PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

SCOURING OF A SAND BED IN FRONT OF A VERTICAL BREAKWATER

Adrianus de Best	graduate student	Delft University of
Eco W. Bijker	professor of coastal engineering	Technology,
Johannes E.W. Wichers	graduate student	The Netherlands

ABSTRACT

The scouring profile of a bed consisting of movable material as this is formed in front of a vertical wall under influence of the standing wave against this wall is discussed. The form of this profile is different for coarse and fine material. In the case of fine material this material is moved in suspension by the mass transport currents from the node towards the antinode. In the case of relatively coarse sand the material is moved by the bed shear towards the node with the result that scouring occurs halfway between the node and the antinode, and deposition at the node. In the case of sand with a medium grainsize - just between the coarse and fine sands - the bed will stay almost flat.

TESTS

The tests have been executed in a flume with a length of 10 m, a width of 0,5 m and a waterdepth h of 0.31 m. Wave periods T of 1.08 s, 1.49 s and 2.00 s with corresponding wavelength L of 1.54 m, 2.40 m and 3.30 m have been applied. Sands with average grainsizes D_{50} of 0.22, 0.16 and 0.13 mm and corresponding fall velocities w of 0.028 m/s, 0.018 m/s and 0.014 m/s have been used.

The following description will be used.

	T_1=1.08 s	T_2=1.49 s	T_3=2.00 s
D_{50} = 0.22 mm	SAI	SAII	SAIII
D_{50} = 0.16 mm	-	SEII	SEIII
D_{50} = 0.13 mm	SBI	SBII	SBIII

After these tests one run is made with a wave period of 2.0 s and much coarser sand, viz. D_{50} = 0.46 mm and a corresponding fall velocity w = 0.067 m/s with the description SHIII.
Further a special test has been made with a wide range of grain-diameters

(D_{50} = 0.175 mm) for estimating the dynamical equilibrium bottom profile. The height H of the standing wave has been 0.12 m for all tests.

DESCRIPTION OF WAVE

The standing wave is described by the second order theory of Miche:

$$u_y = \frac{\pi H}{T} \frac{\cosh k (y + h)}{\sinh k h} \cos kx \cos \omega t$$

$$+ \frac{3}{8} \left(\frac{\pi H}{T}\right) \left(\frac{\pi H}{L}\right) \frac{\cosh 2 k (y + h)}{(\sinh kh)^4} . \sin 2 kx \sin 2 \omega t,$$

in which $k = 2 \pi/L$ and $\omega = 2 \pi/T$.

The origin O is taken at the undisturbed level in the node. Oy is drawn vertically upwards.

In the origin, that is in the node, the second order term is zero, and the orbital velocity at this place is a plain cosine function.

For the wave period of 1.49 s and x = 2/12 L, the resultant orbital velocity and those due to the first and second order term are given as a function of the time in figure 1.

