PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

TECHNIQUE FOR PREDICTION OF WAVE-INDUCED MOTIONS AND LOADS ON OFFSHORE STRUCTURES WITH SPECIAL REFERENCE TO NORTH SEA CONDITIONS

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ABSTRACT

A short survey over methods for predicting the response in regular waves of different types of ships and offshore structures is given. Methods to predict short and long term response in irregular short-crested seas as well as some resulting motions and loads for single hull ships and catamarans are shown.

INTRODUCTION

The motions and loads on a floating structure in waves may be handled in three different steps. The first one is the response in regular waves. The second one is the stationary short term response in irregular waves and the third one is finally the long term response in varying seas. For each step we have to consider three items: wave, calculated response and observed response. Fig. 1 shows the corresponding nine fields and how they are connected logically. Our goal is to obtain the wanted long term statistical distributions of the response by means of calculations. We then need a description of the environment and for each step the method of calculation must be checked against observations i.e. model tests and full scale measurements.

Below I will briefly describe a method to predict long term response. The method has been described in some detail in /1/ where references for further reading also are given. I will here concentrate on principles rather than details. It must be emphasized that this method is strictly valid only for variables which vary linearly with the wave height. This holds for most motions

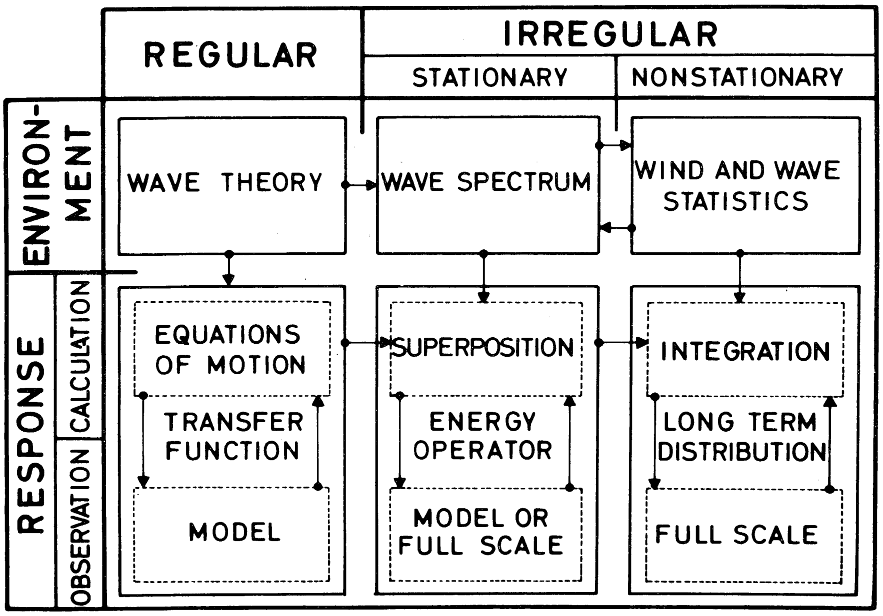

REGULAR
IRREGULAR
STATIONARY
NONSTATIONARY
ENVIRON-MENT
RESPONSE
CALCULATION
OBSERVATION
WAVE THEORY
WAVE SPECTRUM
WIND AND WAVE STATISTICS
EQUATIONS OF MOTION
SUPERPOSITION
INTEGRATION
TRANSFER FUNCTION
ENERGY OPERATOR
LONG TERM DISTRIBUTION
MODEL
MODEL OR FULL SCALE
FULL SCALE

Fig. 1

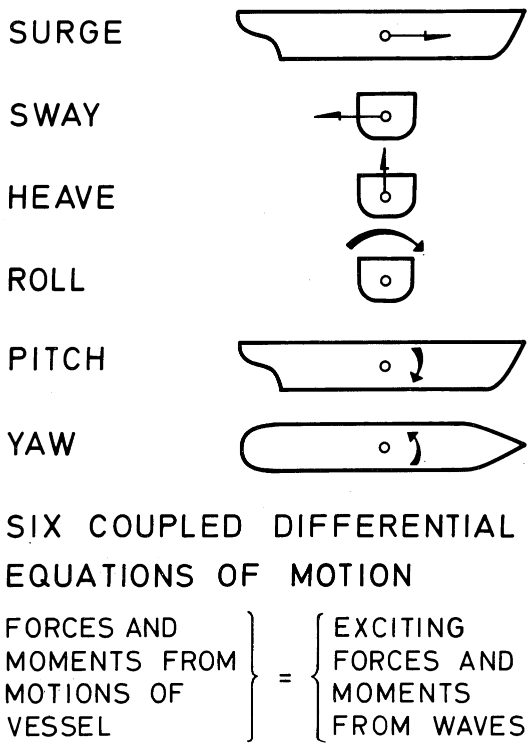

SURGE
SWAY
HEAVE
ROLL
PITCH
YAW
SIX COUPLED DIFFERENTIAL EQUATIONS OF MOTION
FORCES AND MOMENTS FROM MOTIONS OF VESSEL
=
EXCITING FORCES AND MOMENTS FROM WAVES

Fig. 2

and loads on single hull ships and catamarans and for the motions of most drilling platforms. Some loads on drilling platforms, however, may be highly non-linear and this applies particularly to local loads on vertical columns. Therefore, when applying the method outlined here one must check carefully whether the method is adequate or not.

I will here assume that the sea may be described by the methods given by Pedersen /2/. For further details of sea description see also /1/.

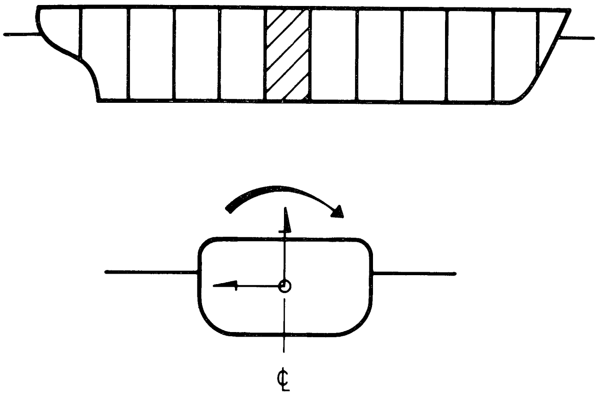
RESPONSE IN REGULAR WAVES

A floating structure, like any other object, has six modes of motion (six degrees of freedom): surge, sway, heave, roll, pitch and yaw as defined in Fig. 2. The motions in regular waves may be found from model tests or by means of theoretical calculations. A theoretical solution may be obtained from a set of six coupled linear differential equations, one for each mode of motion. The coefficients of these equations may be found by applying the so called strip theory. As shown for a ship in Fig. 3, the vessel is then divided into a number of strips in such a way that the flow of water around each strip may be assumed to be essentially two-dimensional. The total forces and moments are obtained as sums of forces and moments on strips.

