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ON THE DETERMINATION OF MANNING'S n FOR FLOOD PROBLEMS OVER A HORIZONTAL INITIAL DRY BED

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1. INTRODUCTION

This paper presents theory and data analysis for determining the extent of long wave run-up flooding over a dry bed. Such can be experienced during tsunamis, tidal waves and tidal bores, all of which at one time or another might occur anywhere in the coastal world. It is only necessary to determine the proper values of Manning's n during past conditions, and then select the proper values of Manning's n, either for similar or improved conditions. It is important to note that for improved conditions Manning's n will be less than that for observed past conditions. Therefore, under improved conditions, the flooding potential will be greater than that for past conditions. This paper gives examples for actual conditions for a Japanese rain storm and also for tsunami flooding at Hilo, Hawaii. For improved conditions, it should be expected that the extent of flooding will be greater than that which occurred during past historical situations.

2. FRICTION LAW RELATIONSHIPS

The frictional shear stress opposing the flow of water in a channel is proportional to the square of the mean velocity over the section and is given by the Darcy-Weisbach (1858) relationship

$$\tau = f \frac{\rho V^2}{2} \tag{1}$$

Here τ is the bottom shear stress, ρ the fluid density, V the mean velocity and f is a non-dimensional frictional coefficient. For a circular pipe of full flow $\rho/2$ is replaced with $\rho/8$, since the hydraulic radius for a pipe is $R = D/4$, where D = diameter.

The frictional coefficient depends not only on the roughness of the surface, but also the depth of fluid. Attempts to separate these effects have led to various laws. These laws are usually formulated in terms of the steady-state discharge rate over a bottom of constant slope, S in this case. The frictional stress is balanced by the component of the gravitational force along the slope. If the wetted perimeter is P the total frictional force per unit length of channel is τP. Similarly, if the submerged area is A, the gravitational force down the slope, per unit length of channel, is ρgSA. Thus,

$$\tau = 1/2\, f \rho V^2 = \rho gSA/P = \rho gSR \qquad (2)$$

Where R is the hydraulic radius, defined by

$$R = A/P \qquad (3)$$

Note that for a wide channel, R is equal to the depth of water. Solving for V

$$V = \sqrt{\frac{2gRS}{f}} \qquad (4)$$

Chezy's formula (1775) states that

$$V = C\sqrt{R\,S} \qquad (5)$$

C is called the Chezy coefficient, and it has dimensions of $(\text{length})^{1/2}/(\text{time})$. From (4) and (5)

$$f = \frac{2g}{C^2} \qquad (6)$$

Thus, the assumption that the Chezy coefficient depends only on the roughness is equivalent to assuming that the frictional coefficient f depends only on the roughness. This is actually not the case, and very often Chezy coefficients are tabulated for different values of the hydraulic radius, and f is given in terms of roughness and different values of pipe diameter.

The most common formulation that attempts to correct this defect is Manning's formula:

$$V = \frac{1}{n} R^{2/3} S^{1/2} \quad \text{metric system} \qquad (7a)$$

V = (m/s) and R = meters

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \quad \text{English system} \qquad (7b)$$

V = ft/sec and R = feet

Where n is a coefficient of dimensions $(\text{length})^{-1/3}$ (time) and is called Manning's n, and the numerical value of n is the same in both the metric and the English systems. Typical values of Manning's n are given in Table I.

TABLE I

TYPICAL VALUES OF MANNING'S n (after Creager and Justin, 1950)

Conditions	Manning's n
New canals in sandy loam to clay loam.	0.020
Medium to large canals in firm earth or gravelly loam operated by organization that will give reasonable maintenance	0.0225
Best conditions of dredged earth canals.	0.0225
Minimum value for canals excavated in rock	0.023
Worst or probable maximum value of concrete linings not subject to rejection because of bad workmanship	0.018
Gunite linings, probable if not scrubbed	0.019
Gunite linings, worst for poor workmanship	0.021
Brick in cement mortar, worst.	0.017
Dressed ashlar surface, worst.	0.017
Bench flume, consisting of natural rock surface for the uphill side, a smooth concrete retaining wall on the downhill side, and with a flow between. Floor lined with concrete and clean. Upside without projecting points	0.020
Cement rubble surface	
best	0.017
worst	0.030
Dry rubble surface	
best	0.025
probable	0.033
worst	0.035
Earth channel with gravel bottom	0.025
Earth channel with clean bottom, brush on sides.	0.05
Flood plain with dense brush	0.100

NOTE: Detailed descriptions, including photographs of actual situations for Manning's n measurements, are given in the U.S. Dept. of Agriculture Bulletin No. 832, "The Flow of Water in Dredged Drainage Ditches" by C.E. Ramser, June 7, 1920.

The frictional law of von Karman, as applied to a wide channel ($R = h$) relates the shear stress τ to the velocity profile $v(z)$ as

$$\frac{\tau}{\rho} = \left[- \ k^2 \ \left| \frac{dv(z)}{dz} \right|^3 \ \frac{dv(z)}{dz} \right] \div \left[\left| \frac{d^2v(z)}{dz^2} \right|^2 \right] \tag{8}$$

where $k = 0.4$ is called von Karman's constant. This integrates to

$$v(z) = \frac{1}{k} \sqrt{\frac{\tau}{\rho}} \ \ln \left(\frac{z + z_o}{z_o} \right) \tag{9}$$

where $\ln$ = natural log (base e) and z_o is a parameter that measures the roughness of the surface and has the dimensions of length.

For a flow of depth h, the mean velocity is

$$V = \frac{1}{h} \int_o^h v(z)\, dz$$

$$= \frac{1}{k} \sqrt{\frac{\tau}{\rho}} \left\{ \frac{h + z_o}{h} \ \ln \left(\frac{h + z_o}{z_o} \right) - 1 \right\} \tag{10}$$

Assuming $h >> z_o$, this becomes

$$V = \frac{1}{k} \sqrt{\frac{\tau}{\rho}} \ \ln \left(\frac{h}{e\ z_o} \right) \tag{11}$$

Thus, the frictional coefficient becomes

$$f = 2 \left(\frac{k}{\ln\ [h/ez_o]} \right)^2 \tag{12}$$

Comparison with equation (6) gives

$$C = \frac{\sqrt{g}}{k} \ \ln \ \left(\frac{h}{ez_o} \right) \tag{13}$$

The k^* and K friction factors for storm surge given by Bretschneider (1967) are related to the Darcy-Weisbach friction factor and Manning's n as follows

$$k^* = f = KR^{-1/6} \ Kh^{-1/6} = (R \text{ in feet}) \tag{14}$$

$$K = \frac{g}{(1.486)^2} n^2 = \frac{32.2}{(1.486)^2} n^2 = 14.6 n^2 (ft^{1/3}) \qquad (15)$$

