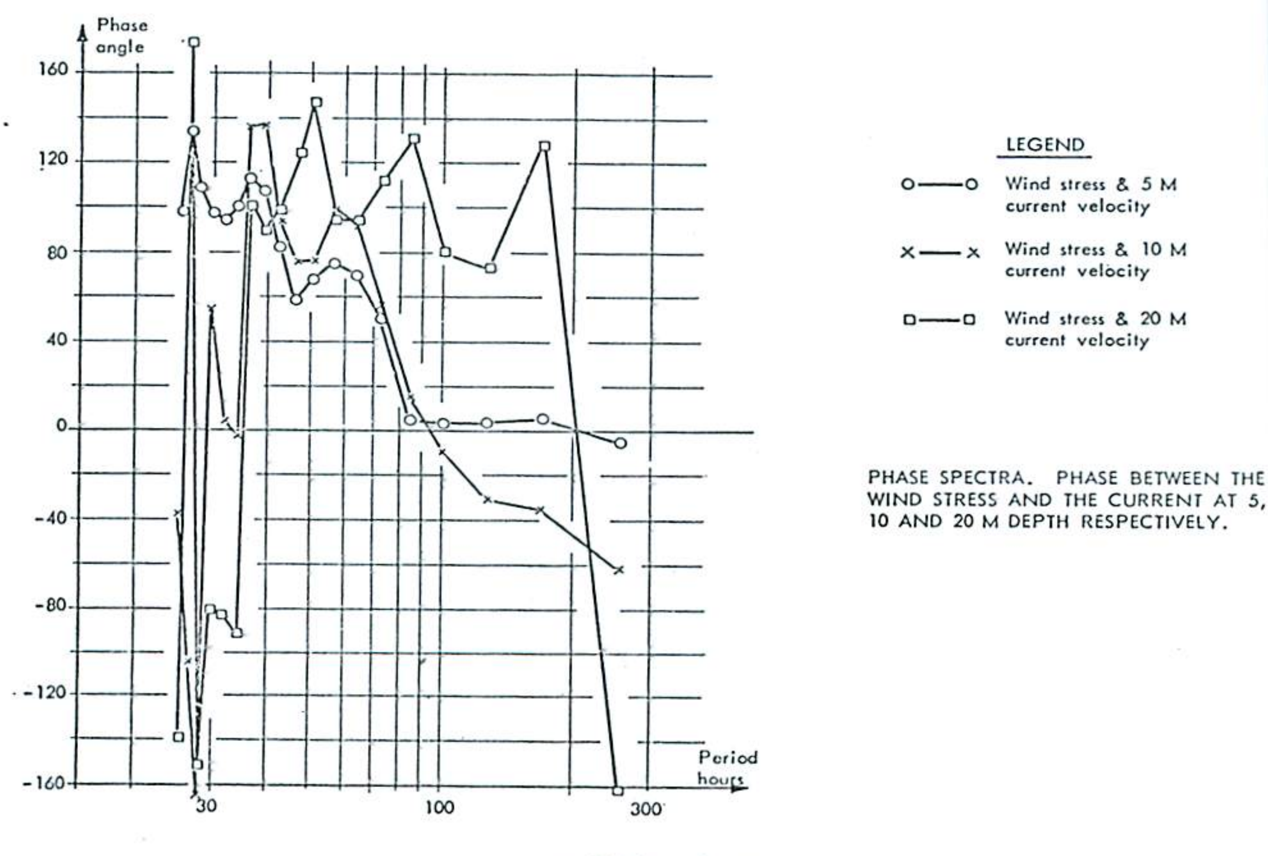
When winds and currents are compared, steady state conditions are often assumed. However, this may be a crude simplification due to rapid changes in the winds, both in strength and direction. It was therefore decided not to use the steady state assumption in this study.

Wave action makes it difficult to record surface current velocities directly. This problem may be handled by actually simulate the surface current velocities on a computer, using a numerical model. This was done, using a numerical version of the simple diffusion



Correlation coefficient
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-100
-50
+50
+100
Hours
Current lag wind
LEGEND
Wind stress & current at 5 M depth
Wind stress & current at 10 M depth
Wind stress & current at 20 M depth
CROSS CORRELATION BETWEEN THE WIND STRESS AND THE CURRENT AT 5, 10 AND 20 M DEPTHS.