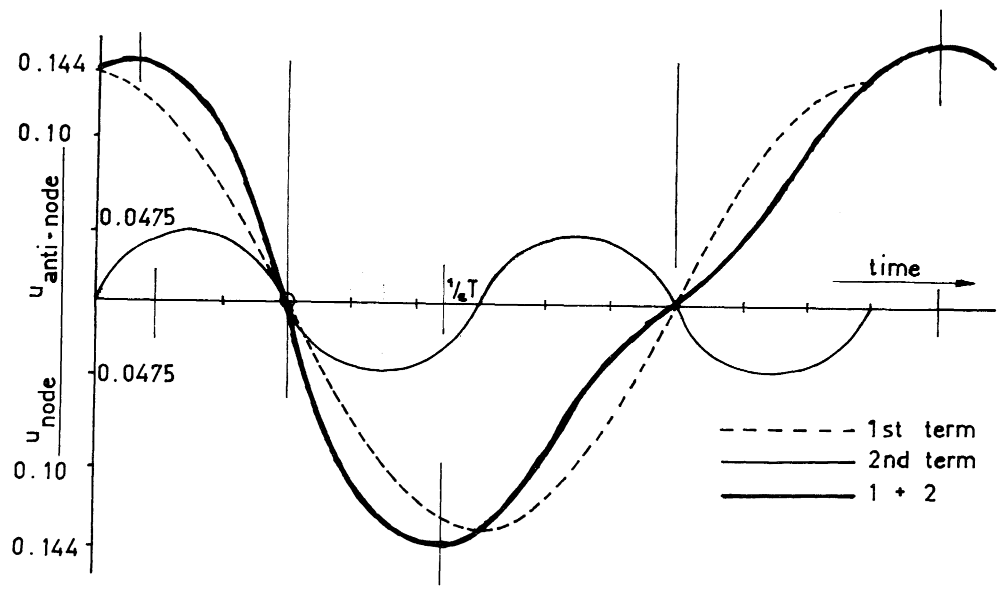

Figure 1. orbital velocity at x = 2/12 L of H = 0.12 m and T = 1.49 s as f(t).

Similar curves have been traced for x = 1/12 L and the period of 1.08 s and 2.00 s.

This curve shows that although the maximum amplitudes of the orbital velocities in both directions are equal, the curve has become assymmetrical. This results in greater acceleration forces towards the node.

The asymmetrical form of the velocity curve has an important influence on the development of the eddies behind the ripples and dunes which determines to a great extend the magnitude and direction of the bed load movement. These points will be discussed in a future publication.

The total maximum orbital velocities just above the bottom, $\hat{u}_b$, are given for the three wave periods as function of x in figures 2, 3 and 4.

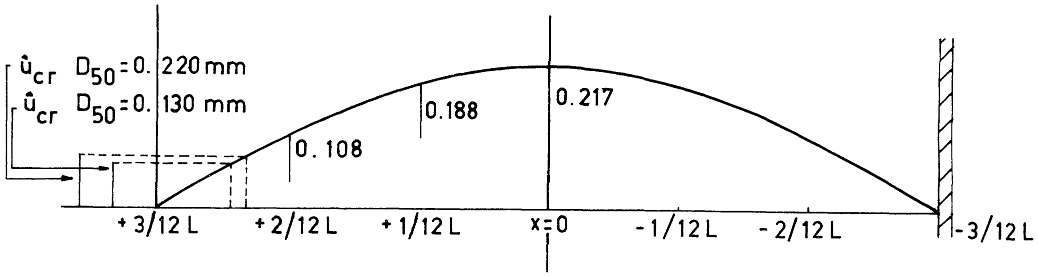

Figure 2. $\hat{u}_b$ as function (x) for T = 1.08 s and H = 0.12 m.

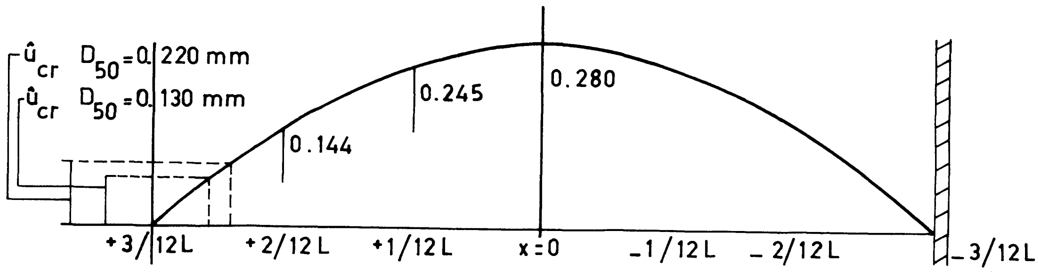

Figure 3. $\hat{u}_b$ as function (x) for T = 1.49 s and H = 0.12 m.