3. ON THE USE OF THE APPROPRIATE FRICTION FACTOR RELATIONSHIPS

Manning's equation in metric units is as follows:

$$V = \frac{1}{n} R^{2/3} S^{1/2} \qquad (16)$$

where

- V = mean velocity in meters per second
- R = hydraulic radius in meters
- S = slope of water surface meters/meter
- n = Manning's n (friction factor)

In the English system for V = feet/sec, R = feet and S = feet/feet equation (16) becomes

$$V = \frac{1.486}{n} R^{2/3} S^{1/2} \qquad (17)$$

It should be noted, definitely, that Manning's n has exactly the same numerical value in both English and metric systems, but the units are $(\text{meter})^{-1/3}$ sec for the metric system and $(\text{feet})^{-1/3}$ sec for the English system. The cube root of 3.28 = 1.486 is the conversion factor from metric to English system, since 1 meter is equal to 3.28 feet.

The Chezy-Kutter formula is given by

$$V = C \sqrt{RS} \qquad (18)$$

where C is the Chezy coefficient, and is related directly to Manning's n in equation (16) and equation (17), respectively, as follows:

$$C = \frac{1}{n} R^{1/6} \qquad \text{metric system} \qquad (19)$$

$$C = \frac{1.486}{n} R^{1/6} \qquad \text{English system} \qquad (20)$$

Since Manning's n is exactly the same in both the metric and the English systems, then the Chezy coefficient C in the metric system must be different from that in the English system. The relationship is as follows:

$$C \text{ (English system)} = 1.486 \sqrt[6]{3.28}\, C = 1.486 \sqrt{1.486}\, C$$

$$= 1.815\, C \text{ (metric system)} \tag{21}$$

Whereas, Manning's n is independent of the hydraulic radius, the Chezy coefficient varies as $R^{1/6}$. It might also be mentioned that the Darcy-Weisbach friction factor f also varies with pipe diameter, water depth, and hence hydraulic radius.

Manning's n is related to the Darcy-Weisbach friction factor (see equation 1) as follows:

$$f = 0.9\, gn^2\, h^{-1/3} \tag{22}$$

Manning's n is related to the von Karman friction length z_o as follows:

$$n = \frac{1.486\, h^{1/6}\, k}{\sqrt{g}\, \ln\,(h/ez_o)} \tag{23}$$

Values of Manning's n are given in Table II as a function of z_o and h.

For water of variable depth, as for example over the continental shelf, or of a surge over water (i.e., rivers and flood plains) and over a dry bed, Manning's n for any particular roughness remains essentially constant for all water depths. This is not true for the other friction relationships. Furthermore, Manning's n has the same numerical value whether used in the metric or the English system. Manning's n does not change for flow in pipes of various diameters, or channels and canals of various depths, hence, it depends only upon the roughness. There is available a tremendous amount of information on Manning's n as a function of various types of roughness. All of the above facts justifies the exclusive use of Manning's n over all other friction factor relationships.

4. BASIC EQUATIONS FOR LONG WAVES

The governing equations for long waves within a confined channel consist of the momentum and continuity equations. These equations are written as:

$$\text{Momentum:} \quad \frac{dv}{dt} = -g\frac{\partial H}{\partial x} - \frac{g}{C^2\,(d_o + H)} V|V| - g(m_1 - m_2) \tag{24}$$

TABLE II

RELATIONSHIP BETWEEN MANNING'S n AND THE FRICTION LENGTH z_o

(z_o BASED ON WIND MEASUREMENTS)

Conditions	z_o (cm)	Manning's n		
		h = 5 (ft)	h = 10 (ft)	h = 50 (ft)
Very smooth (mud flats, ice)	0.001	0.013	0.013	0.015
Lawn grass up to 1 cm high	0.1	0.022	0.022	0.023
Downland, thin grass up to 10 cm high	0.7	0.031	0.030	0.030
Thick grass up to 10 cm high	2.3	0.043	0.040	0.030
Thin grass up to 50 cm high	5.0	0.057	0.049	0.043
Thick grass up to 50 cm high	9.0	0.073	0.061	0.049

NOTE: z_o values are obtained from Proudman (1953) and are based on measured wind speed.

$$\text{Continuity:} \quad B\frac{\partial H}{\partial t} + \frac{\partial (AV)}{\partial x} = 0 \tag{25}$$

where

- H is the water level height above the initial level
- d_o is the natural water depth
- m_1 is the natural slope of the water surface
- m_2 is the slope of the bottom
- B is the surface width of the water stream
- A is the cross-sectional area of the stream
- t is the time
- x is the longitudinal coordinate in direction of flow

and the other symbols are previously defined.

We will consider only the special case of a flat bottom ($m_2 = 0$) and a very wide channel, in which case we do not have to consider continuity. Furthermore, we will use Manning's n instead of Chezy's coefficient and go directly to a very simple formulation of Manning's equation.

5. THE FORMULATION OF MANNING'S EQUATION IN DIFFERENTIAL FORM

For this purpose we will use the English form (namely equation (17)). The slope of the water surface is given by $S = \frac{ds}{dx}$ and the hydraulic radius $R \cong$ depth of water, h, for wide rivers, channels or flood plains. Manning's equation can then be written as follows:

$$\frac{ds}{dx} = \frac{n^2 V^2}{(1.486)^2} h^{-4/3} \tag{26}$$

We can define a Froude number F_N as follows:

$$F_N{}^2 = \frac{V^2}{gh} \tag{27}$$

whence

$$\frac{ds}{dx} = \frac{n^2}{(1.486)^2} \frac{F_N{}^2 h^{-1/3}}{g} \tag{28}$$

Assuming F_N is constant, i.e., $\frac{V^2}{gh}$ = constant, then the solution of the above equation can be given in parametric form as follows:

$$\frac{h}{h_o} = \left[1 - \left(\frac{x}{x_R}\right)\right]^{3/4} \qquad (29)$$

and

$$x_R = \frac{h_o\ ^{4/3}}{19.5\ F_N^{\ 2}\ m^2} \qquad (30)$$

or

$$x_R = \frac{1}{K^*}\ h_o\ ^{4/3} \qquad (31)$$

where

K^* = $\left[19.5\ F_N^{\ 2}\ n^2\right]$

h_o = initial depth of water at location $x = 0$

x_R = distance inland to complete dissipation

The solutions to equations (29) and (30) are shown in Figures 1 and 2 respectively.