The hydrodynamic forces and moments on strips in phase with accelerations and velocities, often called added mass and damping coefficients, may be obtained in different ways. We use the Frank close fit technique. As shown in Fig. 4 the section is then described by offset points and this makes it possible to obtain accurate results for practically any sectional shape. This technique is based on sinks and sources placed on the sections between offset points.

Forces and moments on a shiplike body are shown in Fig. 5. For the sake of completeness the longitudinal force may also be mentioned. For more complex structures the connecting forces between main hulls are often important. As an example the more important connecting forces on the wing of a catamaran are shown in Figs.6 - 8.

STRIP THEORY



TWO-DIMENSIONAL FLOW AROUND EACH SECTION

Fig. 3

ADDED MASS AND DAMPING COEFFICIENTS BY MEANS OF FRANK CLOSE FIT TECHNIQUE

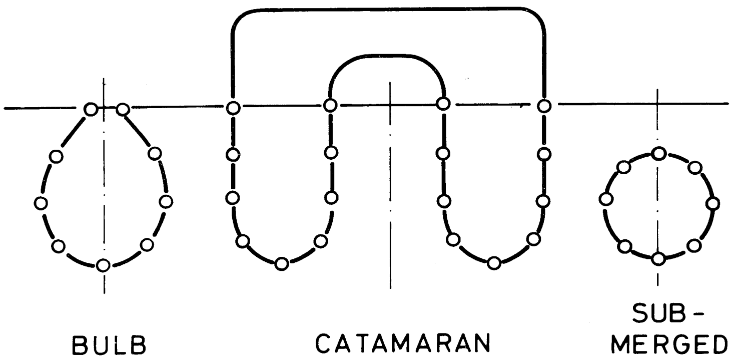


Fig. 4

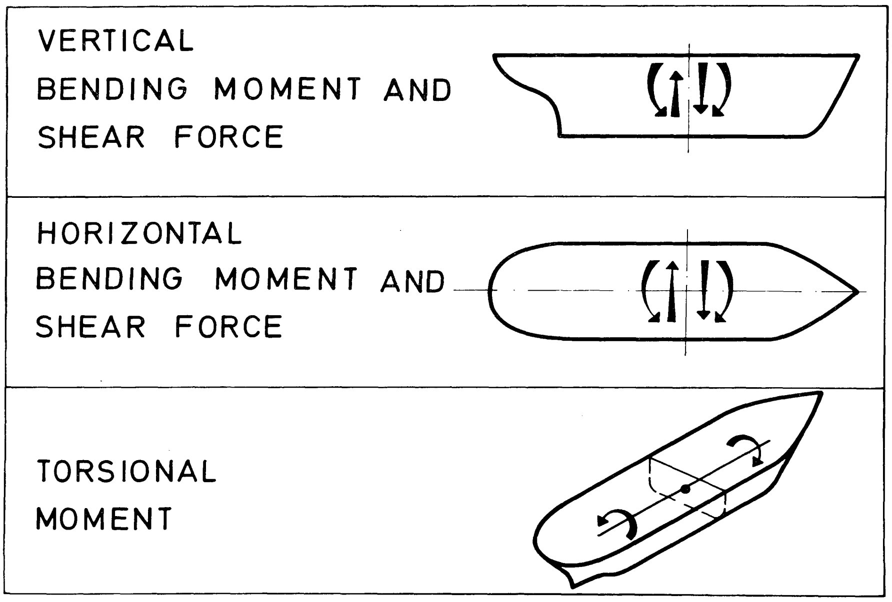

VERTICAL
BENDING MOMENT AND
SHEAR FORCE
HORIZONTAL
BENDING MOMENT AND
SHEAR FORCE
TORSIONAL
MOMENT

Fig. 5

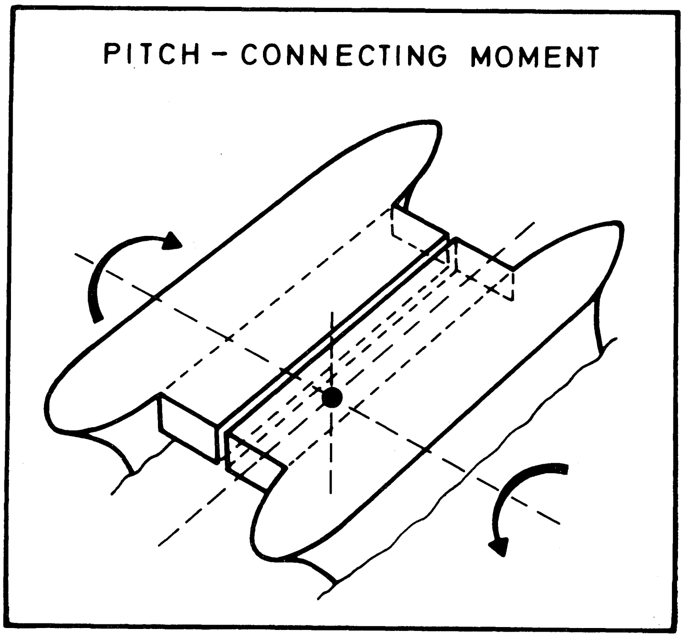

PITCH - CONNECTING MOMENT

Fig. 6

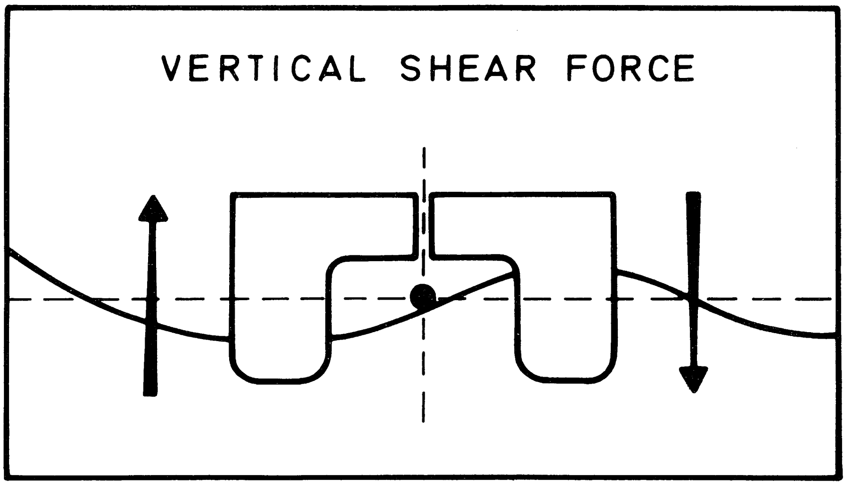

VERTICAL SHEAR FORCE

Fig. 7

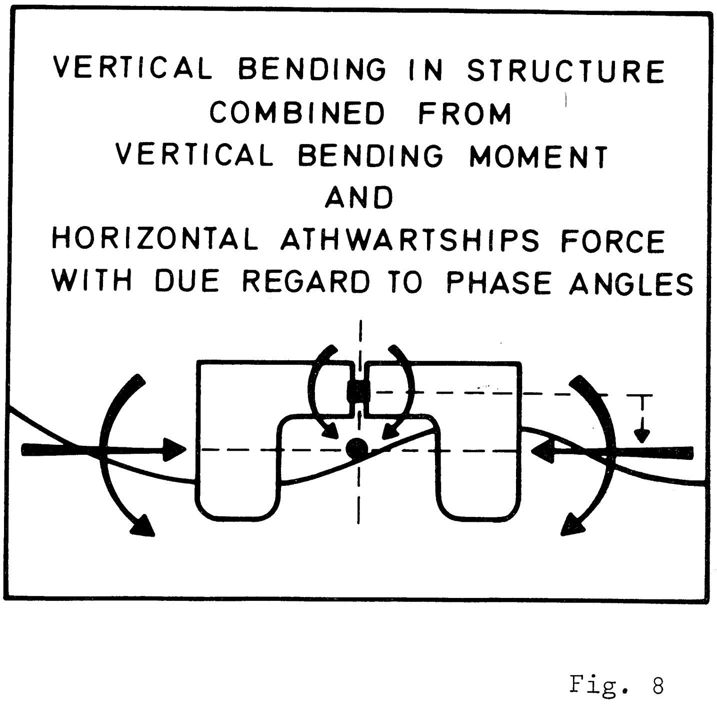

VERTICAL BENDING IN STRUCTURE
COMBINED FROM
VERTICAL BENDING MOMENT
AND
HORIZONTAL ATHWARTSHIPS FORCE
WITH DUE REGARD TO PHASE ANGLES

Fig. 8

In addition to these overall forces and moments there are local loads. As shown in Fig. 9 local loads are external water pressures and internal inertia forces from structures and contents in tanks.

