PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

DIAGRAMS FOR THE PREDICTION OF THE MOTIONS OF SHIPS MOORED IN ARCTIC WATERS

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1. INTRODUCTION

In recent years the offshore exploration industry has made gigantic steps forward. Commercial drilling operations are planned and presently performed in water depths of up to 2000 feet. Most of these operations are to be carried out continuously over prolonged periods of time, and immediate termination of such operations is often impossible.

The two conditions which are of importance with respect to the prediction of motion are:

a. survival condition

b. operational condition

The present paper is oriented toward the latter condition, in other words the question of up to which sea state work from a drilling vessel or a construction barge can be continued. The diagrams showing the motion double amplitudes of ships in particular sea states facilitate the choice of hull forms at an early stage of planning of offshore exploration systems. It is, of course, an important economical factor to know the number of days in a year during which a construction barge is fully operational. The design diagrams will enable the designer to give a synthesized solution to his problem, i.e., he can start out with the allowable motions at a given sea state and find the ship dimensions which will satisfy this condition.

The set of diagrams presented in Chapter 5 allows also to arrive at certain conclusions which will be discussed in Chapter 6.

2. STATEMENT OF THE PROBLEM

The motion of a moored vessel subjected to excitation by an actual sea condition shall be determined. In this paper only the so-called symmetric motions, i.e., surge, heave and pitch shall be determined. Current and wind loads shall be taken into account, as well as the limited water depth as affecting the water particle motions and the mooring system. The maximum tensions in the mooring lines shall be determined. The distinction here is made between the quasi-static and dynamic tension in the mooring lines. The quasi-static tension are those arising in the mooring lines as the vessel is displaced from its equilibrium position under the action of wind, current and waves, whereby the actual frequency of the variation of the wave forces is not taken into account. The derivation of the dynamic tensions takes the actual frequency of the variation of wave forces and surging motion of the ship into account. They, therefore, will account for the transverse oscillations of the mooring lines along these catenary configurations.

3. METHOD OF SOLUTION

The dynamics of a moored vessel can be considered in two mutually coupled subsystems, the vessel herself, and the mooring cables and lines.

Because of the large difference of size of the vessel's mass on one hand and the mass of the mooring lines on the other, the dynamic coupling of the two subsystems is directed in one way only. The motion of the ship excites oscillations in the mooring lines, but the oscillations of the latter do not excite motion of the vessel. This is a basic assumption in the present investigation. Thus, the only influence of mooring lines on the vessel is the restoring reaction.

The two subsystems can now be defined as:

a. Vessel subsystem

The vessel oscillating in an actual seaway and kept on position by elastic restoration forces. These are non-linear in some cases, since they arise from the mooring lines, which change the shape of the catenaries, as the vessel oscillates.

b. Mooring subsystem

The mooring lines--each of them being subjected to a random excitation by its point of attachment to the vessel.

Separate calculations have to be carried out for bow or stern lines, breast lines and the lines amidships.

The solution of the vessel subsystem yields the values of the vessel's motion for a given sea state, while the solution of the mooring subsystem gives the maximum line tensions at the corresponding sea state. The cable diameters are determined by the maximum tensions (static plus dynamic).

3.1 Vessel Subsystem

Equations of Motion.--The dynamic behavior of a moored vessel floating in a regular seaway can be described by a set of six simultaneous second order differential equations, namely

$$\sum_{j=1}^{6} P_{ij}\, r_j\,(t) = F_j(t) \qquad i = 1, 2. . . 6$$

where P_{ij} are operators given by

$$P_{ij} = a_{ij}\, p^2 + b_{ij}\, p + c_{ij} + e_{ij}, \; p = \frac{d}{dt}$$

a_{ij} = virtual mass

b_{ij} = damping coefficient

c_{ij} = buoyant restoring coefficient

e_{ij} = elastic restoring coefficient

$r_j(t)$ = motion displacement, translational or rotational

i = mode of motion

j = mode of motion due to which the coefficient arises

With indices $i = 1$ to 3 denoting the symmetric motion, i.e., surge, heave, pitch; and $i = 4$ to 6 denoting the asymmetric motion, i.e., sway, roll, yaw, we obtain the equations of motion in expanded form as