6. VALUES OF MANNING'S n FROM JAPANESE FLOODING

The problem of the flooding of a city (Tokyo district) by the spilling of a river as caused by a severe rainstorm such as a typhoon was investigated by Hatori (1963) for the eastern part of Tokyo. Here, the Chezy formula (equation (5)) coupled with the Manning's equation in metric form (equation (7a)) were used to determine values of Manning's n at 12 selected stations along the flood line. The data for each station, along with the computed values of Manning's n are shown in Table III. It is deduced that an average of $n = 0.0185$ with a range $0.011 \leq n \leq 0.049$ was obtained.

One might mention that the area, A, in Table III is the sectional area one would obtain in the absence of buildings. Since the Tokyo district has a high density of buildings, the use of an effective area could slightly modify the results in that the hydraulic radius could no longer be approximated by the mean water height.

TABLE III

FLOW DATA AND MANNING'S n AS DETERMINED BY HATORI (1963)

FOR THE TOKYO FLOOD OF SEPTEMBER 15, 1947

Station No.	Area A (m)	Mean Height H (m)	Velocity V (m/sec)	Chezy Coefficient C ($m^{1/2}$/sec)	Manning's n n ($ft^{-1/3}$/sec)
1	2800	1.53	0.44	22.1	0.049
2	2500	1.19	1.20	68.3	0.015
3	3000	1.45	1.00	51.5	0.021
4	1780	0.97	1.69	106.0	0.009
5	2480	1.06	1.21	73.0	0.015
6	2260	0.91	1.33	86.5	0.011
7	4850	1.07	0.62	37.2	0.023
8	5200	0.93	0.58	37.4	0.026
9	3900	0.76	0.77	54.5	0.017
10	2900	0.65	1.03	86.0	0.011
11	2660	0.67	1.13	85.5	0.011
12	2660	0.96	1.13	71.6	0.014

7. APPLICATION AND RESULTS FOR HILO, HAWAII, FOR THE 1946 AND 1960 TSUNAMIS

The coastal land profiles in the vicinity of Hilo, Hawaii are modelled as shown in Figure 3 where the nearshore region is characterized by a beach or berm followed by a relatively flat region which fronts a steep bluff. Because of the large percentage of land near Hilo that can be represented in this manner, the flat bottom equations as developed in Section 5 will be used to predict the inundation profiles of the 1946 and 1960 tsunami surges using recorded observations taken at the time of the particular tsunami. Based on the observed data (i.e., run-up elevations and inundation heights), Manning's n will be determined for each location.

Figure 4 illustrates the general plan of the Hilo vicinity and indicates particular locations where run-up and inundation limits for the two tsunamis were recorded. The land profiles are then determined such that each profile is nearly parallel to the direction of flow (reported by Eaton, et al., 1961)) at each observation point as shown in Figure 5. The land profiles along with the profiles of maximum obtainable water height for each tsunami are shown in Figures 6 and 7. The criteria for the construction of the curves of maximum water height was that the land roughness did not change appreciably during the period 1947 to 1960. Each curve then represents the decay in maximum water height as would be expected from the historical data, whence the value of Manning's n is determined. Table IV lists the Manning's n for each land profiles. Note that profile "L" contains three data points in the case of the 1960 tsunami; namely, the inundation height and two observed run-up elevations.

Figures 8 and 9 show the maximum water level contours for each tsunami as obtained from Figures 6 and 7. Each contour represents a 5 foot difference in elevation.

8. SUMMARY AND CONCLUSIONS

The extent of flooding for long waves, such as tsunamis, tidal waves, tidal bores, etcetera, should take into account the initial wave height, Green's Law, convergences or divergences, and friction losses. The Froude number F_N is a very important criteria. Empirical data are particularly important in order to evaluate the proper values of the friction factor; namely Manning's n. When these values have been determined for existing conditions, then it is possible to predict flooding under existing conditions and also for the extreme conditions. If the existing conditions are changed because of development, then it will be important to select the appropriate value of Manning's n in order to predict the extent of normal and extreme flooding under the improved conditions.

TABLE IV

MEASURED TSUNAMI WAVE HEIGHTS AT HILO

AND THE COMPUTED VALUES OF MANNING'S n

Profile	Observed Elevation		Manning's n
	1946 (ft)	1960 (ft)	($ft^{-1/3}/sec$)
A	32	13	0.020
B	32	10	0.025
C	23	14	0.020
D	29	16	0.030
E	27	13	0.023
F	21	12	0.020
G	10	12	0.025
H	21	21	0.015
I	23	20	0.017
J	-	20	0.016
K	17	26	0.018
L	-,-	35, 20	0.020
M	-	28	0.024
N	25	20	0.024
O	26	22	0.020
P	21	13	0.017

LIST OF SYMBOLS

A	cross-sectional area of the stream
B	surface width of water stream
C	Chezy coefficient
ds	incremental slope
dx	incremental length in direction of flow
dz	incremental elevation
e	constant, base of natural logarithms
F_N	Froude number
f	Darcy-Weisbach friction factor
g	acceleration of gravity
h	water surface elevation at location x
h_o	initial water surface elevation, at $x = 0$.
K	a frictional coefficient
k	von Korman constant
K^*	bottom friction parameter
m_1	natural slope of the water surface
m_2	slope of the bottom
n	Manning's n
P	wetted perimeter of channel
R	hydraulic radius of open channel
S	slope of water surface
t	time coordinate
v(z)	water velocity distribution
V	mean flow velocity
x	coordinate measured in direction of flow
x_R	distance inland that water will travel for complete dissipation
z	vertical coordinate, elevation
ρ	mass density of water
τ	bottom shear stress