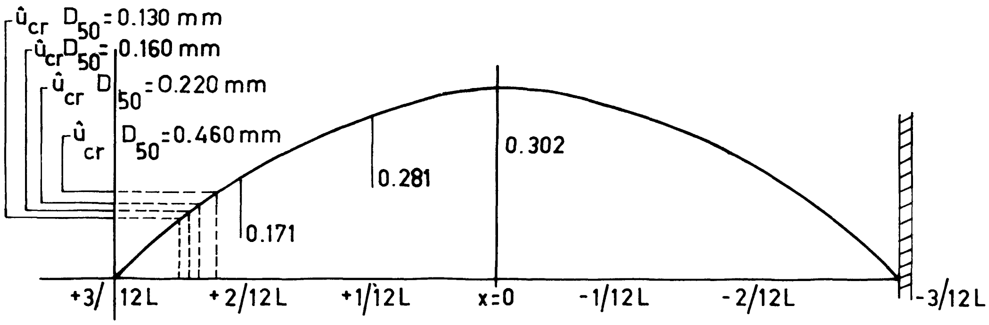

Figure 4. $\hat{u}_b$ as function (x) for T = 2.00 s and H = 0.12 m.

In the standing wave a mass transport current system will be develop as described by Longuet-Higgins [4] and as shown in figure 5.

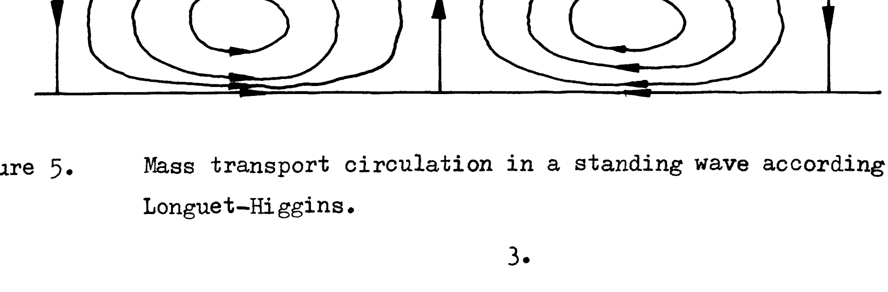

Figure 5. Mass transport circulation in a standing wave according to Longuet-Higgins.

VELOCITY FOR INCIPIENT MOTION

In all cases the ripple and dune formation starts at the place with the maximum orbital motion, that is under the node. The area covered with ripples extends from there towards the antinode as is shown in figure 6.

