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LONGPERIOD WAVES GENERATED BY CALVING GLACIERS

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ABSTRACT

The phenomenon called "kaneling", which occurs in the harbour of Jakobshavn, West Greenland, and the possible relationship between this phenomenon and longperiod waves produced through the calvings of the Jakobshavn Isbrae are mentioned.

A simple method of calculation - viz. that of regarding the glacier as a heavy elastic plate covering the water of the fjord - is described.

The natural frequency calculated for this system is in agreement with the observed period of the "kaneling".

1. INTRODUCTION

In connection with navigation and planning of harbour constructions in regions with tidal glaciers (for example Antarctica, Greenland, Northern Canada) one must take into account the risk for waves generated by calvings. Often the waters in the neighbourhood of tidal glaciers are packed with icebergs and bergy bits, so that construction of harbours - and even navigation - close to the glacier fronts is impossible for that reason. However, a certain type of calving waves - namely longperiod waves produced by oscillating motion of large ice masses - may travel long distances and so have an effect in navigable waters also.

This is probably the case in Jakobshavn, a small town in the southern part of Disko Bugt, West Greenland.

In the harbour bay of Jakobshavn, a seiching phenomenon - called kaneling - occurs, which can probably be explained by assuming the harbour bay to be in resonance with longperiod waves produced through the calvings of the Jakobshavns Isbrae, a large ice stream from the Inland ice, situated some 50 km east of Jakobshavn, (see paper presented at this conference: Torben Sørensen and Hans Schrøder "Longperiod Wave Phenomena in Jakobshavn Harbour Bay, Greenland").

The possible relationship between the kaneling and the calvings of the Jakobshavns Isbrae was pointed out by Hammer as early as in 1893 (see Hammer [2]), and was supported by F. Engelund (Engelund [1]) in connection with preliminary investigations with a view to projection of a quay in Jakobshavn. The suggested explanation of how the longperiod waves are produced is mentioned below.

The frontal part of the Jakobshavns Isbrae, which is more than 600 m thick, is most likely afloat (see the sketch in figure 1). The icebergs produced through the major calvings of the glacier may have a volume of more than 1 km^3, and when icebergs of that size are detached from the glacier, the frontal part of the glacier is brought out of equilibrium and starts performing vertical oscillations. Thereby, a flow - likewise oscillating - is induced in the water below the glacier, which in other words works like a huge pair of bellows, alternately drawing in the water of the fjord and expelling it (see figure 1).

In order to support this theory a calculation of the period of a floating glacier has been undertaken, treating the glacier as a heavy elastic plate covering the water.

2. STATEMENT OF EIGEN-VALUE PROBLEM

For the sake of simplicity, we shall consider the case where the width of the glacier is very large compared to the length of its floating part, (the case of plane strain). The general case has been treated in Reeh [3], to which reference is made. Therefore, the account given here will be very brief.

Consider a cantilever beam of length l and constant thickness h covering water of constant depth d, which is supposed small compared to l (see figure 1).

The differential equation for the oscillating motion of the beam may be written

$$D\frac{\partial^4 u}{\partial x^4} + \varrho g \frac{\partial^2 u}{\partial t^2} - q = 0 \qquad (1)$$

where $D = \frac{\varepsilon I}{I - \nu^2}$ (E = Youngs modulus, ν = the Poisson ratio and I = cross sectional moment of inertia), u = deflection of the beam from equilibrium position, x = coordinate in the longitudinal direction of the beam, ρ = density of beam material, t = time, and q = deviation of the load per unit length from the load in the equilibrium position.

The load q may be divided in two terms, the former accounting for the effect of buoyancy, the latter for the pressure arising from the flow of the water. Hence,

$$q = -\rho_w g u - p \tag{2}$$

where ρ_w = density of water and g = acceleration due to gravity.

Combining eqs. (1) and (2) we obtain

$$D\frac{\partial^4 u}{\partial x^4} + \rho h\frac{\partial^2 u}{\partial t^2} + \rho_w g u + p = 0 \tag{3}$$

The flow under the plate is governed by the equation of continuity and the equation of motion (Newton's second law). These are combined to the wave equation, which reads

$$\frac{\partial^2 p}{\partial x^2} + \frac{\rho_w}{d}\frac{\partial^2 u}{\partial t^2} = 0 \tag{4}$$

By means of eqs. (3) and (4), we finally obtain the following differential equation

$$D\frac{\partial^6 u}{\partial x^6} + \rho h\frac{\partial^4 u}{\partial t^2 \partial x^2} + \rho_w g\frac{\partial^2 u}{\partial x^2} - \frac{\rho_w}{d}\frac{\partial^2 u}{\partial t^2} = 0 \tag{5}$$

The boundary conditions are the following:

At the free end of the beam the bending moment and the shear force are equal to zero. Moreover, the pressure (i.e. the pressure deviation from hydrostatic pressure) is assumed to vanish. At the fixed end of the beam the deflection, the slope of the deflection curve and the flow velocity are equal to zero.

Eq. (5) in connection with the above-mentioned boundary conditions constitutes an eigen-value problem, which can be solved by elementary methods, i.e. by separating the variables writing $u = U(x)\sin\alpha t$, where α is the frequency of the oscillation. α is determined by the condition that a non-zero solution of eq. (5) can be obtained which satisfies the boundary conditions. To each α corresponds a mode. With this determined, the pressure under the plate (deviation from hydrostatic pressure) and the flow velocity can also be obtained.