LIST OF REFERENCES

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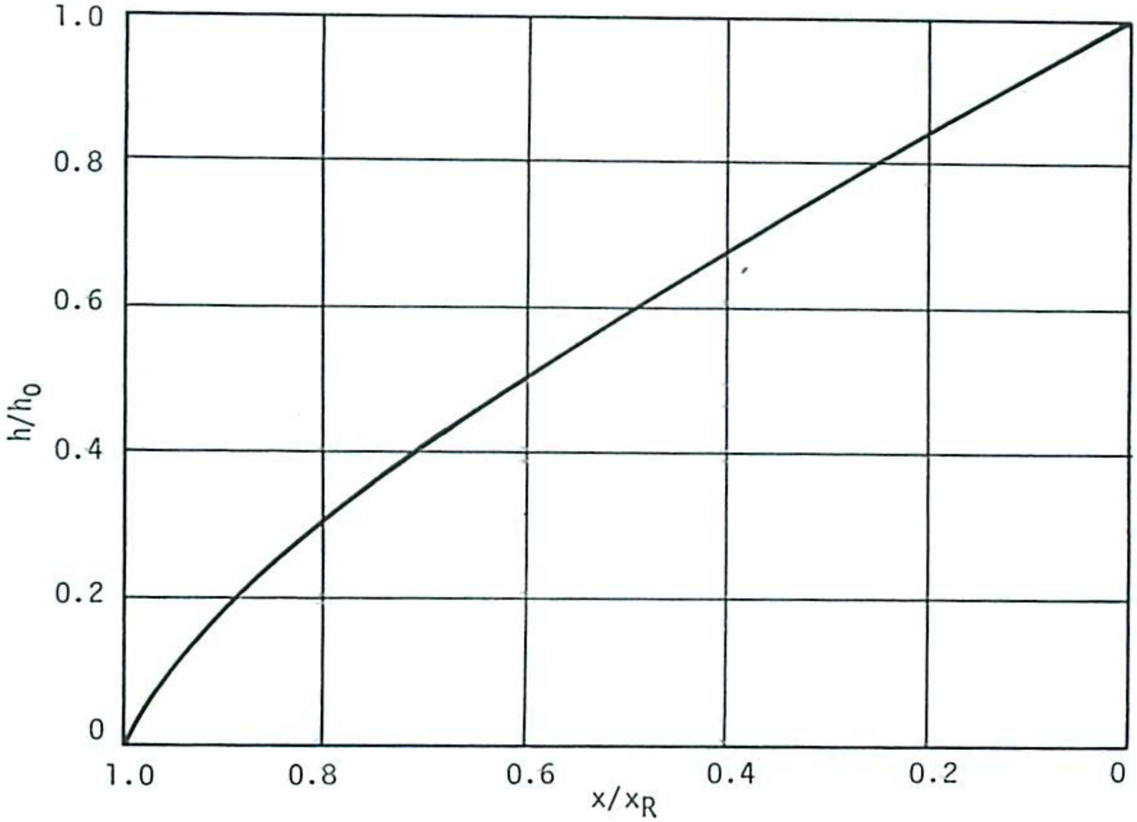


FIGURE 1 WATER SURFACE ELEVATION OF BORE OVER A FLAT DRY BOTTOM: NON-DIMENSIONAL SOLUTION OF h/h_o vs. X/X_R

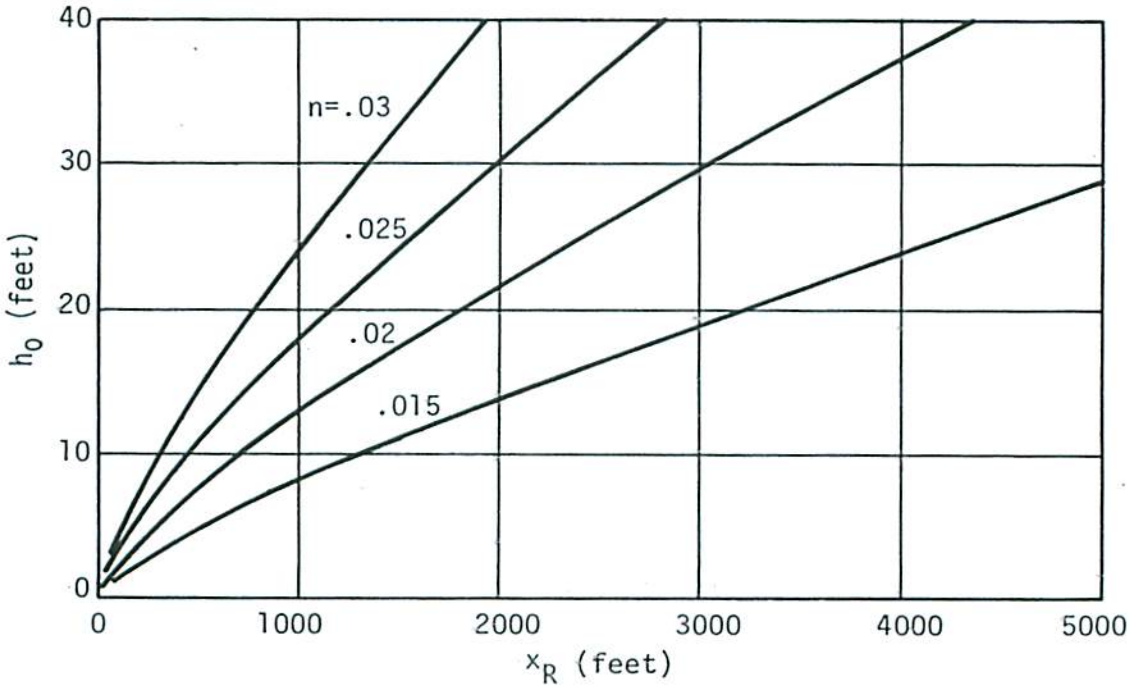
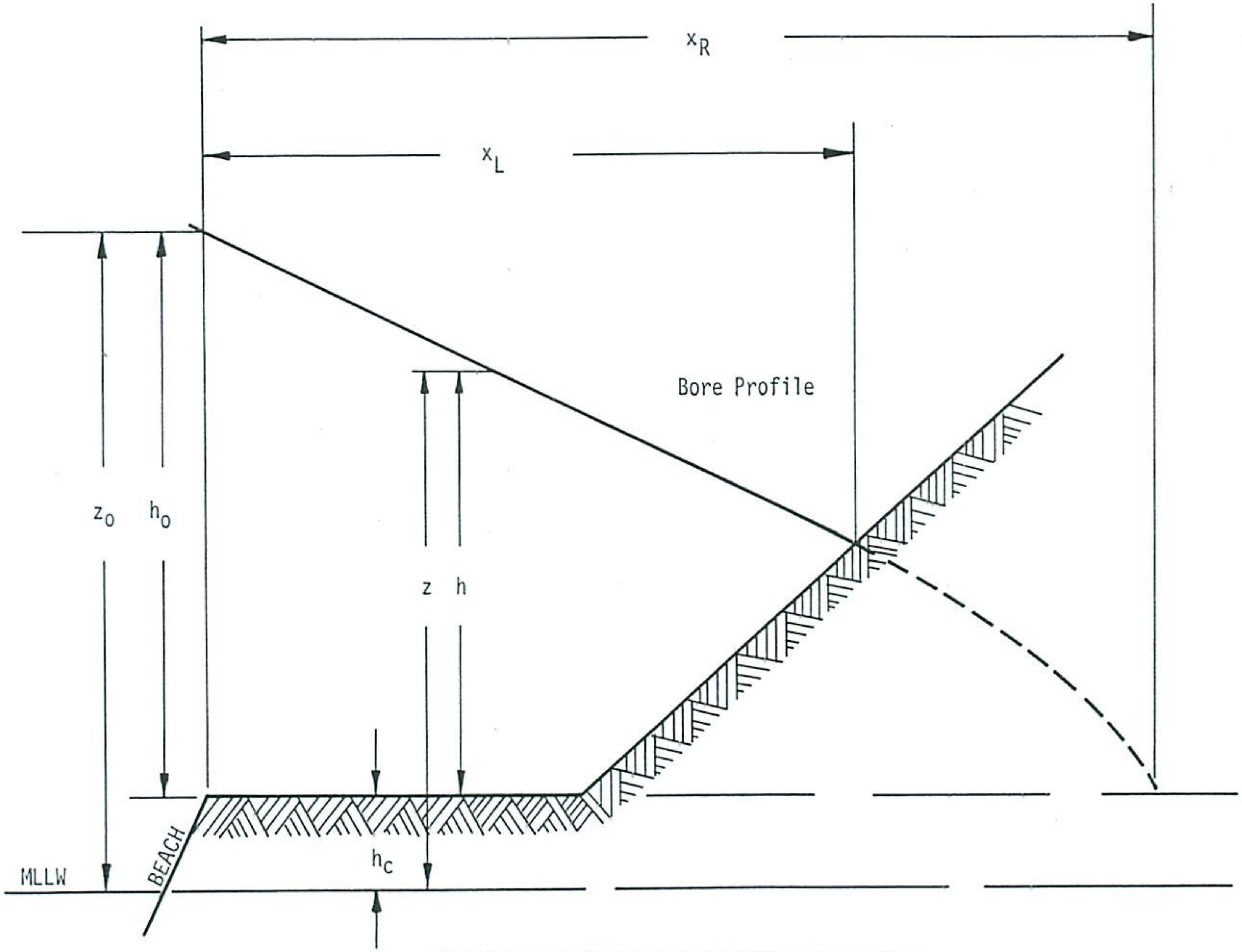


