PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

PREDICTIONS OF LONG-TERM WAVE CONDITIONS WITH SPECIAL EMPHASIS ON THE NORTH SEA

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ABSTRACT

Mathematical methods for describing the long term statistical distributions of wave heights are given. The problems in connection with interpretation of visual data are discussed. A method for predicting long term distributions of individual wave heights are given. Results for the North Sea are presented and compared with measurements.

INTRODUCTION

Calculation of the strength of ships was previously based on experience accumulated through a long time. This was adequate as long as the size of the new ship did not differ much from exsisting ships. The rapid increase in ship size experienced in the sixties, showed clearly that the old methods were insufficient. The necessity of finding theoretical methods for predicting ship motions and loads became apparent. Det norske Veritas took the consequence of this situation, and in the early sixties a group for calculation of waves and wave loads was started within the Research and Development Department.

This paper will give a survey of a simple method developed within this group, for calculating the statistical long term distribution of individual wave heights. The main features of this method are that it is easy to use and that results are obtained quickly. The final proof of its usefulness must be, however, that it gives results which compare favourably with measurements at sea.

It will be shown that this is the case.

SOURCES FOR WAVE DATA

When working with waves and wave loads on offshore structures one of the most difficult problems is to obtain reliable data. For some areas, the amount of data is large. This is the case for instance for the North Atlantic where the ocean weather ships have taken observations regularly for many years.

In addition to the stationary weather ships observations are made from merchant ships, light ships, drilling platforms and so on. Fig. 1 gives a review of these main sources for wave data.

However, it also illustrates a significant problem, namely that most of the wave data are obtained in the form of visual observations. I will return to this problem a little later.

Fig. 2 shows the areas around the world for which visual wave observations have been taken by merchant ships. These data have been presented by Hogben and Lumb /1/. These data appear to be of varying quality, but at least for the North Sea they compare favourably with data from other sources.

The positions of the weather ships in the North Atlantic are shown in Fig. 3. Data from these ships will partly be used to illustrate our method for predicting wave heights.

DESCRIPTION OF DATA

In Fig. 4 a table is given which illustrates the most usual way of presenting wave data. This table gives the percentage of the total number of wave observations which at the same time fall within a given wave height and period group. In this way the table represents a large number of short term wave conditions. For example it is found that 6.3% of the observations had a wave height in the range 0.75 to 1.75 metres and at the same time a period in the interval of 7 to 9 sec. The given table is based on data for the North Atlantic.