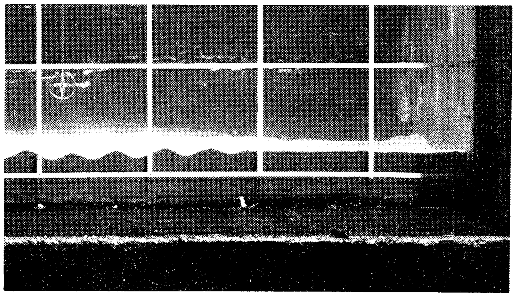

fig. a - AI after 3 minutes

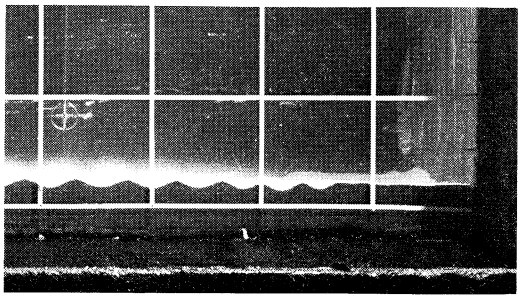

fig. b - AI after 15 minutes

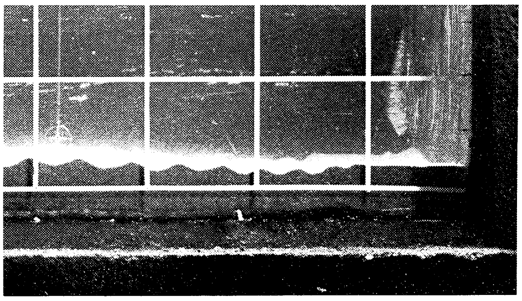

fig. c - AI after 45 minutes

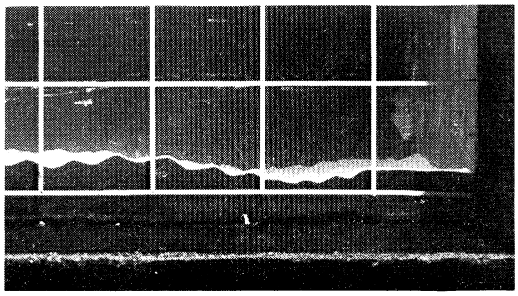

fig. d - AI after 5 hours

Figure 6. Extension of the area covered with ripples.

The values of the velocity for incipient motion have been determined from the place to where the area covered with ripples extends after equilibrium is reached. This place is reached for all tested grainsizes and waves in some 30 to 45 minutes.

For the finest material the most extended ripples will be covered later on by sand deposits brought there by the mass transport currents.

In figure 7 the bed profiles are represented which are characteristic for medium (SE), fine (SB) and coarse (SA) sand and for waves of 0.12 m height, 2.00 s period and a waterdepth of 0.31 m.

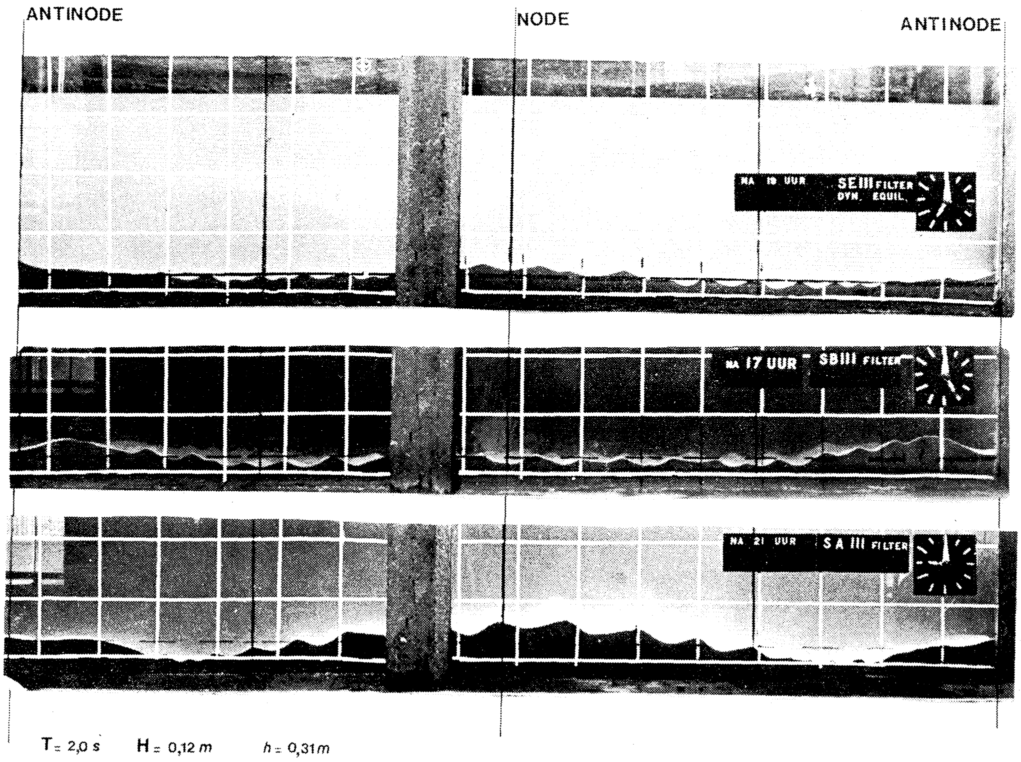

Figure 7. Characteristic bed profiles.