$$\begin{bmatrix} P_{11} & P_{12} & P_{13} & 0 & 0 & 0 \\ P_{21} & P_{22} & P_{23} & 0 & 0 & 0 \\ P_{31} & P_{32} & P_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & P_{44} & P_{45} & P_{46} \\ 0 & 0 & 0 & P_{54} & P_{55} & P_{56} \\ 0 & 0 & 0 & P_{64} & P_{65} & P_{66} \end{bmatrix} \cdot \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix} + \begin{bmatrix} e_{11} & e_{12} & e_{13} & e_{14} & e_{15} & e_{15} \\ e_{21} & & & & & e_{26} \\ & & & & & \\ & & & & & \\ & & & & & \\ e_{61} & e_{52} & & & & \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \\ F_6 \end{bmatrix}$$

For mooring patterns symmetric about the longitudinal plane of symmetry, the elasticity matrix reduces to

$$\begin{bmatrix} e_{11} & 0 & e_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & e_{44} & e_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e_{66} \end{bmatrix}$$

This shows, that also the elasticity matrix allows partitioning of the six simultaneous equations into two sets of three coupled differential equations each, namely:

Symmetric motions

$$
\begin{array}{l}
\text{(surge)} \\
\text{(heave)} \\
\text{(pitch)}
\end{array}
\begin{bmatrix}
(P_{11}+e_{11}) & P_{12} & (P_{13}+e_{13}) \\
P_{21} & P_{22} & P_{23} \\
P_{31} & P_{32} & P_{33}
\end{bmatrix}
\begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}
=
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix}
$$

Asymmetric motions

$$
\begin{array}{l}
\text{(sway)} \\
\text{(roll)} \\
\text{(yaw)}
\end{array}
\begin{bmatrix}
(P_{44}+e_{44}) & (P_{45}+_{45}) & P_{45} \\
P_{54} & P_{55} & P_{56} \\
P_{64} & P_{65} & (P_{66}+e_{66})
\end{bmatrix}
\begin{bmatrix} r_4 \\ r_5 \\ r_6 \end{bmatrix}
=
\begin{bmatrix} F_4 \\ F_5 \\ F_6 \end{bmatrix}
$$

Now each of these sets can be solved separately, which makes for simplification of the computational procedure. In this paper the results for the symmetric motions only are presented. Samples of the corresponding response operators $|H_i\ (j\sigma)|$ are plotted in Figure 1. The coefficients of the equations of motion were derived by using a strip theory, the added mass and damping coefficients of the equations of motion were derived by using a strip theory, the added mass and damping coefficients derived for Lewis-form approximations of the actual sectional shapes. Twenty-one stations were used. The derivation of the elastic restoration coefficients is given in Appendix A.

3.2 Mooring Subsystem

Because of the complexity and nonlinearity of the mooring system, statistical treatment of the transverse oscillations of a mooring line has not been attempted. Each mooring line is assumed to be excited by its point of attachment to the ship, as well as by current and wave forces acting directly on the mooring line. In lieu of a better method, the point of attachment of the mooring line to the ship is assumed to make periodic oscillations. The amplitudes in the three spatial directions are obtained from geometric considerations and the significant hull oscillations. The phase lags between the various motions are taken as the phase lags that would be obtained if the hull were excited by train of regular waves of the significant wave period (corresponding to the given sea state). Making these assumptions, the upper chain terminal would appear to move on the surface of an ellipsoid.

The resulting oscillations of the mooring line will be termed three-dimensional. Considerable simplification of the problem is obtained by assuming that the upper cable terminal describes an ellipse which lies in the vertical plane passing through the anchor point and the upper cable terminal. The cable configuration is then always coplanar with the vertical plane. At this time it has not been proven by comparison of test runs between two- and three-dimensional cases that this simplification can be justified. The main reason for using the two-dimensional case lies in the excessive computer time required. Walton and Polachek (1959) give for the coplanar case the figure of 20 hours per run, using a UNIVAC computer. To avoid this difficulty, resort can be taken to electronic analog computers, but here again the equipment necessary for a solution of the three-dimensional case is excessive. The author has used both analog and digital computer techniques to solve these problems and it appears, that for an engineer probably the IBM-CSMP (Continuous System Model Program) is the one most readily applied and with computer time costs within reasonable limits. The results plotted in Figure 10 to 13 required on the average about 20 minutes per run.