MAIN SOURCES FOR WAVE DATA

VISUAL DATA FROM MERCHANT SHIPS

VISUAL AND INSTRUMENTAL DATA FROM
- WEATHER SHIPS
- LIGHT SHIPS
- DRILLING PLATFORMS

INSTRUMENTAL DATA FROM BUOYS

VISUAL DATA FROM LIGHTHOUSES

Fig. 1

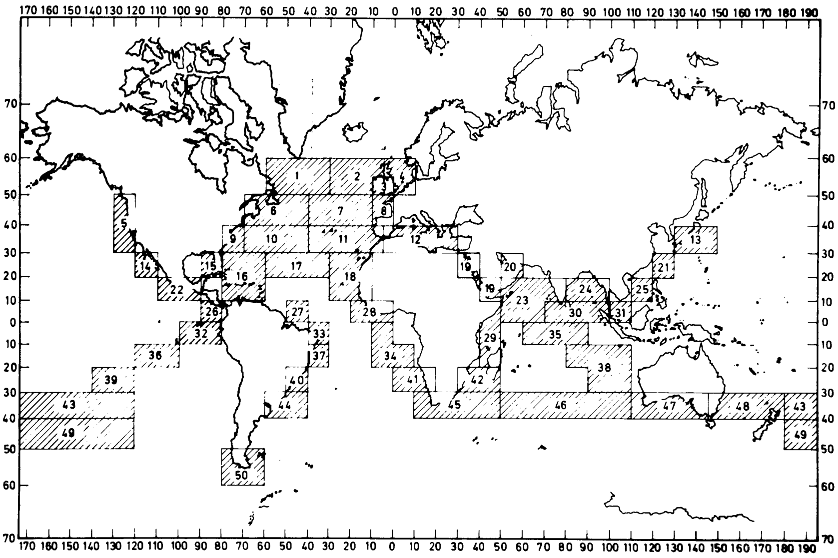


Fig. 2

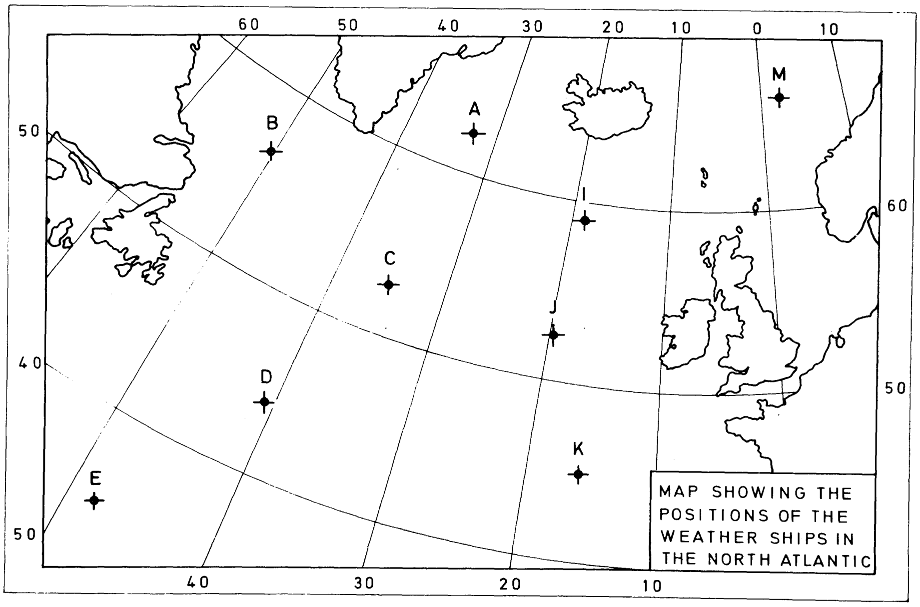


Fig. 3

LONG TERM WAVE DATA							
PERCENT WITHIN CLASS		WAVE PERIOD					
		< 5	5 - 7	7 - 9	• • •	ALL PERIODS
WAVE HEIGHT	0 - 0.75	2.09	1.18	0.46	• • •	4.06
	0.75-1.75	7.28	13.10	6.30	• • •	28.77
	1.75-2.75	2.13	12.66	11.83	• • •	30.12
	⋮	⋮	⋮	⋮	⋮	⋮
	ALL HEIGHTS	11.90	34.55	35.86	• • •	100.0

Fig. 4

It should be emphasized that it is not the largest wave height that governs in which interval the observation is counted. According to the instructions to the observers, the average of the larger well-formed waves of the wave system is reported /2/. Similarly the average wave period is found by counting a number of waves and measuring the time by a stop watch. The only way of describing these data mathematically is by using statistical methods.

As an example the conditional distribution of wave height for the period interval 7 - 9 sec is shown in Fig. 5. On the horizontal axis the wave height in metres is given, while the vertical axis gives the probability that the wave height is smaller than or equal to a given value. It may for instance be seen that the probability that the wave height is smaller than or equal to 4.75 metres is approximately 0.9 or 90%, when the wave period lies in the 7 - 9 sec interval.

It should be made quite clear at this point that when we talk about wave height we mean the vertical distance between crest and trough.

The particular plotting paper used in Fig. 5 is so designed that a Weibull distribution will appear as a straight line. Mathematically the Weibull distribution is given as

$$P(H_v) = 1 - \exp(-(H_v/H_c)^{\gamma}) \qquad (1)$$

Here

$P(H_v)$ - the probability that the visual wave height (index v) is smaller than or equal to H_v.

H_c, γ - parameters of the distribution.

γ is determined as the slope of the straight line, while H_c is given by the location along the wave height axis. In Fig. 5 a straight line has been fitted by eye to the points.