In the following table the values of the amplitude $\hat{u}_b$ of the orbital motion at the bed for the incipient motion as found by Bagnold [1] and Goddet [3] and as they follow the present tests are presented.

	SBI m/s	SBII m/s	SBIII m/s	SEII m/s	SEIII m/s	SAI m/s	SAII m/s	SAIII m/s	SHIII m/s
Bagnold	0.068	0.075	0.082	0.082	0.092	0.088	0.096	0.107	0.152
Goddet	0.130	0.146	0.162	0.155	0.172	0.146	0.165	0.184	0.231
Present tests	0.065	0.073	0.091	0.092	0.099	0.086	0.094	0.104	0.129

These results show a good correspondence between the results of Bagnold and the present test. Only the results for the very coarse sand deviate.

The moment of incipient motion will also be compared with the criterium of Shields although this has been derived for uniform flow. For this reason only one value for each of the used D_{50} grain-diameters will be found, independent of the period of the waves. They are found to be for coarse sand 0.182 N/m^2, for fine sand 0.159 N/m^2, for medium sand 0.169 N/m^2 and for the much coarser sand 0.262 N/m^2.

These data are obtained from figure 8, giving the relationship between the grainsize and v_*.

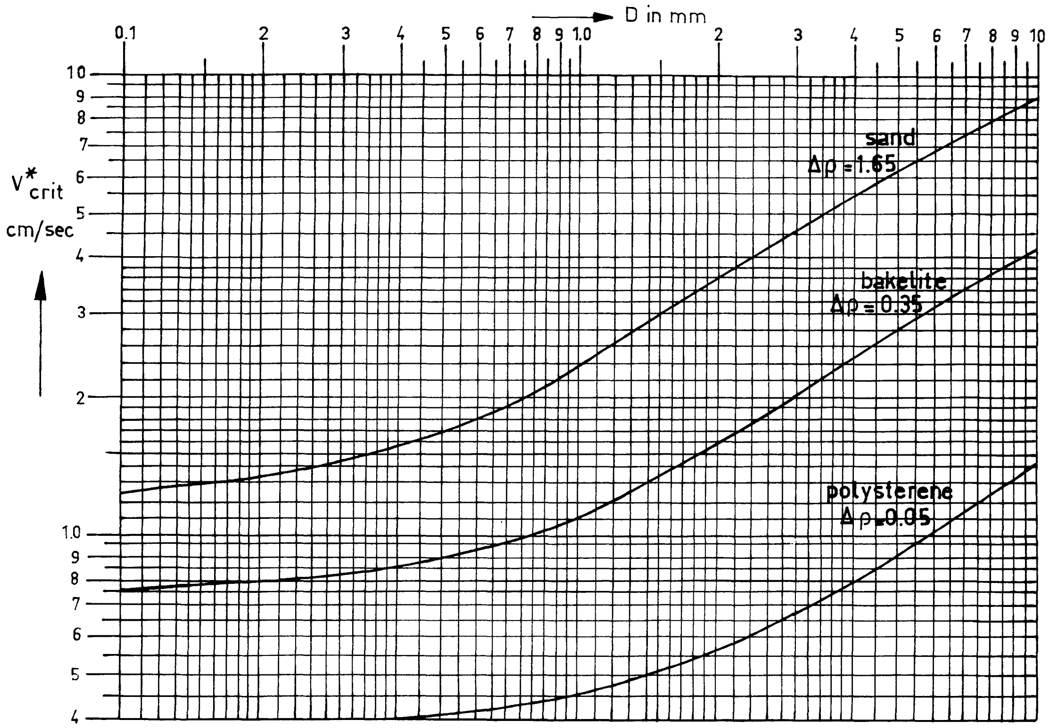

Figure 8. Bed shear stress velocity v_* versus D for various materials.