Equations of Motion of a Mooring Line.--The mooring line is modeled by a lumped parameter system, as depicted in Figure 8. The straight line segments are assumed weightless but elastic. Weight, mass, hydrodynamic mass, drag, wave and current forces of the segments are centered at the stations (Figure 9). The equations of motion for this system are a set of ordinary differential equations:

$$[I_j] \, \{\ddot{x}_j\} = \{F_j\}$$

where

$[I_j]$ = inertial matrix at station j

$\{\ddot{x}_j\}$ = acceleration vector at station j

$\{F_j\}$ = force vector at station j

The tension in a generic straight line segment is given by:

$$T_{j+1/2} = \frac{A_{j+1/2} \, E_{J+1/2}}{l^{o}_{j+1/2}} \, \Delta l_{j+1/2}$$

$A_{j+1/2}$ = net cross sectional area of the cable

$E_{j+1/2}$ = Young's modulus

$l^{o}_{j+1/2}$ = original length of the segment, without tension

$\Delta l_{j+1/2}$ = change in length of the segment under tension

From the above set of equations it can be seen that the materials of the segment do not need to be identical. Also, weights added at the stations or to the segments, such as ice loads near the splash zone, can be tested. This would readily allow to investigate theoretically the effect of replacing a certain length of mooring line by a line of different material or cross section. Also, the effect of the addition of a sinker can readily be obtained.

Results.--A few sample outputs of the program for one particular case are shown. Figures 10 to 13 are for coplanar cables, i.e., waves and currents are collinear with the vertical plane of the cable.

Conclusion.--The maximum tension derived from such a detailed investigation of the dynamic of a single mooring line may be larger than the one expected from the quasi-static case, where the largest static current and wind load was applied. In this case the mooring line should be dimensioned according to this load.

4. ENVIRONMENTAL PARAMETERS

In the present study the sea state is represented by the Spectrum given by Bretschneider (1970), which is a slightly revised version of the original Bretschneider spectrum (1959). The advantage of this spectrum lies in the fact, that it readily conforms to the characteristic quantities of a sea state, namely the significant wave height and period. In this paper the wave data were chosen from the most common conditions recorded on the Ocean Weather Ship stations "Juliet" (52° 30' N, 20° 00' W) and "India" (59° N, 19° W) as given by Draper and Squire (1967) and Draper and Whitaker (1965) respectively. In particular, the following combinations of significant wave heights and periods were used to define the sea states:

Sea State	Significant Wave Height [Ft]	Significant Wave Period [Sec]
3	3.3	6.9
4	7.0	7.8
5	10.0	9.0
6	18.0	10.5
7	30.0	13.6

The mean zero crossing periods as used in the above two reports is related to the significant wave period as used in the Bretschneider (1970) spectrum by

$$T_z = \sqrt[4]{\frac{\pi}{4}}\; T_s$$

hence the significant wave period is

$$T_s = \sqrt[4]{\frac{4}{\pi}}\ T_s$$

The non-dimensional Bretschneider frequency spectrum is given by:

$$S\ (\nu) = 4.0\ \nu^{-5}\ \exp\ (-\nu^{-4})$$

where

$$\nu = \frac{T_s}{T} \equiv \text{dimensionless wave frequency}$$

From this the frequency spectrum is obtained by

$$S\ (f) = H_s^2\ T_S \cdot S\ (\nu)$$

where

$$f = \frac{1}{T}$$

and

$$\int S\ (f)\ df = H_s^2$$

Applying the spreading function

$$\frac{2}{\pi}\ \cos^2\ \theta$$

The Bretschneider spectrum for two-dimensional or directional sea is obtained as

$$S_{\eta\eta}\ (\sigma,\ \theta) = (\frac{8}{\pi}\ H_s^2\ \sigma_s^4)\ \sigma^{-5}\ \exp\ [-(\frac{\sigma}{\sigma_s})^{-4}] \cdot \cos^2\theta \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

5. STATISTICAL ANALYSIS

Knowing the complex frequency response operator $H_i\ (j\ \sigma\ )$ for all modes of motion, the corresponding significant double amplitudes of motion are then obtained by integrating the motion spectra over the range of frequencies.