All the different types of motion and loads mentioned above may for a vessel in regular waves be described by transfer functions. A transfer function gives the ratio between the amplitudes of response and wave for constant ship speed and heading, and may be obtained from model tests and strip theory calculations as mentioned above. As an example transfer function for heaving of a catamaran /1/ is shown in Fig. 10. A theoretical solution from our computer program for catamarans is here compared to model test data.

SHORT TERM DISTRIBUTIONS

With short term we here mean a period of time which is short enough to make it possible to describe waves and response as stationary random processes. This period of time is of the order of one hour and is thus very short as compared to the life of a ship. A random process is stationary when its statistical properties remain unchanged with time. This does not mean that the process repeats itself. A particular pattern of the sea within a certain area at a given instant is unique and does never occur again. It is not possible to predict the appearance of the sea surface or the response at a certain time in the future. The statistical properties of sea and response may, however, be predicted and this is all we need in order to make predictions regarding the behaviour of a floating vessel at sea.

The transfer function which gives the response in regular waves may be used to predict the response in irregular waves. A combination of wave spectrum and transfer function by means of the linear superposition principle gives a response spectrum. This principle is illustrated in Fig. 11. The response spectrum is for all frequencies obtained as the product of wave spectrum and squared transfer function. As also shown in Fig. 11 the irregular waves may be regarded as sums of regular waves. Each regular wave gives a regular response and the irregular response is the sum of the regular responses.