\- FIG. 5 -



Phase angle
160
120
80
40
0
-40
-80
-120
-160
30
100
300
Period hours
LEGEND
Wind stress & 5 M current velocity
Wind stress & 10 M current velocity
Wind stress & 20 M current velocity
PHASE SPECTRA. PHASE BETWEEN THE WIND STRESS AND THE CURRENT AT 5, 10 AND 20 M DEPTH RESPECTIVELY.

\- FIG. 6 -

equation

$$\nu_\varepsilon \frac{\partial^2 U_w}{\partial z^2} - \frac{\partial U_w}{\partial t} = 0 \qquad (3)$$

where $U_W(z,t)$ is the water velocity. The surface stress recorded (proportional to wind velocity squared) was used as the surface boundary condition. The terms in (3) were approximated by the following finite differences:

$$\frac{\partial^2 U_w}{\partial z^2} = \frac{1}{(\Delta z)^2}\left[w_{i+1,j-1} - 2w_{i,j-1} + w_{i-1,j-1} \right] \qquad (4)$$

$$\frac{\partial U_w}{\partial t} = \frac{1}{\Delta t}\left[w_{i,j} - w_{i,j-1} \right] \qquad (5)$$

where

$w_{i,j}$ = the current computed.

i = index referring to actual depth level.

j = index referring to actual time step.

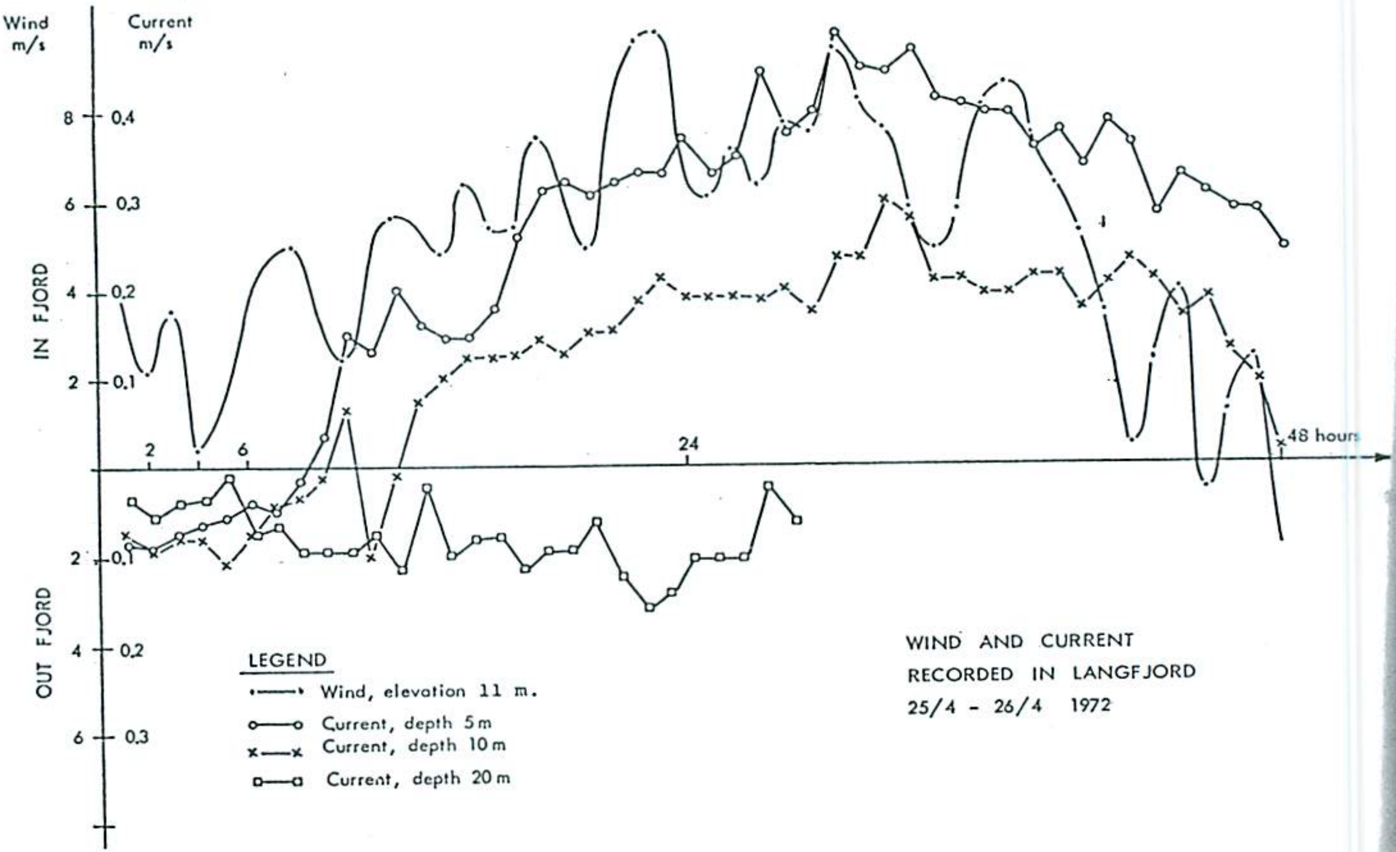
The surface boundary condition is described by the relation