The motion spectra are obtained from

$$S_{x_i x_i}\ (\sigma) = S\ (\sigma) \cdot |H_i\ (j\ \sigma)|^2$$

or in the case of a two-dimensional sea spectrum with principal wave heading χ

$$S_{x_i x_i}\ (\sigma,\ \chi) = \frac{2}{\pi}\ S\ (\sigma) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\ \theta\ |H_i\ (j\sigma,\ \chi + \theta)|^2\ d\theta$$

The significant double amplitude for a particular principal wave heading is then given by

$$[\overline{X}_i\ (\chi)]_{\ sign} = \sqrt{\int_{\sigma_{min}}^{\sigma_{max}} S_{x_i x_i}\ (\sigma,\ \chi)\ \ d\ \sigma}$$

These values are given in Figures 2a, b, c, for sea states 3 through 7.

6. RESULTS

The response operators and the significant double amplitudes for the modes of motion surge, heave and pitch have been calculated and plotted for Series 60 hull forms of the following parameters:

Block coefficients = C_B = 0.60, 0.70, 0.80

Beam-draft ratios = B/H = 2.50, 3.50, 4.50

Length-beam ratios = L/B = 5.50, 7.00, 8.50

Samples of the response operators for heave and pitch are plotted in Figures 1a and 1b and the response operators for surge is plotted in Figure 1c. The water depth in the case of surge was d = 400 ft. A spread mooring system as depicted in Appendix A, Figure 6/4, consisting of 10 mooring lines was used.

The significant double amplitudes were obtained and plotted for ships of the above parameters, but in addition calculations for the ship lengths L_{pp} = 200, 400, 600, 800, 1000 ft., and sea states 3, 4, 5, 6, 7, were carried out (sea states as defined in Chapter 4).

Sample heaving and pitching double amplitudes are plotted in Figures 2a and 2b. Surging double amplitudes are given in Figure 2c.

Finally, envelope curves for the significant double amplitudes of heaving and pitching motions are given in Figures 3 through 5 for ships with the block coefficients C_B = 0.60, 0.70, 0.80, respectively. All variations of the parameters B/H and L/B as given above are included. A compendium of the response operators in surge, heave and pitch for all 27 combinations of the above ship parameters is given in Ref. [6] and the corresponding diagrams for the significant motion double amplitudes in irregular seas are given in Ref. [7].

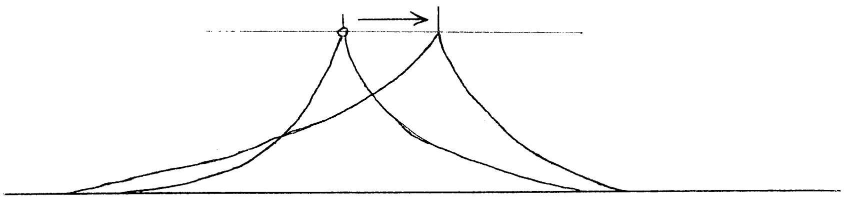
7. CONCLUSIONS

The very interesting fact appears that the significant double amplitudes of motion of ships do not depend very largely on the ship parameters, C_B, B/H, L/B, even though much greater significance of these parameters on the response of ships to regular waves (on the response operators) can be shown.

8. APPENDIX A

Elastic Restoration

The mooring lines are dimensioned according to the maximum current and wind loads. Once the length and scope of the mooring lines are determined, and a mooring pattern is decided upon, a pretension of the lines in calm water is assumed. For purpose of illustration, let us assume the mooring pattern in Figure 6/1, i.e., bow and stern line only. The pair of mooring lines yield a horizontal reaction, if the ship is displaced from its equilibrium position.