In Fig. 6 a comparison between different data **is** shown. The distributions given are marginal distributions, i.e. they represent the sum of all periods.

The curve marked S2 is based on data from the lightship S2 in

LONG TERM WEIBULL DISTRIBUTION OF WAVE HEIGHT

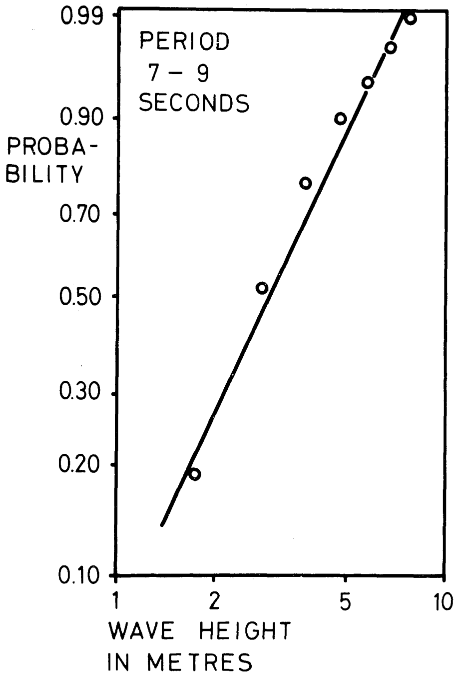


Fig. 5

LONG TERM WEIBULL DISTRIBUTION OF WAVE HEIGHT

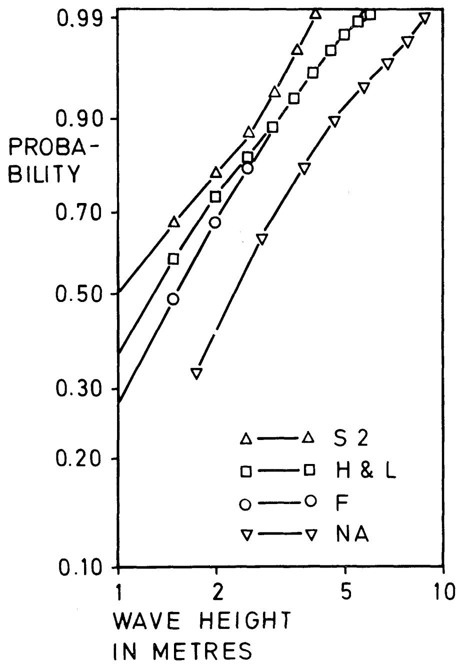


Fig. 6

the southern part of the North Sea as published by Roll /3/. H & L denotes data according to Hogben and Lumb /1/ for the North Sea (area 4), while F means data from the rescue ship "Famita" and NA data for the North Atlantic according to Walden /4/.

In order to obtain a better fit between the theoretical Weibull distribution and the data, a third parameter may be introduced in Eq.1. This will give a better fit at the lower wave heights, but it will have no practical influence on the extrapolation to large wave heights.

It should be mentioned that the scale exaggerates the difference between the different curves for the lower wave heights.

As was mentioned before most data are given in terms of visual observations. It has been found by comparing simultaneous measurements and visual observations that a formula of the form

$$H_v = cH_{1/3}^{d} \qquad (2)$$

may be used for translating visually observed wave heights to significant wave heights.