FIGURE 2 HORIZONTAL DISTANCE OF RUN-UP FOR FLAT DRY BOTTOM (F_N = 2)



SCHEMATIC DIAGRAM FOR DEFINITION OF SYMBOLS

FIGURE 3

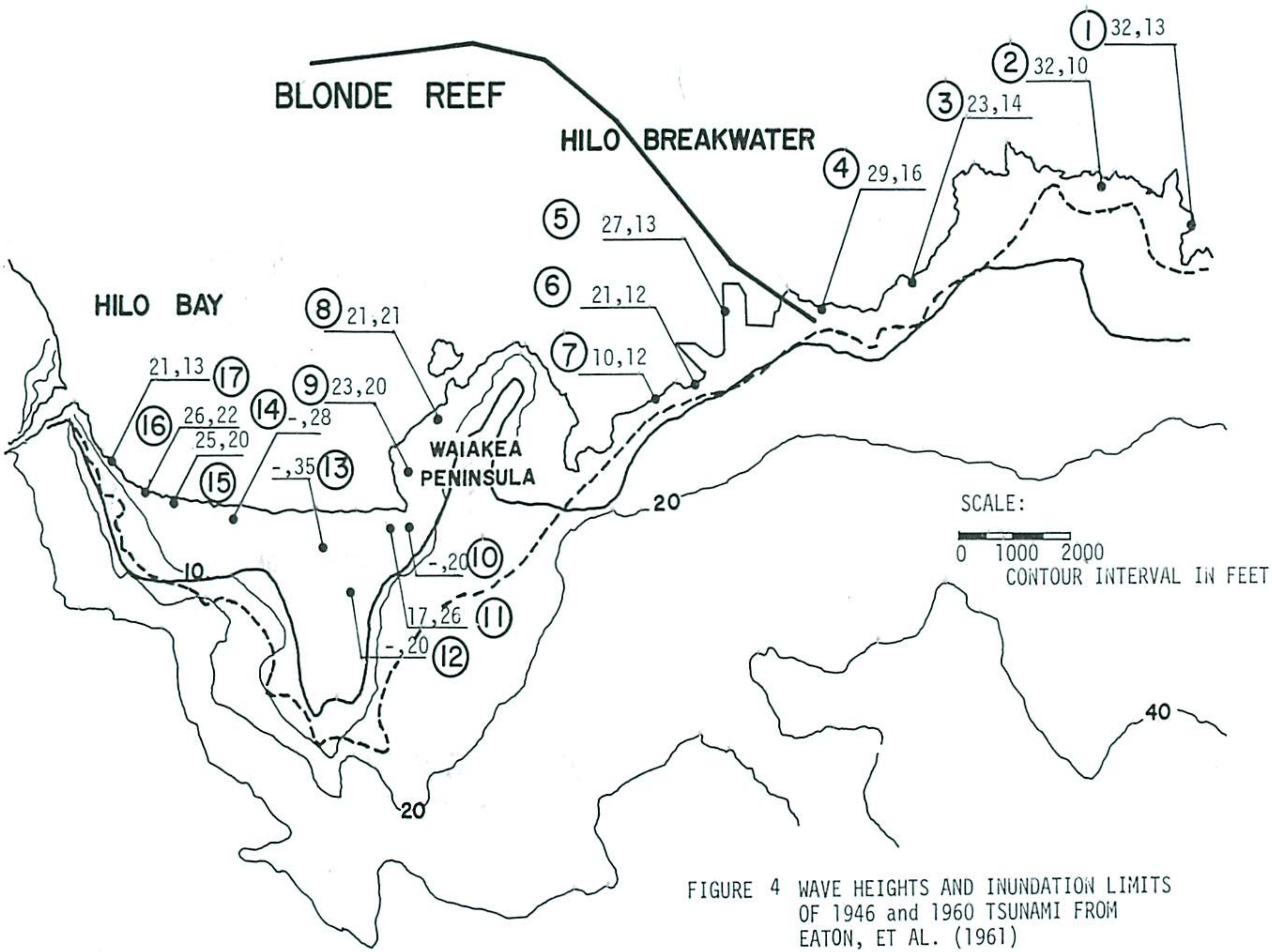
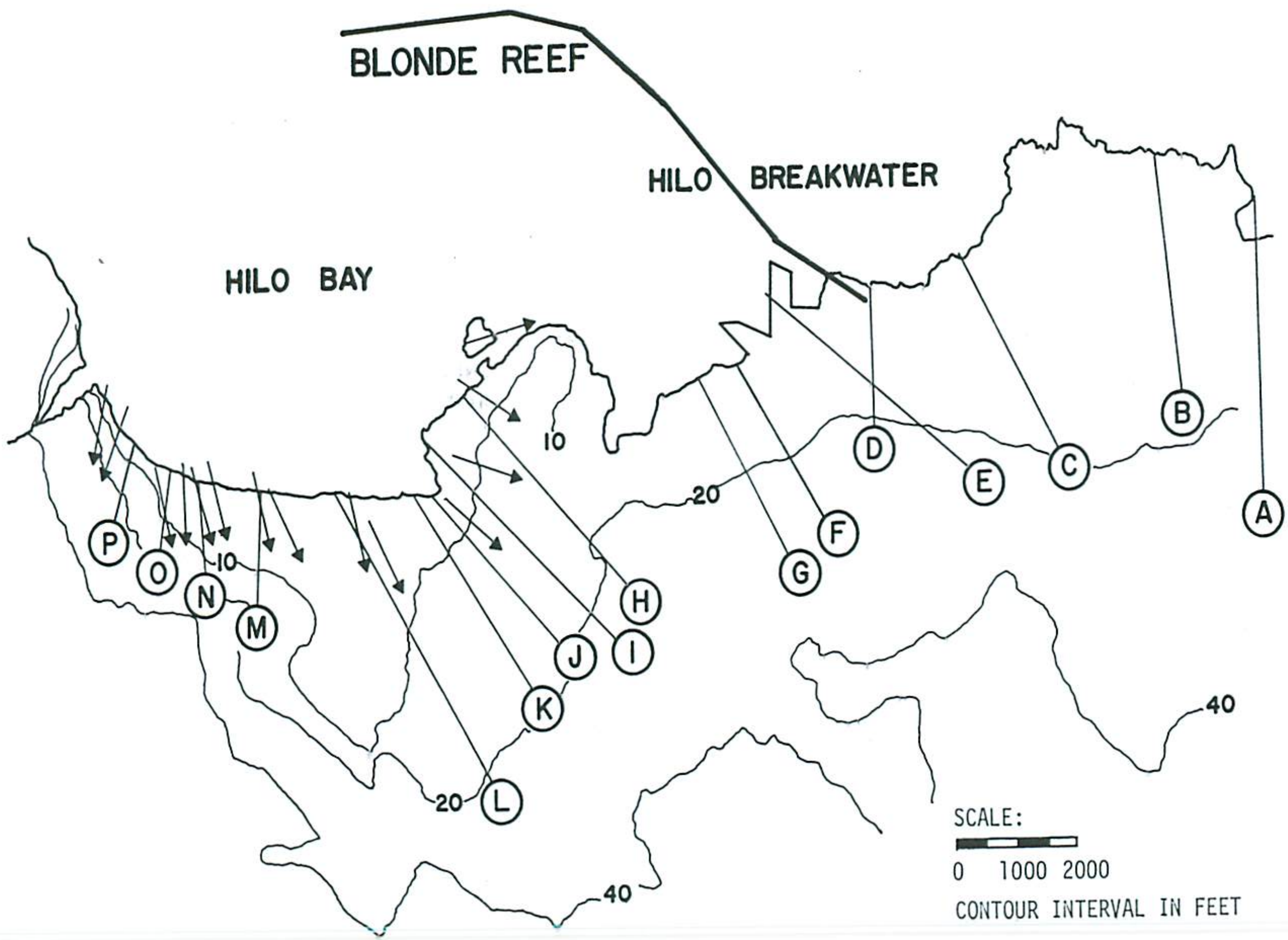
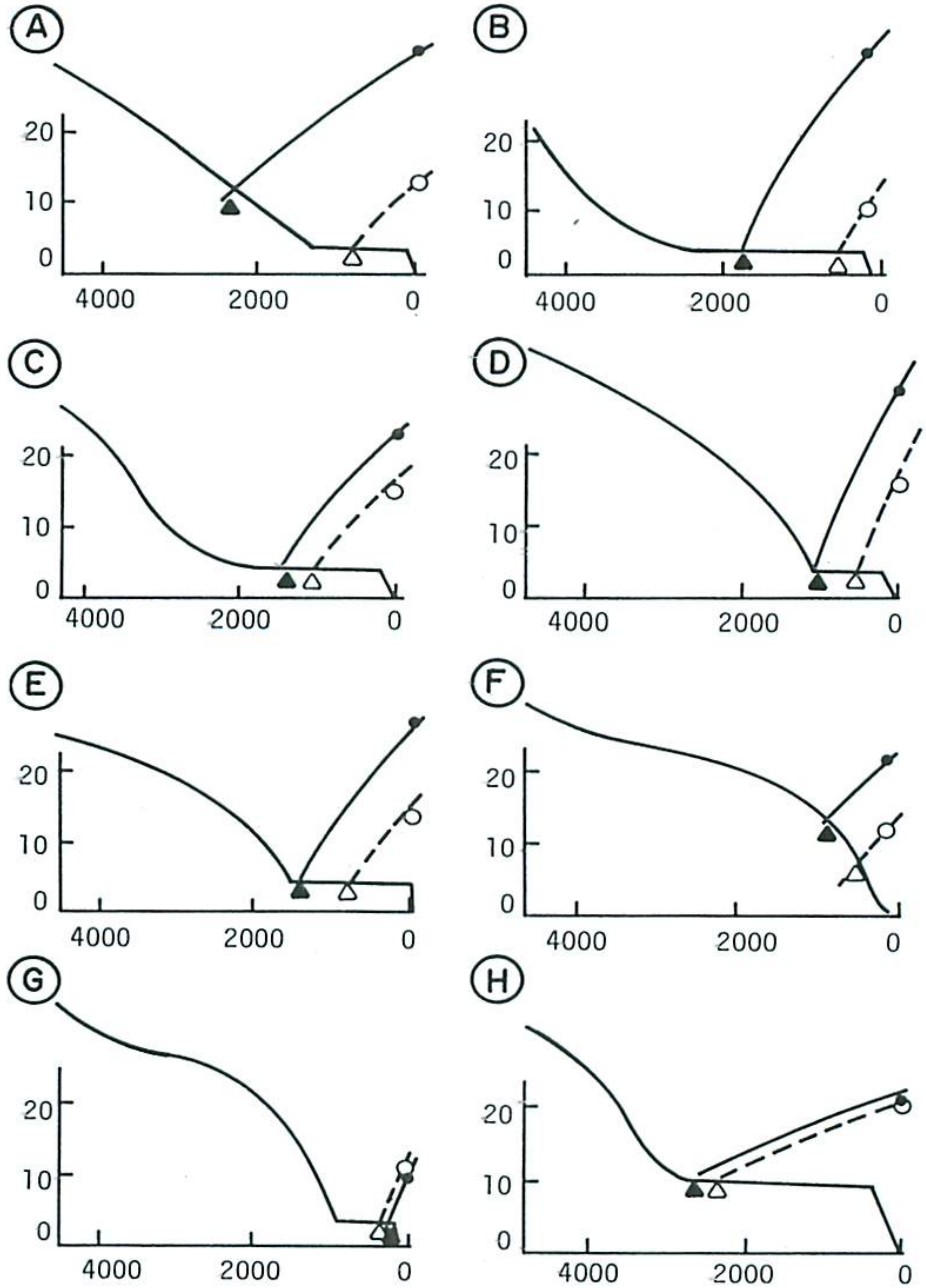