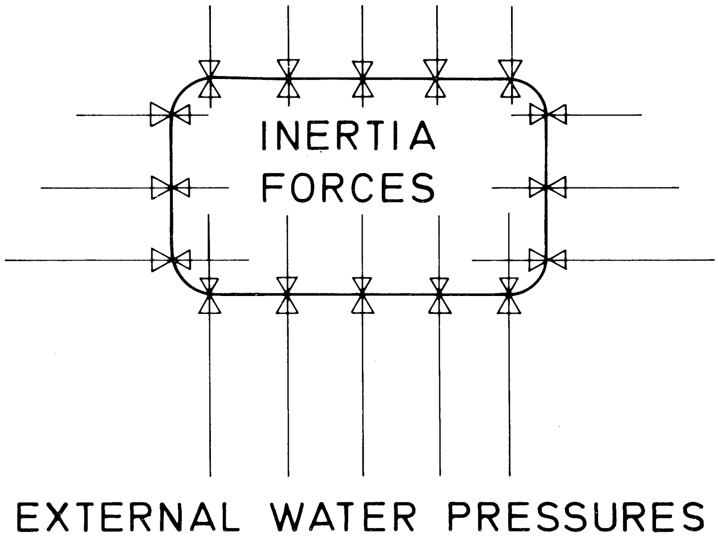

INERTIA
FORCES
EXTERNAL WATER PRESSURES

Fig. 9

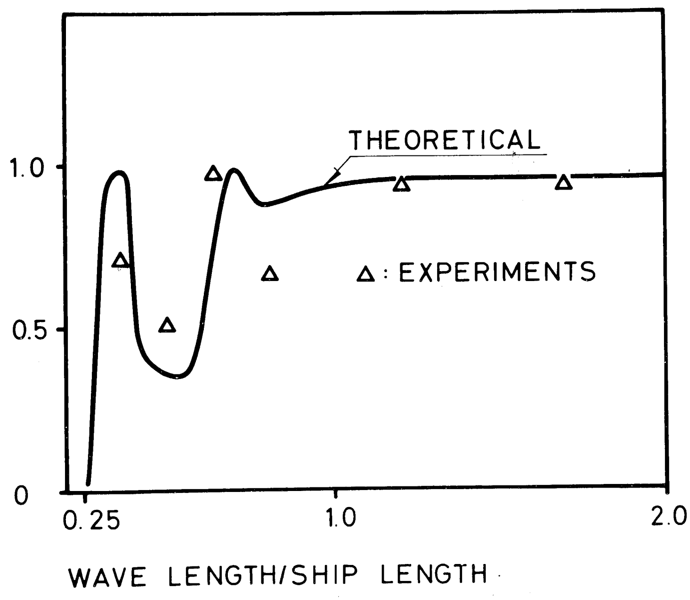

THEORETICAL
△ : EXPERIMENTS
1.0
0.5
0
0.25
1.0
2.0
WAVE LENGTH/SHIP LENGTH

Fig. 10

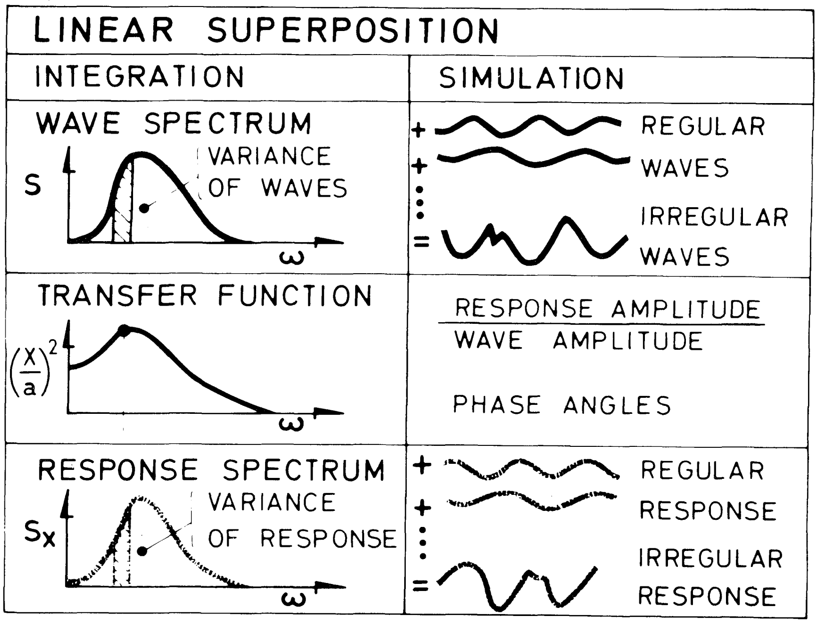


Fig. 11

ASYMPTOTES OF SHORT TERM RESPONSE

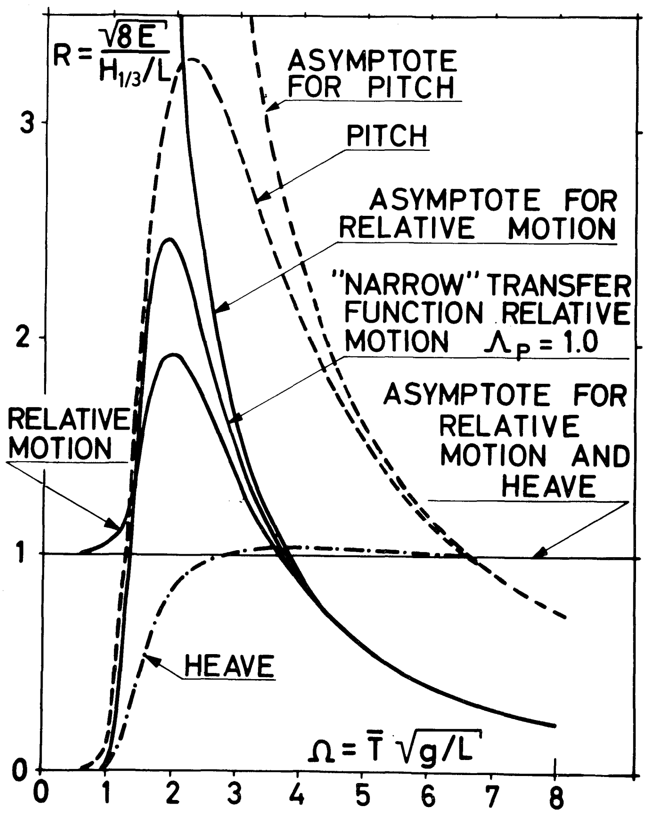


Fig. 12

The ratio between response amplitudes and wave amplitudes in irregular seas may be obtained from the ratio between the areas of response spectrum and wave spectrum. This ratio may be presented non-dimensionally as shown in Fig. 12, which is taken from /3/. A curve in such a figure gives the parameter E of the short term response in any weather characterized by significant wave height $H_{1/3}$ and average apparent wave period $\overline{T}$ and for any size L of vessel provided the geometry is the same.

The asymptotes of R as function of Ω shown in Fig. 12 were handled in some detail in /3/ where useful asymptotic formula were developed. It was shown that the area under the squared transfer function is an important parameter. It was also shown how the asymptotes of R are dependent on the asymptotes of transfer functions and wave spectra.

As for wave heights the irregular response follows the Rayleigh distribution function and E is the single parameter of this distribution. See Fig. 13 where X denotes departures of response maxima (non-dimensional e.g. for heave X = H/L where H is heave) from zero level (half peak-to peak).

E equals twice the area under the response spectrum and this area equals the variance of the response.

The short term Rayleigh distribution is

$$P_s(X) = 1 - \exp(-X^2/E) \qquad (1)$$

where

$P_s(X)$ is the probability that the departure from the zero level (mean) of a "crest" (or " trough") of the non-dimensional response variable does not exceed X.

LONG TERM DISTRIBUTION

As described in the previous section the parameter E which characterizes the short term response, may be found for any weather defined by significant wave height $H_{1/3}$ and average apparent wave period $\overline{T}$. We have also established a description of the long term joint probability distributions of $H_{1/3}$ and average apparent wave period $\overline{T}$. This description is based on Weibull distributions of wave heights (for details see /1, 2, 3/). It is thus clear that the long term distribution of E may be found by combining short term response in given weather and long term weather statistics. This principle is illustrated in Fig. 14. We have found that this procedure produces a Weibull distribution of $\sqrt{E}$ which we write as follows

$$P(\sqrt{E}) = 1 - \exp(-(\sqrt{E}/a)^m) \quad (2)$$

Details for the practical procedure of calculating the long term distribution of $\sqrt{E}$ and finding the parameters a and m are given in /1/.

The Weibull distribution was also found to fit long term full scale measurements of $\sqrt{E}$ /4/.

As mentioned above each value of E represents a short term Rayleigh distribution of X and the long term distribution of X may thus be found by means of a summation of all short term distributions with due regard to their different probabilities of occurrence as given by Eq. (2).

$$P(X) = 1 - \int_0^\infty \exp(-X^2/E)\, dP(\sqrt{E}) \quad (3)$$

Eq. (3) was studied in some detail in /3, 4/ where it was shown that P(X) is approximately a Weibull distribution.

$$P(X) \simeq 1 - \exp(-(A/b)^k) \quad (4)$$

where

$$A = (X/a)^2 \quad (5)$$

and where b and k are functions of m given in Table 1.

It is thus possible to find directly the parameters of the long term Weibull distribution of X from the parameters of the long term Weibull distribution of $\sqrt{E}$.

