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THE INFLUENCE OF AN ICE COVER ON THE THICKNESS OF THE SURFACE FLOW IN A STRATIFIED FIORD.

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ABSTRACT

A simple model for the motion of a surface layer on top of a heavier, motionless bottom layer has been used to study the influence of an ice cover on the thickness of the top layer. Sufficiently far from the downstream ice edge, the increase of thickness due to the ice cover will approach a fixed percentage of the thickness without ice. Using available friction data, it has been found that the increase may vary within the range of 20 to 80 per cent.

We consider a simplified model with a surface layer of thickness h and density ρ_1, flowing with velocity V_1 over a motionless bottom layer of density ρ_2. The total depth of both layers is H. Both H and h will vary with the horizontal coordinate x due to friction. Slopes S_1 and S_2 and shear stresses τ_1 and τ_2 at the surface and interface will occur as shown in fig. 1.

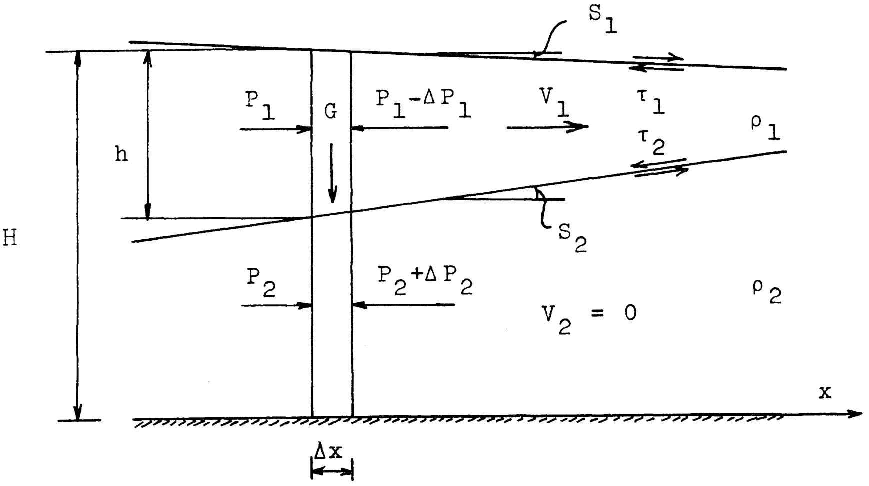

Fig. 1

Assuming constant flow per unit width = q, and referring to fig. 1, where P and G are forces defined below, we find for the horizontal equilibrium of elements of length = Δx and width = 1:

Surface layer:

$$\Delta P_1 = G \cdot S_2 + V_1 \frac{\Delta V_1}{\Delta x} \frac{G}{g} + (\tau_1 + \tau_2) \cdot \Delta x \tag{1}$$

$$\left.\begin{aligned} &\Delta P_1 = g \rho_1 h \,(S_1 + S_2) \cdot \Delta x \;, \qquad G = g \rho_1 h \cdot \Delta x \\ &\Delta V_1 = \frac{q}{h - (S_1 + S_2)\Delta x} - \frac{q}{h} = \frac{q}{h} \frac{(S_1 + S_2) \cdot \Delta x}{h - (S_1 + S_2) \cdot \Delta x} \\ &\frac{\Delta V_1}{\Delta x} \approx V_1 \frac{S_1 + S_2}{h} \end{aligned}\right\} \tag{2}$$

Combining (1) and (2):

$$g \rho_1 h S_1 = \tau_1 + \tau_2 + V_1^2 \rho_1 (S_1 + S_2) \tag{3}$$

$$S_1 = (\tau_1 + \tau_2 + V_1^2 \rho_1 S_2) / (g \rho_1 h - V_1^2 \rho_1) \tag{4}$$

Bottom layer:

$$\Delta P_2 = G \cdot S_2 + \tau_2 \cdot \Delta x \tag{5}$$

$$\begin{aligned} \Delta P_2 = &\left[g \rho_1 (h - \Delta x (S_1 + S_2)) + g \rho_2 \Delta x \, S_2 - g \rho_1 h \right] (H - h) \\ &+ g \rho_1 h \Delta x \, S_2 = g \Delta x (\rho_2 S_2 - \rho_1 (S_1 + S_2))(H - h) + G \cdot S_2 \end{aligned} \tag{6}$$