$$\nu_\varepsilon \cdot \frac{\partial U_w}{\partial z} = \rho C_D \cdot WIND \cdot |WIND|$$

i.e. the current velocity gradient at the surface is determined by the wind stress. ρ is the density of air and C_D is the drag coefficient. In finite differences,

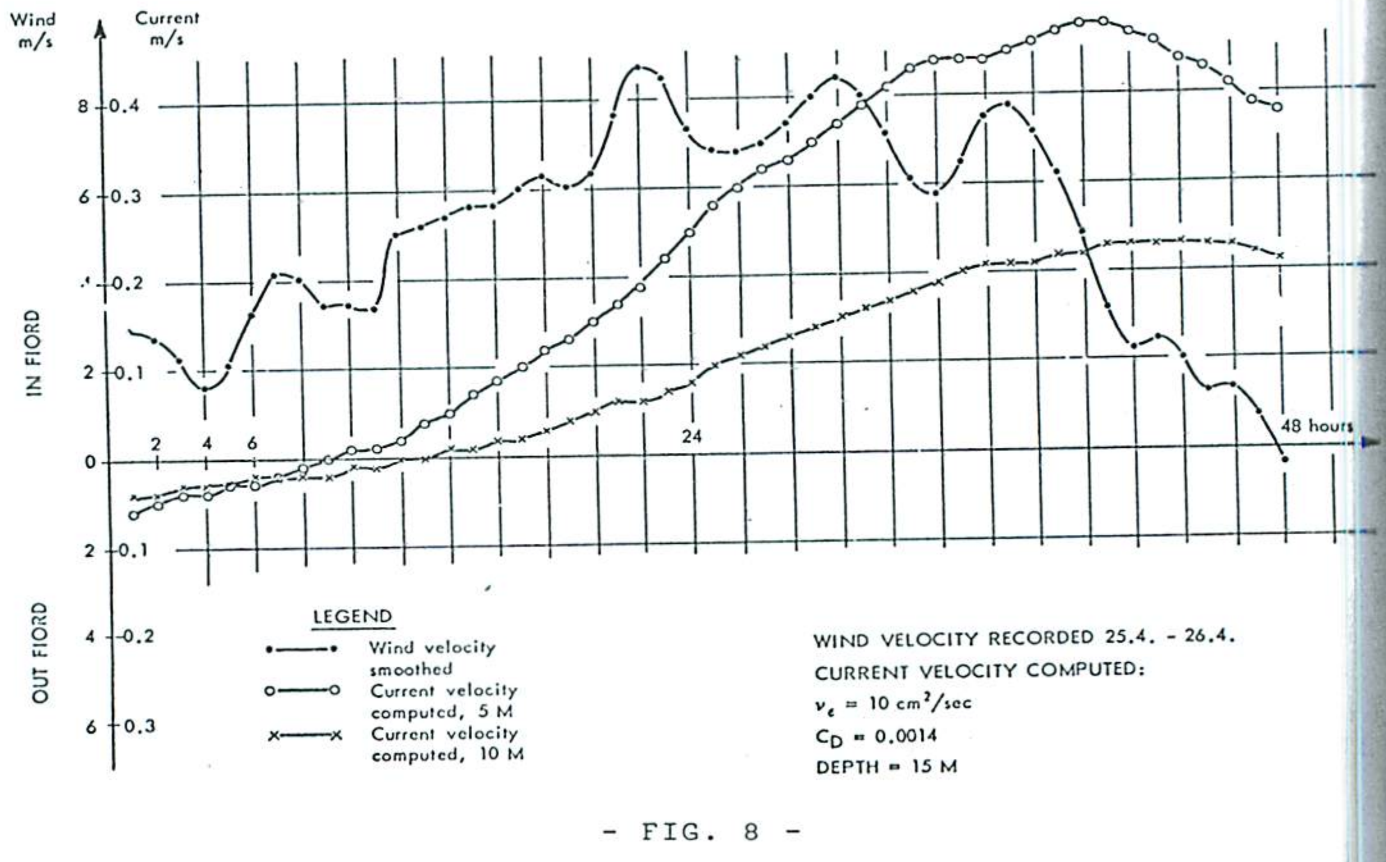
$$\left.\frac{\partial U_w}{\partial z}\right|_{surface} = \frac{1}{\Delta Z}\left[w_{o,j} - w_{1,j} \right]$$