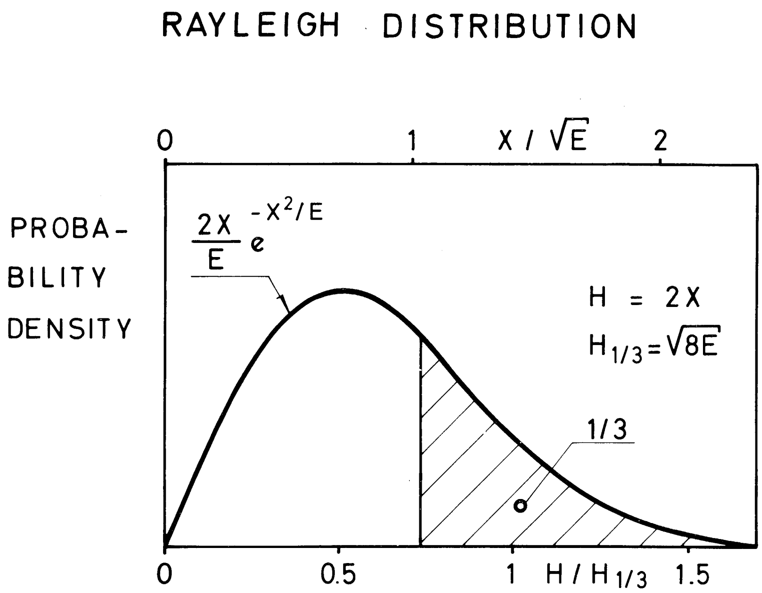

RAYLEIGH DISTRIBUTION
0
1
X / √E
2
PROBA-
BILITY
DENSITY
2X/E e^(-X²/E)
H = 2X
H1/3 = √8E
1/3
0
0.5
1
H / H1/3
1.5

Fig. 13

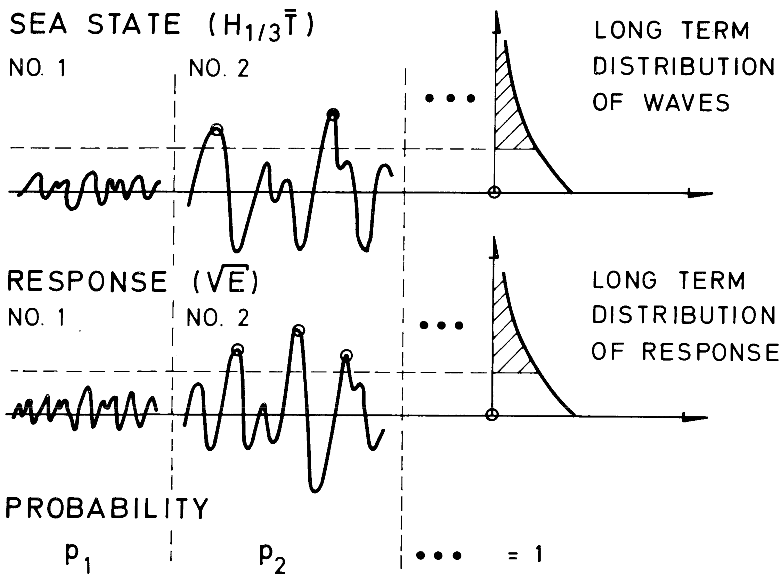

SEA STATE (H1/3T̄)
NO. 1
NO. 2
• • •
LONG TERM
DISTRIBUTION
OF WAVES
RESPONSE (√E)
NO. 1
NO. 2
• • •
LONG TERM
DISTRIBUTION
OF RESPONSE
PROBABILITY
p1
p2
• • • = 1

Fig. 14

The probability level $Q = 1 - P(X)$ and the number N of response cycles are related as follows

$$Q = 1/N \tag{6}$$

By assuming an average response period it is thus possible to interpret the probability level as a period of time. As shown in Fig. 15 a ship life of 20 years corresponds approximately to the probability level $Q = 10^{-8}$. The corresponding value of the response variable is the most probable largest value during this lifetime.

It would lead too far to discuss how the distribution of heading angles etc. is taken into consideration so for this and other details reference is made to /1, 3,/.

Some examples of most probable largest values on probability level $Q = 10^{-8}$ are shown in Figs. 16 and 17. These figures show pitch and yaw angles of a catamaran in North Sea weather conditions /1/.

Relationships between the long term distributions of responses and waves are shown in Fig. 18. This figure which is taken from /3/ shows a non-dimensional ratio between response and wave as a function of the relationship between ship size and dominating wave period. This is the long term equivalent to the short term response ratio for irregular conditions shown in Fig. 12 and the response ratio for regular condition (transfer function) shown in Fig. 10. Figs. 10, 12 and 18 represent the three steps mentioned in the introduction.

USE OF STATISTICAL LOAD DISTRIBUTIONS IN STRENGTH ANALYSIS

Firstly I will briefly comment upon the role of load analysis in connection with analysis and design of structures in general. Fig. 19 shows four important items to be covered in connection with a strength analysis. Such a list may be written in different ways but it is important to note that strength always is equally dependent on demand and capability. The dimensions are functions of the ratios between demands and capabilitis. I emphasize this list because of the fact that the quality of the strength analysis never

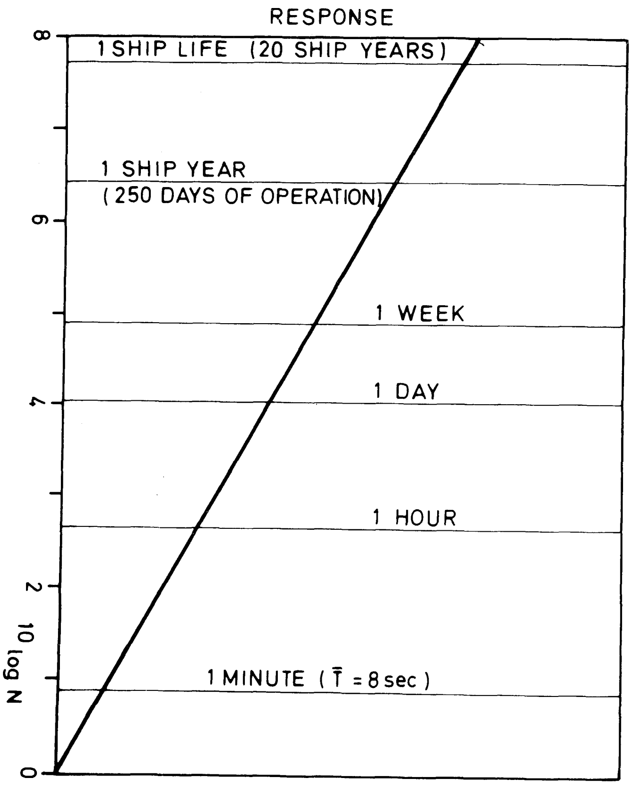

RESPONSE
1 SHIP LIFE (20 SHIP YEARS)
1 SHIP YEAR
(250 DAYS OF OPERATION)
1 WEEK
1 DAY
1 HOUR
1 MINUTE ($\bar{T}$ = 8 sec)
$^{10}\log N$
0
2
4
6
8