The bed shear as result of the orbital motion, will be calculated with a experimentally derived formula for the combination of waves and current and a turbulent boundary layer [2] .

The formula reads:

$$\tau = \rho \varkappa^2 \; p^2 \; u_b^2 ,$$

in which: $\varkappa$ is the constant of von Karman = 0.4 and p is a coefficient which is determined experimentally to be 0.43.

The most serious objection against this procedure is that the formula which is developed for the combination of waves and current is applied for mere waves. A mathematical exploration by the second author showed, however, that also under mere waves a formula of this type can be expected [2] .

According to quite commonly used practice the bed shear is also calculated via the normal bed resistance coefficient C. In this case $\tau = \rho \; g \; u_b^2/C^2$. It should be stated, however, that this must be fundamentally wrong since the coefficient C which is calculated for a fully developed boundary layer with a thickness equal to the water depth, is applied now for a boundary layer of a very limited thickness.

The following results are obtained in N/m^2:

	SBI	SBII	SBIII	SEII	SEIII	SAI	SAII	SAIII	SHIII
"Shields"	0.159	0.159	0.159	0.169	0.169	0.182	0.182	0.182	0.262
"Bijker"	0.125	0.158	0.245	0.250	0.290	0.219	0.261	0.320	0.493
"Chézy"	0.004	0.024	0.031	0.038	0.039	0.025	0.043	0.046	0.073

The results show a good correspondence between the criteria via Shields and Bijker, and no correspondence at all with the criterium via Chézy.

The fact that different values according to the "Bijker" criterium are found for different period indicates that the movement of material is not a simple function of the orbital velocity, but that the period of the waves, and by so the acceleration has an important influence.

MOVEMENT OF MATERIAL IN SUSPENSION

In the case the majority of the material is moved as suspended load it will be moved according to the circulation pattern of Longuet-Higgins. In this case scouring will occur under the node and shoaling between the node and the anti-node. This is just as occurs with the fine material (see fig. 7). The next step was to study the predictability of bed material movement as suspended load.

Shinohara and Tsubaki [6] determined the criterium for uniform flow for which the material moved for a considerable part as suspended load as $v_*/w=5/3$, with $v_* = \sqrt{\tau/\rho}$.

In order to see in which state the movement of materials has to be placed the value $u_{*\,crit}/w$ and $\hat{u}_*/w$ for the various grain sizes and periods will be computed and are listed here below.

The values of $u_{*\,crit}$ are calculated from u_{crit} via the criterium of Shields and with the formula of Bijker. The values of $\hat{u}_{*\,max}$ are calculated only via the formula of Bijker from $\hat{u}$.

		SBI	SBII	SBIII	SEII	SEIII	SAI	SAII	SAIII	SHIII
$u_{*\,crit}/w$	"Shields"	0.91	0.91	0.91	0.72	0.72	0.49	0.49	0.49	0.24
$u_{*\,crit}/w$	"Bijker crit"	0.80	0.90	1.12	0.88	0.95	0.53	0.58	0.64	0.33
$u_{*\,max}/w$	"Bijker"	2.71	3.47	3.72	2.67	2.89	1.35	1.73	1.86	0.78

When these values are compared with the criterium of Shinohara and Tsubaki of 1.67 for a considerable quantity of material moving in suspension, the following conclusions may be drawn.

For the coarse sand A, the values of $u_{*\,crit}/w$ are far below 1.67 and the values of $\hat{u}_{*\,max}/w$ equal or only just above 1.67. It may be expected therefore that the movement of the material takes place mainly as bed load.

For the fine sand the values of $u_{*\,crit}/w$ are closer to the Shinohara and Tsubakis value of 1.67 and the values of $\hat{u}_{*\,max}/w$ are well above 1.67. It is to be expected, therefore, that in this case the transport takes place mainly as suspended load. This explains why, with fine sand the profile as shown on fig. 7 is formed.