The drag coefficient was chosen 0.0014 according to recent values found in the literature. (Denman and Miyake 1973, K. Hasselmann et. al. 1973). The momentum input at the surface was calculated from the actual wind recordings. The water velocity was set to zero at 15 meter depth due to the possible existence of reversing currents at 20 meters. The eddy viscosity was chosen 10 cm^2/sec. This value should represent an average of the eddy viscosities calculated for the upper 15 meters, see the table.



Wind m/s
Current m/s
IN FJORD
OUT FJORD
48 hours
LEGEND
Wind, elevation 11 m.
Current, depth 5 m
Current, depth 10 m
Current, depth 20 m
WIND AND CURRENT
RECORDED IN LANGFJORD
25/4 - 26/4 1972

- FIG. 7 -



Wind m/s
Current m/s
IN FIORD
OUT FIORD
48 hours
LEGEND
Wind velocity smoothed
Current velocity computed, 5 M
Current velocity computed, 10 M
WIND VELOCITY RECORDED 25.4. - 26.4.
CURRENT VELOCITY COMPUTED:
ν_e = 10 cm²/sec
C_D = 0.0014
DEPTH = 15 M

- FIG. 8 -

Fig. 7 shows a part of the actual wind and current recordings and Fig. 8 shows the currents computed from the numerical model with the given constants. In Fig. 8, the wind input has been smoothed to obtain computational stability. Note that in Fig. 7, the maximum current recorded at 5 meter depth is approximately 5% of the maximum wind velocity recorded.

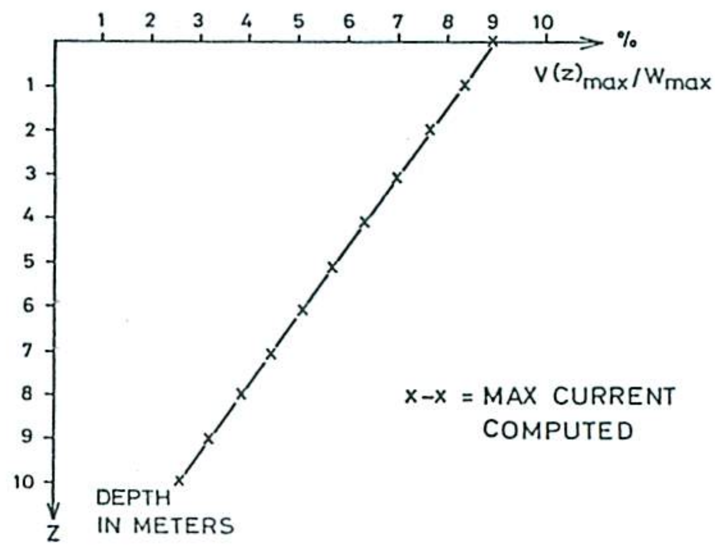
By comparison, it was concluded that the maximum currents recorded were modelled reasonably well by the maximum currents computed. The maximum surface currents velocities could therefore be determined from the numerical model.

Fig. 9 shows the maximum currents $V(z)_{max}$ computed at different depths, in per cent of the maximum wind velocity W_{max} recorded. Note that the maximum surface current computed was found to be 9% of the max. wind! This number is sharply contrasted to the expected number of 2 - 3% usually experienced.

The large value of the surface current is related to the value of the eddy viscosity of 10 cm^2/sec., which was found to be very low. For an eight to ten m/sec. wind (see Fig. 7), the eddy viscosity of surface waters are usually closer to 100 than to 10 (see Neumann and Pierson 1966). The numerical computations were therefore repeated with different choices of the eddy viscosity, using the same (observed) wind input. The results are shown on Fig. 10. The horizontal shows the eddy viscosity, and the vertical shows the max. surface current velocity computed, in per cent of the max. wind velocity recorded. For an eddy viscosity of 60 - 100 cm^2/sec., a max. surface current of approximately 2 - 3% was obtained. But for an eddy viscosity of 10, about 9% was obtained. The max. surface current of 9% is therefore attributed to the low value of the eddy viscosity found in the surface waters.