Fig. 15

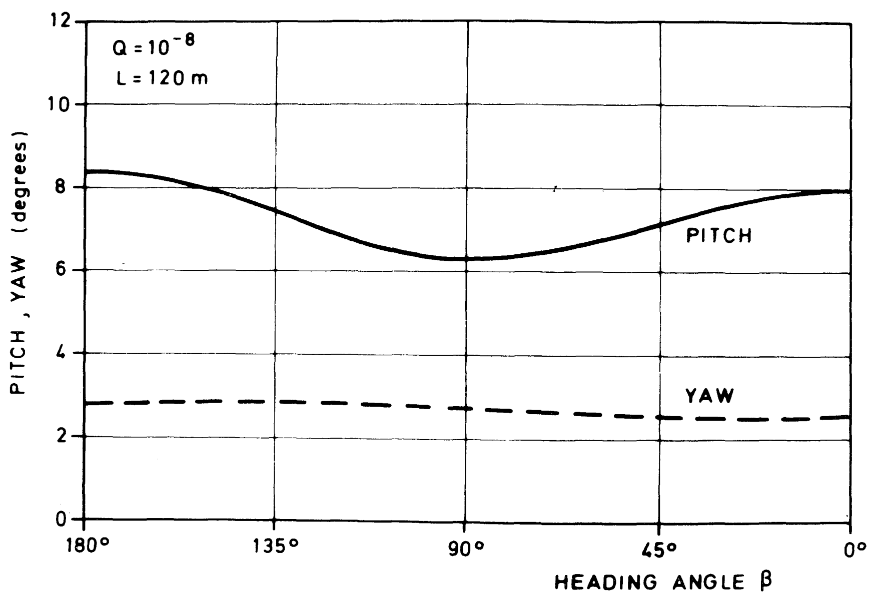
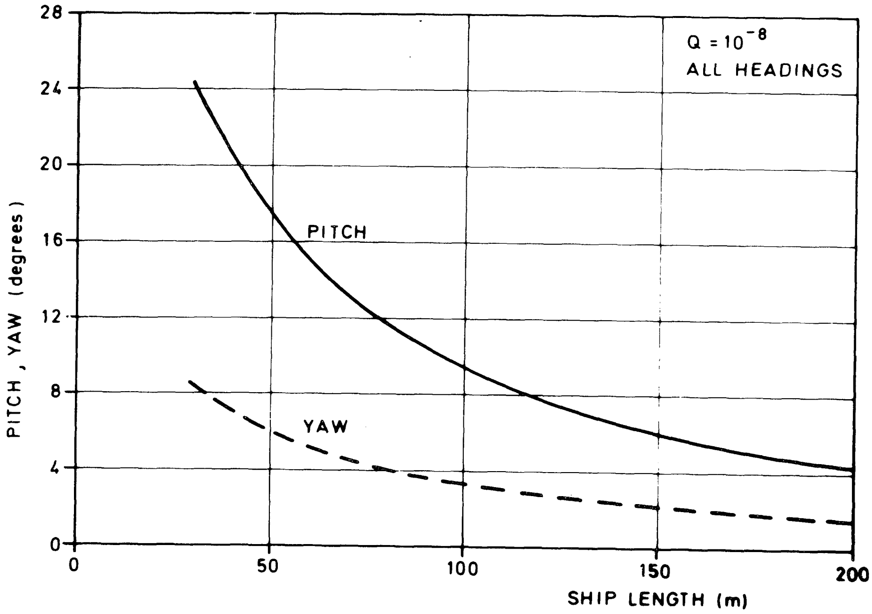
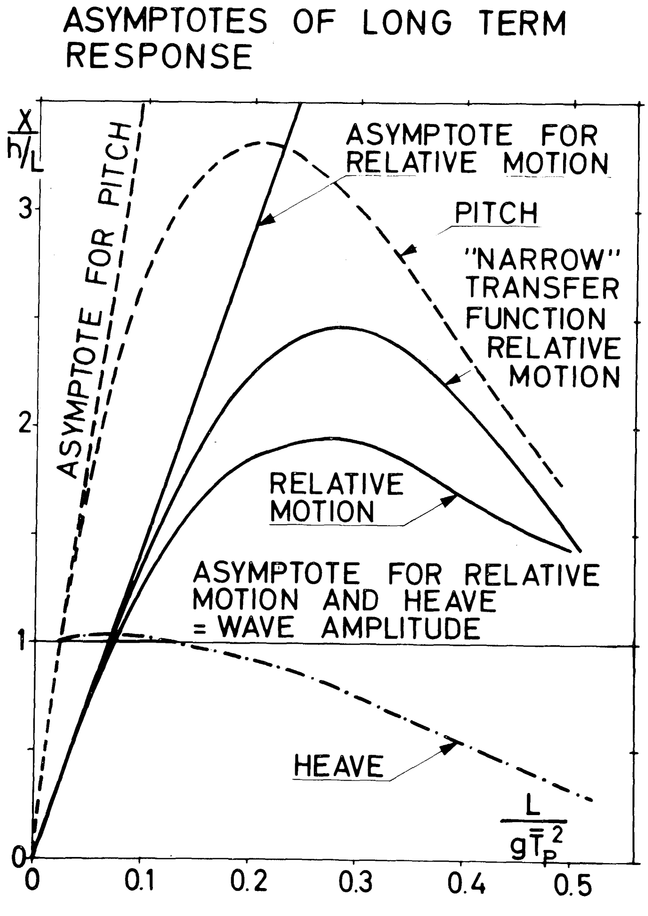

$Q = 10^{-8}$
$L = 120$ m
PITCH
YAW
PITCH , YAW (degrees)
0
2
4
6
8
10
12
180°
135°
90°
45°
0°
HEADING ANGLE β

Fig. 16



Q = 10^-8
ALL HEADINGS
PITCH
YAW
PITCH , YAW (degrees)
SHIP LENGTH (m)

Fig. 17



ASYMPTOTES OF LONG TERM RESPONSE
ASYMPTOTE FOR PITCH
ASYMPTOTE FOR RELATIVE MOTION
PITCH
"NARROW" TRANSFER FUNCTION RELATIVE MOTION
RELATIVE MOTION
ASYMPTOTE FOR RELATIVE MOTION AND HEAVE = WAVE AMPLITUDE
HEAVE

Fig. 18

is better than that of the poorest of these four items. This self-evident truth is often overlooked to a surprisingly large extent. It would be incorrect to say that one item is more important than the others - you need them all for an optimum design.