The most serious objection against this method is the fact that a criterium for the rate of suspension for uniform flow (Shinohara and Tsubakis) is applied to an oscillatory motion. In this motion sufficient suspended load may not have the time to develop. After the comparison of the moment of incipient motion according to the different criteria the attention has already been drawn to this phenomenon. For this reason the value of 1.67 could be taken somewhat higher. However, since the results obtained with the criterium of Shields correspond rather well with the test results, the deviation will not be great.

MOVEMENT OF MATERIAL AS BEDLOAD

In order to study this type of movement the force on the grains will be considered. This force can be written as:

$F = F_1 + F_2$, where F_1 is the normal drag force anf F_2 is the force as result of dynamic effects.

$$F_1 = 1/2 \ \rho_w C_D (1/4 \ \pi D^2) u_b^2$$

$$F_2 = \rho_w C_M (1/6 \ \pi D^3) \frac{\partial u}{\partial t}$$

With values of $C_D = 0.7$ and $C_M = 1.59$ the following values for F_1 and F_2 for coarse sand "A" are obtained:

$$F_1 = 1.33 \ 10^{-5} \ u_b^2 \text{ and } F_2 = 8.9 \ 10^{-9} \frac{\partial u}{\partial t}$$

The values of u_b and $\frac{\partial u}{\partial t}$ can be obtained from the approximation of Miche of which an example is given on fig. 1.

The results which are obtained for two crosssections viz.: x = 1/12 L and x = 2/12 L are given in the following table. The forces are given in Newtons.10^{-8}.

	crosssection x = 1/12 L				crosssection x = 2/12 L			
	SA I		SA II		SA I		SA II	
	to node	from node	to node	from node	to node	from node	to node	from node
F_1	26.0	24.2	83.0	19.2	8.3	6.1	26.7	3.6
F_2	1	0.9	1.8	0.9	0.6	0.5	0.9	0.4
F_{tot}	27.0	25.1	84.8	20.1	8.9	6.6	27.6	4.0

These results show that the force towards the node is always greater than towards the antinode so that net motion of bed load towards the node can be expected.

TEST WITH SAND WITH D_{50} = 0.175 mm.

After these tests, a test has been executed with sand mixed from the two sands with mean graindiameters of 120 and 130 mm. The result of this test is shown in figure 9.

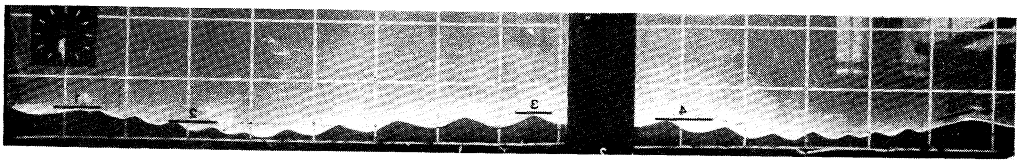

Figure 9. Bed profile due to sorting out of grains.

The coarse part of the material is moved to the node (3) and the fine part is moved to the antinode (1 and 5). The grain size distributions are shown in figure 10.