Nordenstrøm /5/ has shown that the formula should read

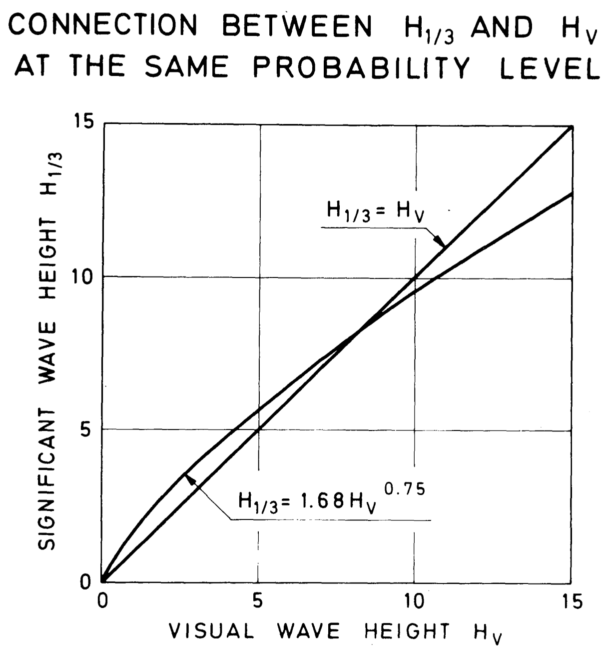
$$H_v = 0.5\ H_{1/3}^{1.33} \qquad (3)$$

when H_v and $H_{1/3}$ have the same probability of exceedance. See Fig. 7.

This formula will be used when predicting the distribution of individual wave heights.

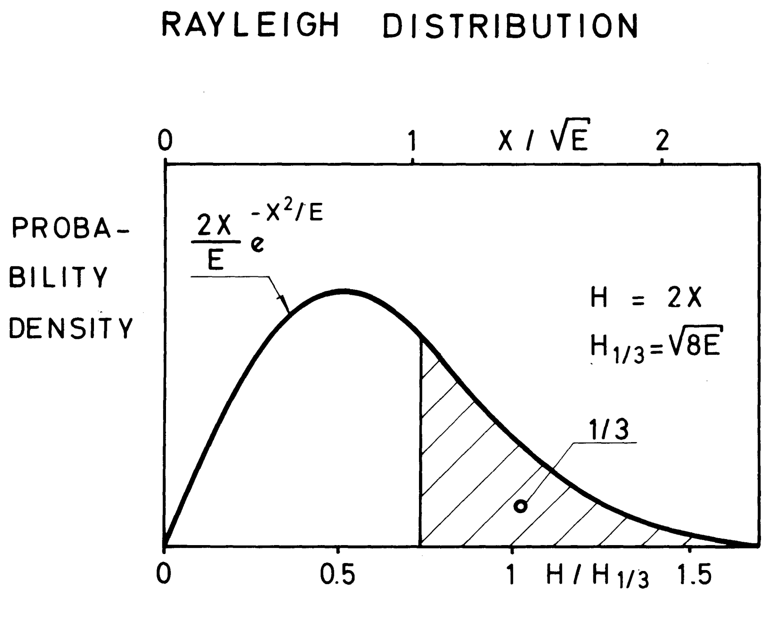
The statistical distribution of wave heights in a short term state of sea (stationary conditions) is found to be well described by a Rayleigh distribution given as

$$P(H) = 1 - \exp(-2(H/H_{1/3})^2) \qquad (4)$$



CONNECTION BETWEEN $H_{1/3}$ AND H_V
AT THE SAME PROBABILITY LEVEL
SIGNIFICANT WAVE HEIGHT $H_{1/3}$
VISUAL WAVE HEIGHT H_V
$H_{1/3} = H_V$
$H_{1/3} = 1.68 H_V^{0.75}$
0
5
10
15

Fig. 7



RAYLEIGH DISTRIBUTION
PROBA-
BILITY
DENSITY
$X / \sqrt{E}$
$\frac{2X}{E} e^{-X^2/E}$
$H = 2X$
$H_{1/3} = \sqrt{8E}$
1/3
$H / H_{1/3}$
0
0.5
1
1.5
2

Fig. 8

Here

$P(H)$ - the probability that an individual wave height is smaller than or equal to H.

$H_{1/3}$ - significant wave height.

The probability density, $p(H) = d\,P(H)/dH$, is given in Fig. 8.

It should be mentioned that the significant wave height is defined as the average of the 1/3 highest waves.

We have now the necessary basic equations for calculating the distribution of the individual wave heights.

PREDICTION OF INDIVIDUAL WAVE HEIGHTS

By looking at Fig. 9 one may visualize how a distribution of individual wave heights is obtained.