FIGURE 4 WAVE HEIGHTS AND INUNDATION LIMITS OF 1946 and 1960 TSUNAMI FROM EATON, ET AL. (1961)

FIGURE 5 PROFILE CONSTRUCTION AND DIRECTION OF 1960 WAVE ADVANCE

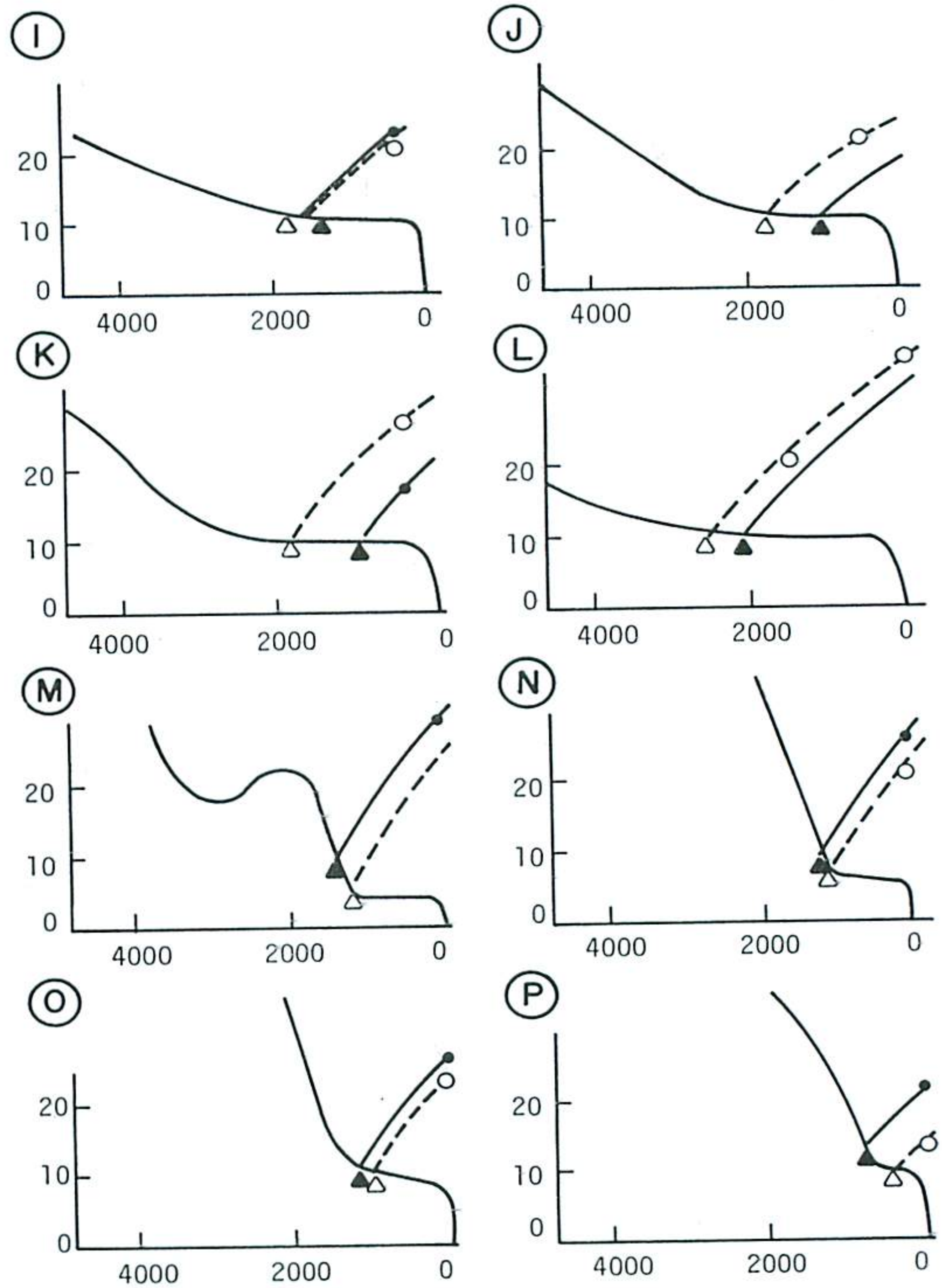




KEY:

▲ Inundation Limit, 1946 Tsunami
△ Inundation Limit, 1960 Tsunami
● Observed Elevation, 1946 Tsunami
○ Observed Elevation, 1960 Tsunami

FIGURE 6



KEY:

▲ Inundation Limit, 1946 Tsunami
△ Inundation Limit, 1960 Tsunami
● Observed Elevation, 1946 Tsunami
○ Observed Elevation, 1960 Tsunami