3. RESULTS

As an illustration figure 1 shows the deflection curve correspond-

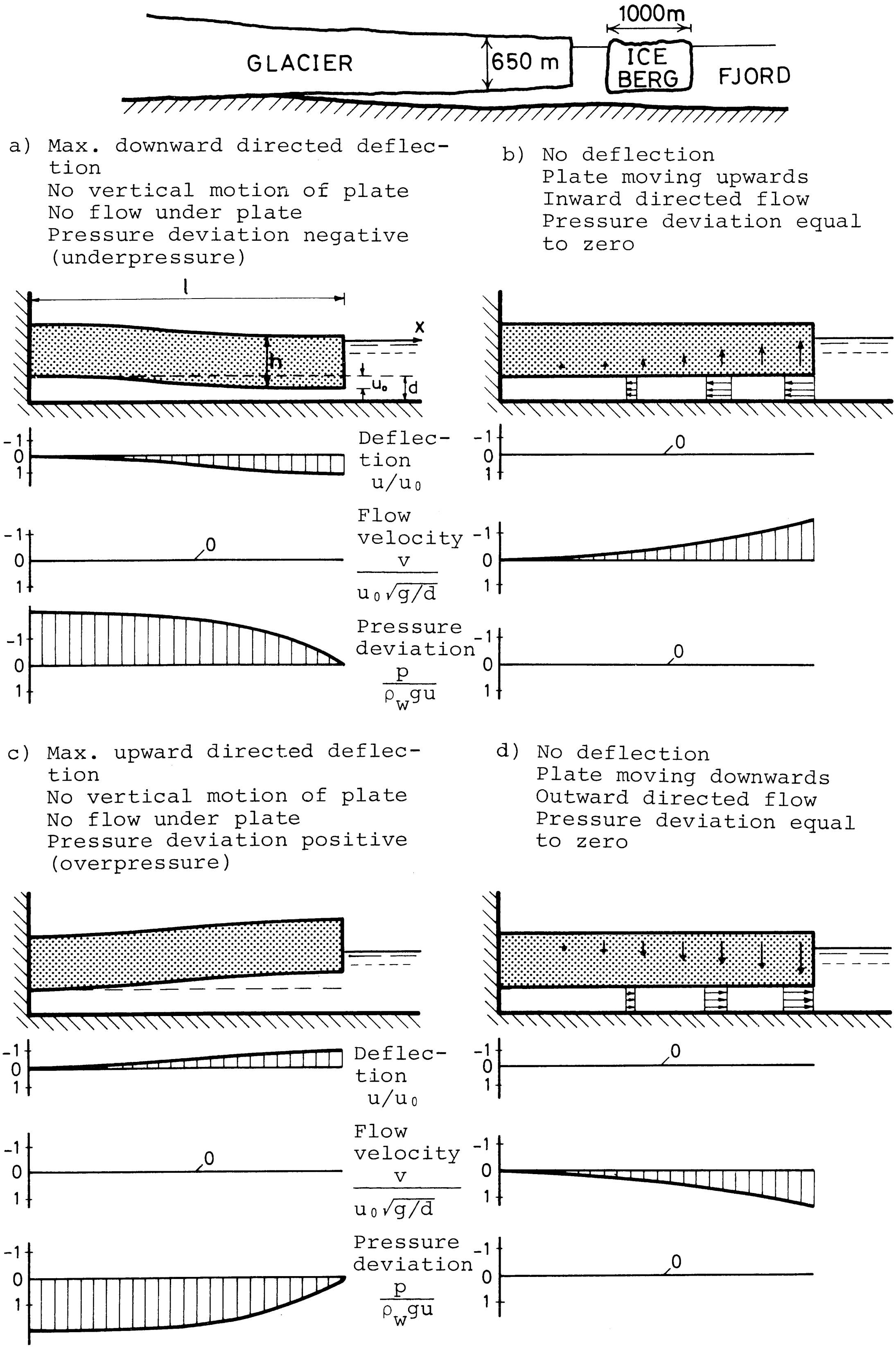

Figure 1. Characteristic situations of the oscillating motion of the glacier

ing to the fundamental vibration of the system, the velocity of the flow of the water under the beam, and the pressure deviation under the beam. Four characteristic situations during a cycle are considered, (see text in the figure).

According to eq. (2), the total load on the beam is composed of a buoyancy-term ($-\rho_w gu$) and a pressure deviation term $-p$. Figure 1 shows that the pressure deviation term is greater than the buoyancy term, thus illustrating the importance of taking into account the flow of the water in the calculation of the natural frequency of the system.

Finally, it should be mentioned that applying reasonable values of glacier dimensions and material properties a time of oscillation of the same order of magnitude as the "kaneling" (approximately 6 min) is obtained.

4 REFERENCES

[1] Engelund, Frank: Rapport om forundersøgelsen vedrørende atlantkaj Jakobshavn 1957. Grønlands Tekniske Organisation, Ingeniørkontoret.

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[3] Reeh, N.: Natural frequency of the system - a heavy elastic plate covering shallow water. Bygningsstatiske Meddelelser. Vol. 41, No. 3, 1970.