The application of statistical wave loads in connection with a strength analysis is not a straight forward procedure. One of the first problems for the designer is the problem of putting the loads on the structure model. This is difficult mainly because of the fact that the maxima of the different types of loads do not occur simultaneously. For example on a ship the vertical bending moment on probability level 10^{-8} does not occur at the same time as the horisontal bending moment on the same probability level, and a stress which is dependent on both moments (e.g. in the deck corner) may not be obtained by simply adding the stresses from the two moments. This problem is of utmost importance as statistical wave loads may not be extensively used in strength analysis unless practical solutions are obtained. Fig. 20 gives a list of four different solutions and Figs. 21 - 25 give som details of each type of solution.

Quasi static "equivalent" wave in Fig. 21 denotes the traditional approach. One chooses a "most important load" which for a ship has been vertical bending moment amidships and determines the "maximum" long term value of this load. The next step is to place the ship statically in a wave with a length which equals that of the ship and determine a wave height which gives the same value for the most important load in the quasi static calculation. The wave obtained is called equivalent wave and is used to calculate forces and moments on the entire hull. It is obvious that dynamics and phase angles are poorly represented by this method and I have therefore developed some other methods which solve these problems.

The first development was the "square root method" which now has been successfully used in Det norske Veritas for a few years. As shown in Fig. 22 we start with transfer functions for all loads and calculate the corresponding long term load distributions. We then calculate the stresses for each load separately e.g. stresses from vertical bending moments, horizontal bending moments, torsional moments and local water pressures. For each point consider-

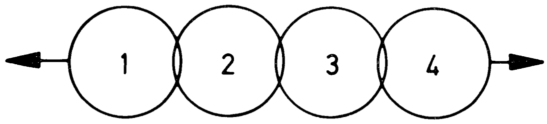
STRENGTH ANALYSIS

1 DESCRIBE ENVIRONMENT

2 PREDICT LOADS (DEMAND)

3 CALCULATE STRESSES ETC.

4 APPLY CRITERIA
(CAPABILITY)



A CHAIN IS NEVER
STRONGER THAN
THE WEAKEST LINK

Fig. 19

ALTERNATIVE APPLICATIONS OF STATISTICAL WAVE LOADS

1 QUASI STATIC
"EQUIVALENT" WAVE

2 SQUARE ROOT METHOD

3 LONG TERM STRESS
DISTRIBUTIONS

4 DYNAMIC EQUILIBRIUM
A: IN REGULAR WAVES
B: IN IRREGULAR WAVES
FROM SIMULATION

Fig. 20

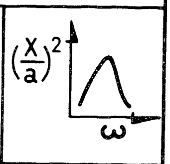
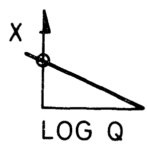
QUASI STATIC "EQUIVALENT" WAVE.	
TRANSFER FUNCTION FOR MOST IMPORTANT LOAD.	
LONG TERM DISTRIBUTION FOR MOST IMPORTANT LOAD.	
QUASI STATIC CALCULATION IN "EQUIVALENT" WAVE.	
DYNAMICS AND PHASES POORLY REPRESENTED.	

Fig. 21

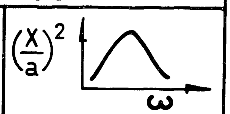
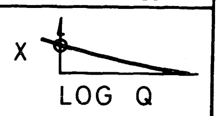
SQUARE ROOT METHOD	
TRANSFER FUNCTIONS FOR ALL LOADS.	
LONG TERM LOAD DISTRIBUTIONS.	
STRESS ANALYSIS FOR EACH TYPE OF LOAD $X_1 \rightarrow \sigma_{x1}\ \sigma_{y1}\ \tau_1 \quad X_2 \rightarrow \sigma_{x2}\ \sigma_{y2}\ \tau_2 \ \ldots$	
COMBINATION OF STRESSES FOR σ_x σ_y AND τ $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2 + \ldots + 2\rho\sigma_1\sigma_2 + \ldots}$ UNCERTAINTIES DUE TO PHASES.	
EQUIVALENT STRESSES UNCERTAIN σ_x σ_y AND τ NOT IN PHASE.	

Fig. 22

ed we then get a number of stresses which are combined by the square root formula. This formula is correct provided we know the correlations ρ between all stresses. Today we use zero correlation, which is true for most variables but not all of them. The uncertainties lie here in the correlation which is related to phase angles and simultaneous occurence of different loads. Equivalent stresses are uncertain because the stress components σ_x, σ_y and τ are not in phase.

The next development was the "long term stress distributions". As shown in Fig. 23 the difficulties due to the square root formula were overcome and σ_x, σ_y and τ may be correctly estimated. These components are, however, not in phase and equivalent stresses are still difficult to obtain. Another difficulty is the necessity to perform a larger number (200 - 300) of stress calculations but we are now developing computer programs which automatically couple load and stress analysis so this will soon be feasible and we look at it as an important development.

The "dynamic equilibrium in regular waves" shown in Fig. 24 is in a way similar to the traditional approach with a quasi static calculation in an equivalent wave but the dynamics of the situation are introduced and the phase angles between all loads are correct. This makes it possible to calculate equivalent stress as a function of time and find maximum for this stress.

"Dynamic equilibrium in irregular waves" shown in Fig. 25 is similar to "Dynamic equilibrium in regular waves" but instead of regular wave we use a simulated irregular wave system. This method may be the most promising one because it is the only method which gives equivalent stress in irregular seas.

Both dynamic equilibrium methods have the advantage that the loads are in equilibrium when they are applied to the structure and this is a great advantage in connection with the stress analysis. On the other hand, however, these methods give difficulties in connection with the choice of equivalent wave system but this is considered to be a relatively small problem.