If one at a particular location in the sea measured the waves, the recordings should be interpreted as representing a large number of short term sea states, each characterized by the significant wave height. For each sea state all wave heights are measured and a distribution obtained. When measuring during a long period of time, the contributions from each sea state accumulate to give the final long term distribution.

In principle, therefore, what one should do mathematically is to sum a large number of short term Rayleigh distributions taking account of their probability of occurence.

The last mentioned probability is obtained from the observed data as described by Eqs. 1 and 2.

The following equation is thus obtained

$$P(H) = \int_{0}^{\infty} \left[1 - \exp(-2(H/H_{1/3})^2)\right] \cdot \frac{dP(H_{1/3})}{dH_{1/3}}\, d\,H_{1/3} \qquad (5)$$

Here P(H) is the probability that the individual wave height is smaller than or equal to H.

This type of integral has been solved by Nordenstrøm /6/ and is given by

$$P(H) = 1 - \exp(-(H/CH_c^{1/d})^D) \tag{6}$$

where

$$C = c^{-1/d} \sqrt{b/2}$$
$$D = 2k \tag{7}$$

b and k are functions of $\gamma \cdot d$ and given by Nordenstrøm /7/ (Table 1). With the notations used by Nordenstrøm $\gamma \cdot d = m$.

It is seen that the parameters of the Weibull distribution for individual wave heights (Eq. 6) are determined from

a) The parameters of the original distribution describing the observed data (Eq. 1)

b) The parameters of the formula transforming visual wave height to significant wave height (Eqs. 2 and 3)

c) A table giving b and k as a function of $\gamma \cdot d$.

In Fig. 10 the relationship between the different parameters are shown graphically.

The curves marked "Visual" were obtained applying the formula given in Eq. 3, while the curves marked "Significant" should be used when the distribution of significant wave height is known.

The described procedure has been applied to North Sea data from Famita and Hogben and Lumb. The result of this analysis is shown in Fig. 11. The upper curve for the North Sea is obtained from the Famita-data and the lower from Hogben and Lumb-data.

It has been stated by Draper /8/ that based on measurement he considered a wave of 61 feet (18.6 metres) as being representative of a typical winter. As can be seen from Fig. 11 this corresponds well with our predictions.

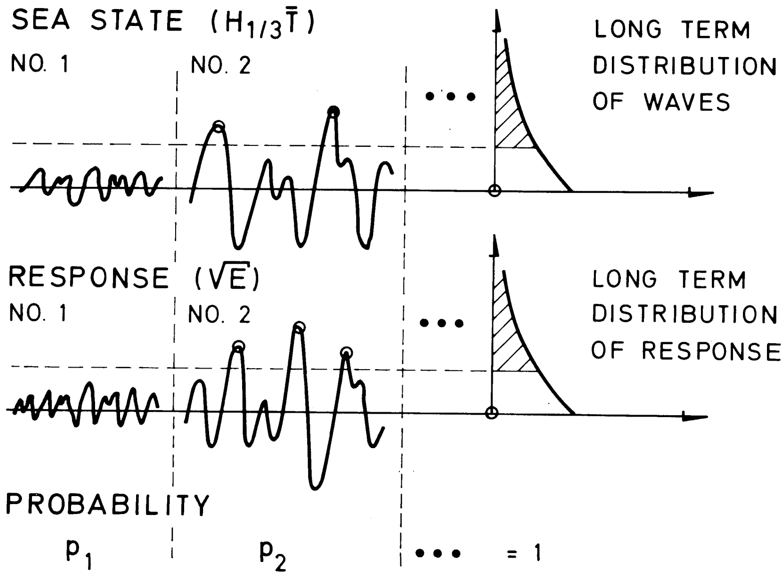

SEA STATE ($H_{1/3}\bar{T}$)
NO. 1
NO. 2
LONG TERM DISTRIBUTION OF WAVES
RESPONSE ($\sqrt{E}$)
NO. 1
NO. 2
LONG TERM DISTRIBUTION OF RESPONSE
PROBABILITY
p_1
p_2
••• = 1