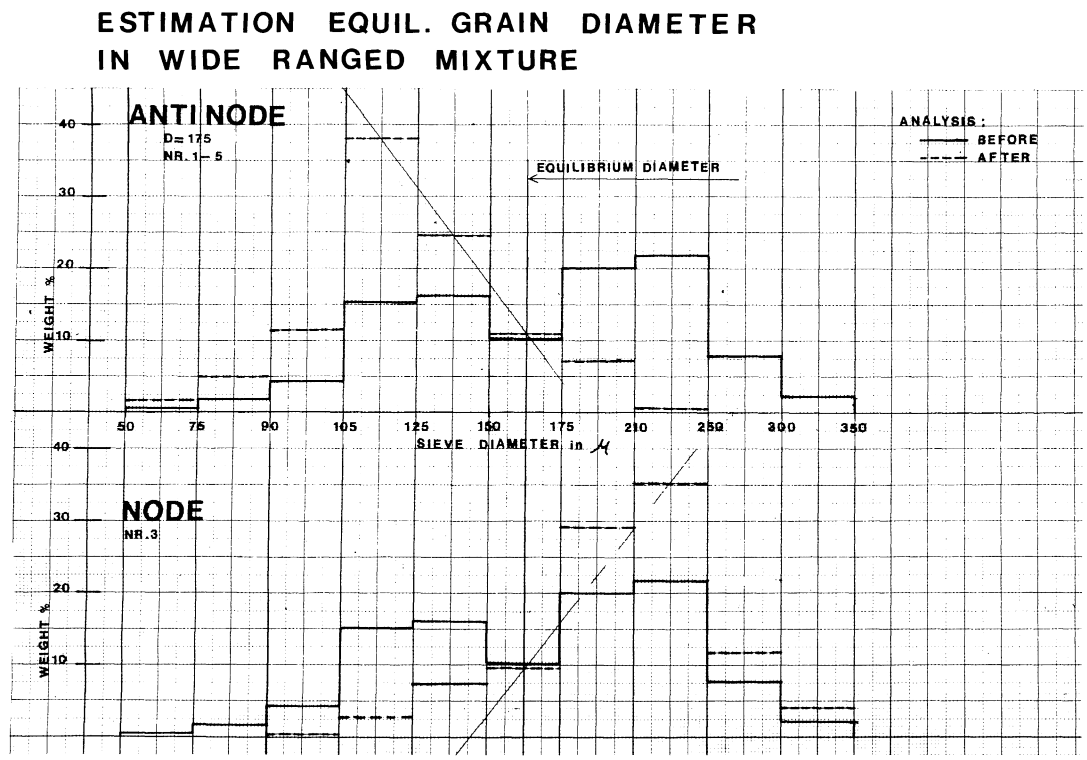

Figure 10. Grainsize distributions for and after the test (Duration $17\frac{1}{2}$ hours, T = 2.00 s)

From the grain size distribution and the bed profile as it developed under the standing wave a graindiameter can be chosen for which the acceleration forces which tend to drift the material towards the node, are in equilibrium with the

mass transport forces which tend to move the material towards the antinode. The result will be in this case a almost flat bed.
Figure 7 SE shows that this is indeed the fact. All profiles of figures 7 are taken after an approximately equal duration of the tests.

CONCLUSIONS

These tests show that there is not one fixed profile that may develop under scouring in front of a breakwater. The final profile that will develop depends on the ratio between bed shear resulting from the orbital velocity and the fall velocity of the bed material. Since with increasing wave period the bed shear due to the orbital motion increases it may be expected that for longer waves the criterium for which scouring under the node will occur will be reached more rapidly. It will be wise, therefore, to extend the protection in front of a vertical breakwater over a distance slightly longer than one half of the length of the longest wave that may be expected and that is sufficiently high to reach the Shinohara and Tsubakis criterium.

References

1. Bagnold, R.A. — Motion of waves in shallow water. Interaction between waves and sand bottom. Proc. Royal Soc. Series A vol 187 1946

2. Bijker, E.W. — Some considerations about scales for coastal models with movable bed. Publ. 50. Delft Hydraulics Laboratory 1967

3. Goddet, J. — Etude du débit d'entrainement des matériaux mobiles sous l'action de la houle. La Houille Blanche no. 2 1960

4. Longuet-Higgins, M.S. — Mass transport in water waves. Phil. Trans. Roy. Soc. Series A no. 903 vol. 245 1953

5. Shields, A. — Mitt. Preusischen Versuchsanstalt f. Wasserbau und Schiffbau. Helft 26 1936

6. Shinohara, K. and Tsubaki, T. — Reports Res. Inst. of Appl. Mech. Kyushu Univ. Japan 7 (25) pp 15-45 1959