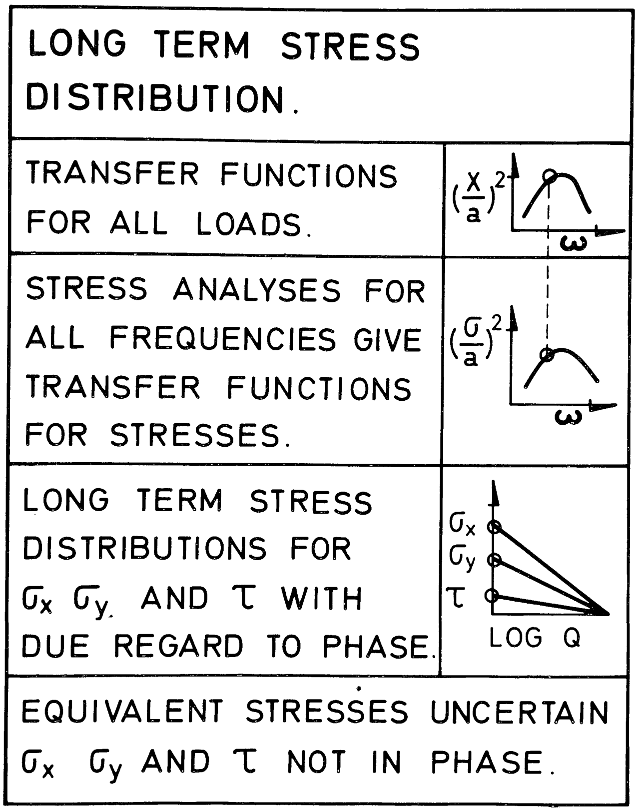

LONG TERM STRESS DISTRIBUTION.
TRANSFER FUNCTIONS FOR ALL LOADS.
$(\frac{X}{a})^2$
ω
STRESS ANALYSES FOR ALL FREQUENCIES GIVE TRANSFER FUNCTIONS FOR STRESSES.
$(\frac{\sigma}{a})^2$
ω
LONG TERM STRESS DISTRIBUTIONS FOR σ_x σ_y AND τ WITH DUE REGARD TO PHASE.
σ_x
σ_y
τ
LOG Q
EQUIVALENT STRESSES UNCERTAIN σ_x σ_y AND τ NOT IN PHASE.

Fig. 23

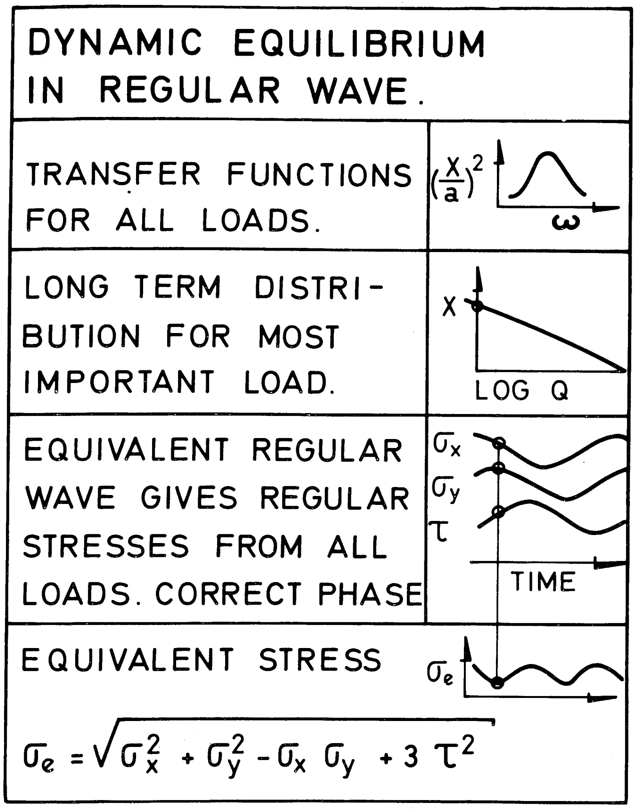

DYNAMIC EQUILIBRIUM IN REGULAR WAVE.
TRANSFER FUNCTIONS FOR ALL LOADS.
$(\frac{X}{a})^2$
ω
LONG TERM DISTRIBUTION FOR MOST IMPORTANT LOAD.
X
LOG Q
EQUIVALENT REGULAR WAVE GIVES REGULAR STRESSES FROM ALL LOADS. CORRECT PHASE
σ_x
σ_y
τ
TIME
EQUIVALENT STRESS
σ_e
$\sigma_e = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau^2}$

Fig. 24

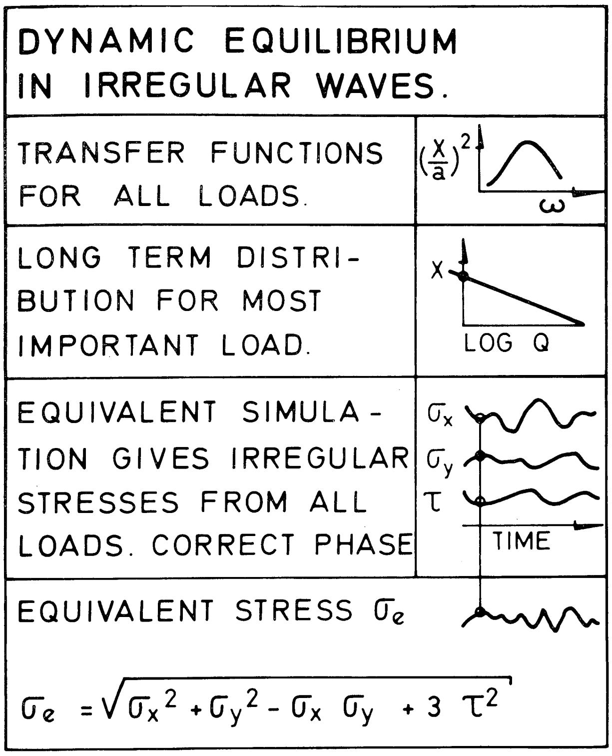

DYNAMIC EQUILIBRIUM IN IRREGULAR WAVES.
TRANSFER FUNCTIONS FOR ALL LOADS.
$(\frac{X}{a})^2$
ω
LONG TERM DISTRIBUTION FOR MOST IMPORTANT LOAD.
X
LOG Q
EQUIVALENT SIMULATION GIVES IRREGULAR STRESSES FROM ALL LOADS. CORRECT PHASE
σ_x
σ_y
τ
TIME
EQUIVALENT STRESS σ_e
$\sigma_e = \sqrt{\sigma_x{}^2 + \sigma_y{}^2 - \sigma_x \; \sigma_y \; + 3 \; \tau^2}$

Fig. 25

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Table 1

Short term distribution of X:

$$P(X) = 1 - \exp(-X^2/E)$$

Long term distribution of E:

$$P(\sqrt{E}) = 1 - \exp(-[\sqrt{E}/a]^m)$$

Long term distribution of X:

$$P(X) \simeq 1 - \exp(-[(X/a)^2/b]^k)$$

2/m	k	b	2/m	k	b
0	1.000	1.000	1.3	0.472	0.433
0.5	0.722	0.614	1.4	0.452	0.421
0.6	0.677	0.574	1.5	0.434	0.411
0.7	0.638	0.543	1.6	0.417	0.402
0.8	0.604	0.517	1.7	0.401	0.394
0.9	0.572	0.495	1.8	0.387	0.386
1.0	0.543	0.476	1.9	0.373	0.379
1.1	0.517	0.460	2.0	0.361	0.373
1.2	0.494	0.446	3.0	0.269	0.336
			4.0	0.214	0.314
			5.0	0.178	0.304