Fig. 9

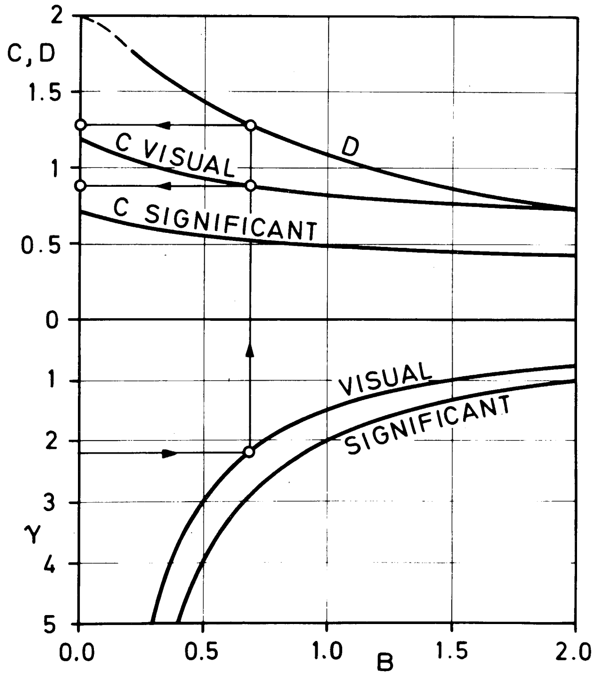

PARAMETERS OF LONG TERM DISTRIBUTION OF WAVE HEIGHT
C, D
2
1.5
1
0.5
0
1
2
3
4
5
Y
D
C VISUAL
C SIGNIFICANT
VISUAL
SIGNIFICANT
0.0
0.5
1.0
B
1.5
2.0

Fig. 10

SUMMARY

Our procedure may in short be summarized in the following steps

1. Plot the observed data on a Weibull probability paper (marginal distribution)
2. Fit a straight line to these data and find the parameters H_c and γ.
3. By using Fig. 10 find the parameters of the Weibull distribution of individual wave heights
4. Finally put the obtained values into Eq. 6.

This is clearly a very simple and straightforward method. It may be argued that one should take into account for instance the probability of occurence of the different wave periods and that the integral in Eq. 5 should have an upper limit different from infinity. This is of course correct. However, in view of the variable quality of available wave data we are of the opinion that the given procedure will give results which are sufficiently accurate for practical purposes. And last, but not least, it is found as indicated in Fig. 11, that predictions according to this method compare favourably with measurements.

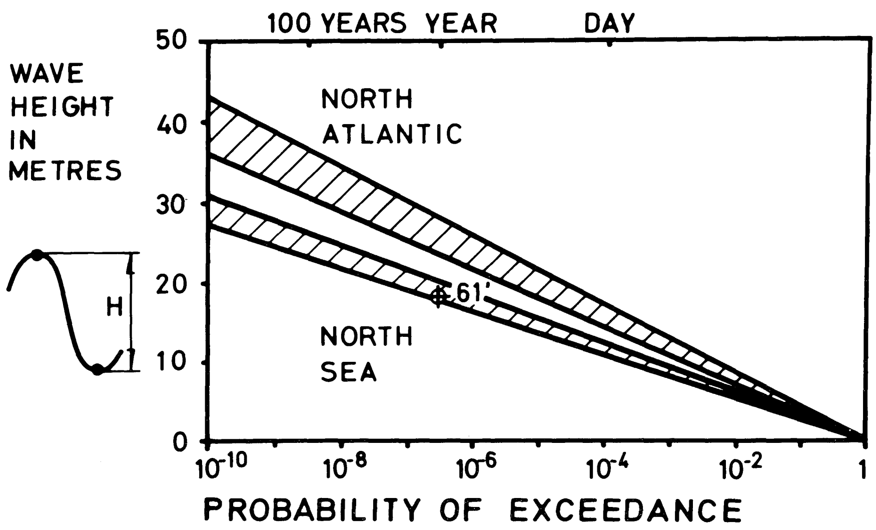


Fig. 11

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