FIGURE 7

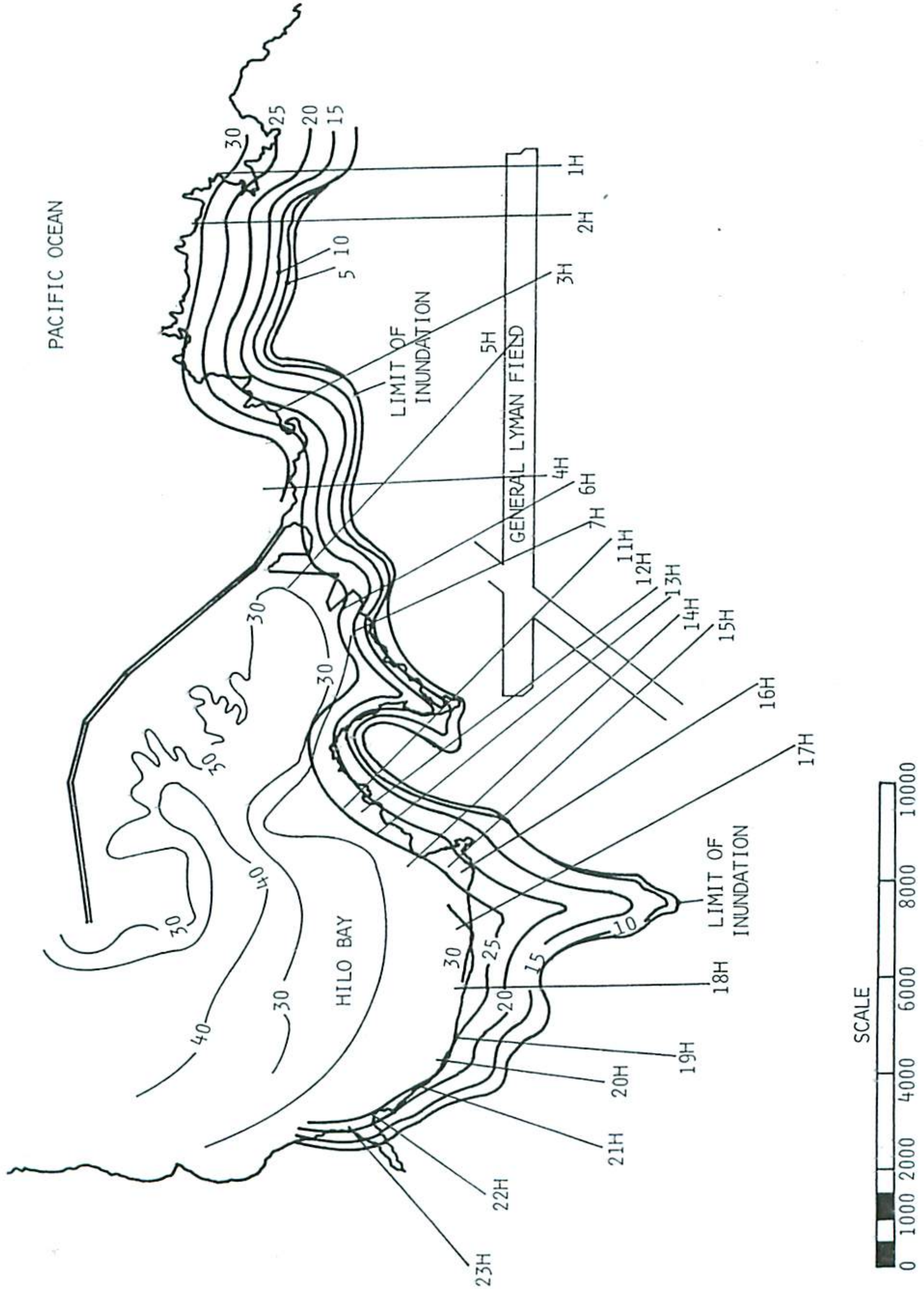


FIGURE 8
WATER SURFACE ELEVATION (1946 TSUNAMI)

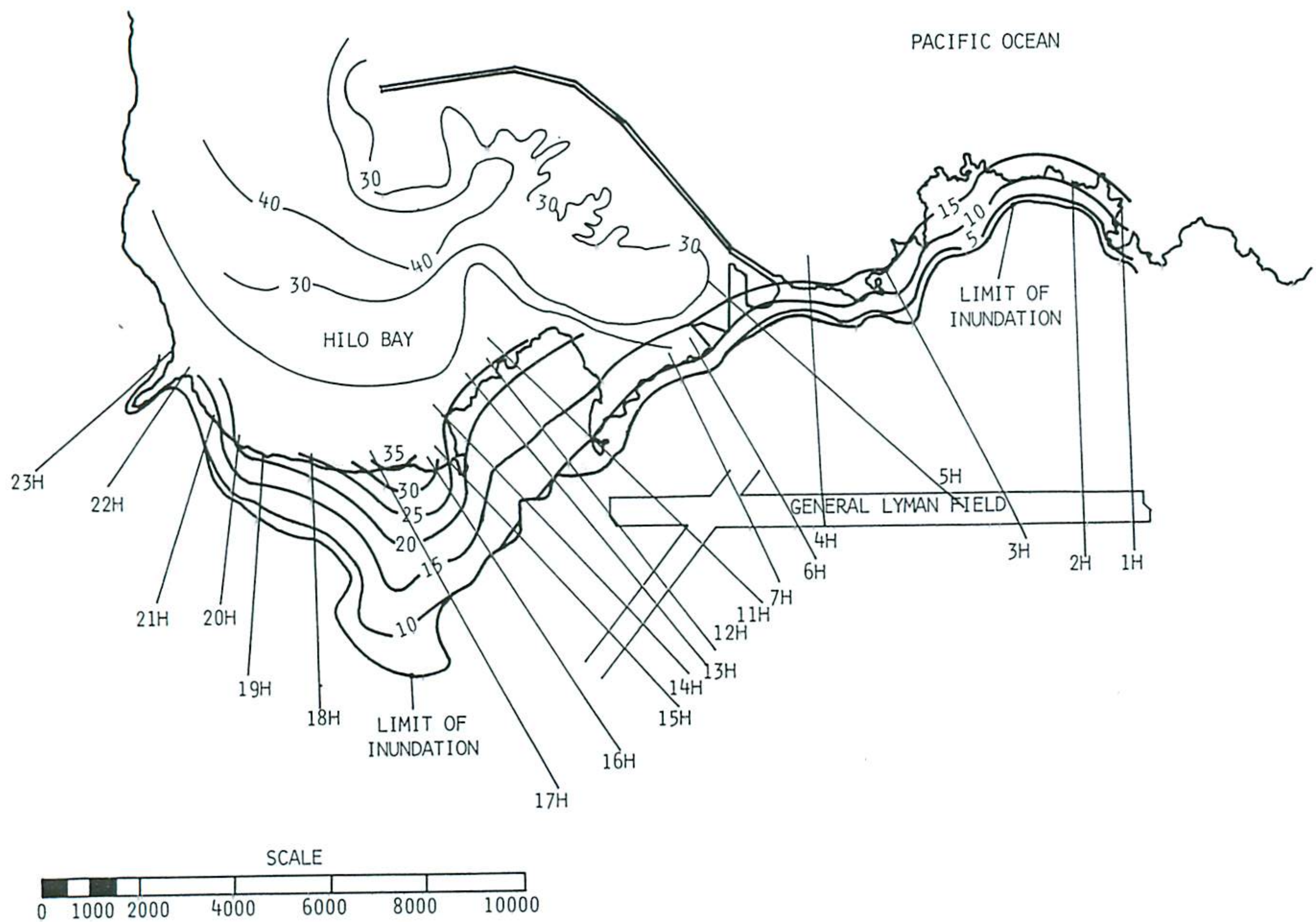


FIGURE 9

WATER SURFACE ELEVATION (1960 TSUNAMI)