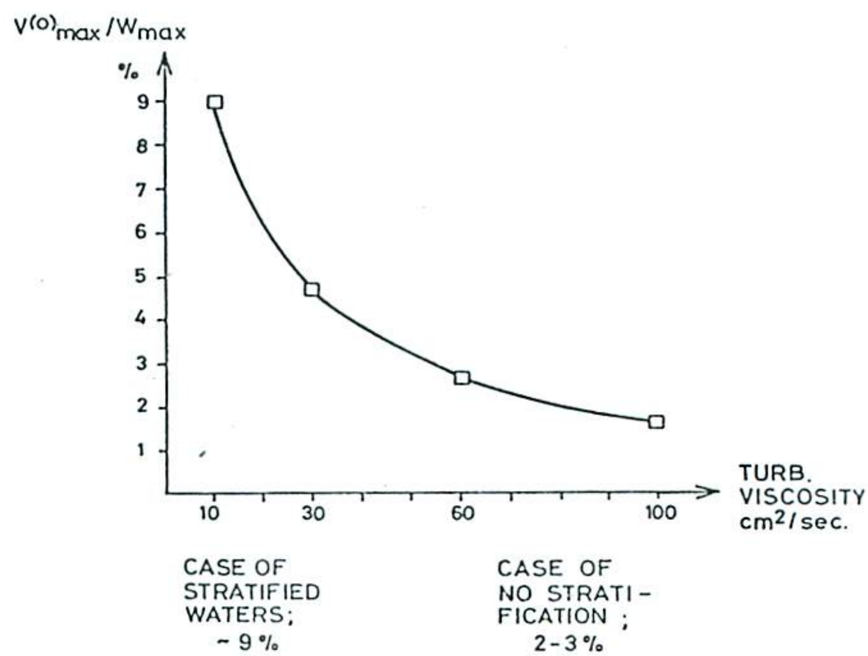
The explanation to the low value of the eddy viscosity coefficient is probably stable stratified water masses in the fiord. Fresh water discharge from the rivers mixes with the saline water into the stable stratified surface layer. Measurements of the vertical temperature and salinity profile showed some stratification; the salinity difference between 1 m and 20 m depth was about 1,5 o/oo. This stable stratification prevents the water masses from vertical mixing, which is also expressed by the low value of the eddy viscosity found. (A physical interpretation of ν_ε is that ν_ε expresses

VARIATION OF MAXIMUM CURRENT WITH DEPTH :



- FIG. 9 -

NUMERICAL SIMULATION OF VARIATION OF MAXIMUM SURFACE CURRENT WITH VISCOSITY :



- FIG. 10 -

the water masses' ability of vertical mixing). When the vertical exchange is reduced, the momentum transfer from the atmosphere is prevented from penetrating into the deeper layers of the fiord. The momentum transferred will therefore be concentrated to the upper layers of the surface waters, causing the extreme water velocities recorded.

For the design of floating bridges, this effect is important because frictional forces due to currents are proportional to the square of the current velocity. Further study on the wind effect on stratified waters is therefore necessary because of its practical implications.

CONCLUSIONS

The wind and the currents in 5, 10 and 20 meter depths have been recorded simultaneously in a deep and narrow fiord during a 22 day period. Correlations, spectral and cross-spectral analysis were applied to the data. The currents redorded were simulated by means of a numerical diffusion model. The results from the computations may be summarized as follows.

1. The cross-spectral method developed by D.M. FARMER for evaluation of diffusion coefficients has been successfully applied. The eddy viscosity of the surface waters of the fiord was found to be within the range 1 - 20 cm^2/sec.

2. To extrapolate recorded wind current velocities to the surface, numerical simulation of the wind currents has proven to give reasonable results.

3. The maximum surface current velocity computed was found to be about 9% of the maximum wind velocity recorded. This large number is explained to result from the stable stratified surface waters of the fiord.

4. A stratification of the surface waters was found to have considerable influence on wind-generated currents. The study of its influence is important in order to obtain design criteria for construction of floating bridges in stratified waters and for pollution control.

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