Combining (5) and (6):

$$S_2 = \left[S_1 + \frac{\tau_2}{g \rho_1 (H - h)} \right] \frac{\rho_1}{\rho_2 - \rho_1} \tag{7}$$

S_1 and S_2 can be solved explicitely from (4) and (7). The expressions are rather complicated, and we will only look at the simplified situation when $\frac{\delta V_1}{\delta x}$ is so small that it can be neglected. In this case (4) reduces to

$$S_1 = \frac{\tau_1 + \tau_2}{g \rho_1 h} \tag{8}$$

and we obtain for S_2:

$$S_2 = \frac{H(\tau_1+\tau_2) - h\tau_1}{gh(H-h)(\rho_2-\rho_1)} \tag{9}$$

In the particular case when $H >> h$ equation (9) reduces further to

$$S_2 = \frac{\tau_1+\tau_2}{gh(\rho_2-\rho_1)} = S_1 \frac{\rho_1}{\rho_2-\rho_1} \tag{10}$$

Introducing Darcy-Weissbach's friction coefficient f through the common expression $\tau = \frac{f}{8}\rho V^2 = \frac{f}{8}\rho(q/h)^2$ gives the differential equation for $H >> h$ and $\delta V/\delta x \approx 0$

$$\frac{dH}{dx} = -(S_1+S_2) = -K\left(\frac{h_o}{h}\right)^3 \tag{11}$$

where $K = \frac{(f_1+f_2)\, q^2 \rho_2}{8g(\rho_2-\rho_1)h_o^3}$ and h_o is a reference value.

With $h = h_o$ for $x = 0$ and K invariant with x the solution to (11) becomes

$$\boxed{h = h_o(1-4Kx/h_o)^{1/4}} \tag{12}$$

The surface friction f_1 may be the result of wind forces on an open surface, but we will here consider the effect of a continuous ice cover and compare it with the situation $f_1 = 0$ when no wind or ice is present.

We consider a fiord or channel where the conditions for (12) apply, with a downstream ice edge at $x = 0$, i.e. ice covered for negative x-values, and assume that h_o is not influenced by the ice cover. For $(-x)>>h_o$, (12) reduces to

$$h = h_o(-4Kx/h_o)^{1/4} \tag{13}$$

or inserted for K

$$h = \left(\frac{f_1+f_2}{2g}\,\frac{\rho}{\rho_2-\rho_1}\right)^{1/4}(-x)^{1/4}\,q^{1/2} \tag{14}$$

i.e. h is proportional to $q^{\frac{1}{2}}$ and independent on h_o for sufficiently large negative x-values.

If K' and h' represent the situation with $f_1 = 0$ we obtain from (13) and (14)

$$\frac{h}{h'} = \sqrt[4]{\frac{K}{K'}} = \sqrt[4]{\frac{f_1+f_2}{f_2}} \qquad (-x) \gg h_o \qquad (15)$$

Little is known about the values of f_1 and f_2. According to experiments by Larsen (1) and Sjöberg (2) f_2 = 0,005 - 0,01 may be assumed. Measurements made by the author 1969-70 in Norwegian rivers (3) indicate f_1 = 0,01 - 0,05. If these values are introduced in (14) we find

$$\boxed{1.2 < \frac{h}{h'} < 1.8} \qquad (16)$$

The analysis above neglects important factors as diffusion and density gradients occuring with a natural density stratification. Numerical values obtained from eq. (14) may therefore be only informative, but it is likely that the equation describes quite well the relations between the variables involved.

Based on the scarce information available about interface and ice friction it can be concluded that the formation of an ice cover may increase the depth of a surface flow of fresh water in a deep fiord by about 50 percent on the average.

Literature:

1. Ian Larsen: Tolagsstrømninger. (Thesis). D.T.H. Laboratoriet for Havnebygging. Copenhagen 1962.

2. Anders Sjöberg:Reducering av tätthetsberoende utbytestrømning med hjälp av luftbubbelridå. Användning av luftbubbelridå som hinder mot saltvattenupptrengning i flodmynningar. CTH, Inst. för vattenbyggnad. Medd. no. 40. Gothenburg 1967.

3. Einar Tesaker: Measurements of Ice Roughness and the Effect of Ice Cover on Water Levels in three Norwegian Rivers. IAHR, Ice Symposium, Reykjavik 1970.