The wind load according to each sea state and some assumed current are used to calculate next the "quasi-static" displacement of the ship from its calm water equilibrium position. The quasi-static displacement of the ship changes the mooring line catenaries, and therefore the effective "spring constant" for wave included oscillations about the "quasi-static equilibrium position" is different from the original spring constant for surging oscillations in absence of current and wind. In other words the spring constant is not constant. However, inspection of results of calculations show that the amplitude of oscillation does not affect the spring constant appreciably.

9. REFERENCES

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2. Bretschneider, C. L., 1970, "Forecasting Relations for Wave Generation" Look Lab Quarterly Report, Vol. 1 No. 3, July 1970.
3. Draper, L. and Squiare, E.M., 1967, "Waves at Ocean Weather Ship Station INDIA" Trans. of the Royal Institution of Naval Architects, Vol. 109, 1967.
4. Draper, L. and Whitaker, M.A.B., 1965, "Waves at Ocean Weather Ship Station JULIETT" Deutsche Hydrographische Zeitschrift, Band 18, 1965 Heft 1.
5. Walton, T. S. and Polachek, H., 1959, "Calculation of Non-linear Transient Motion of Cables" David Taylor Model Basin, Report 1279, July 1959.
6. Seidl, L., 1971, "Surging, Heaving, and Pitching Response Operators of Moored Ships", J.K.K. Look Lab Technical Report No. 20, University of Hawaii 1971.
7. Seidl, L., 1971, "Significant Double-Amplitudes of Surging, Heaving and Pitching Motion of Ships Moored in Irregular Seas" J.K.K. Look Laboratory Report No. 21.

Figure 1 a

Unit Amplitude Response Operator in Heave

Note:

C_B = 0.70

L/B = 5.5

B/H = 3.5

i_ψ = 0.247 x L_{pp}

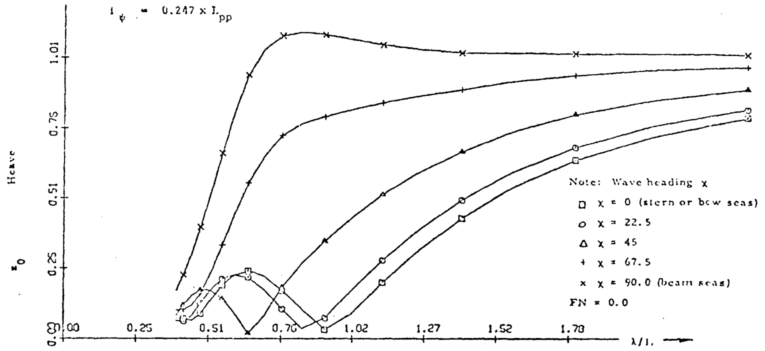


Figure 1 b

Unit Amplitude Response Operator in Heave

Note:

C_B = 0.70

L/B = 5.5

B/H = 4.5

i_ψ = 0.247 x L_{pp}

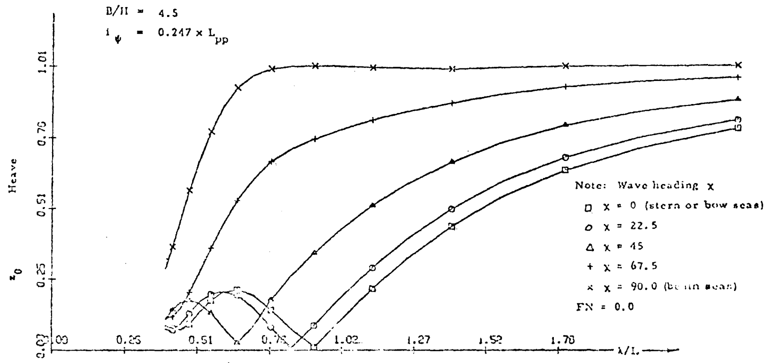


Figure 1 c

Unit Amplitude Response Operator in Surge

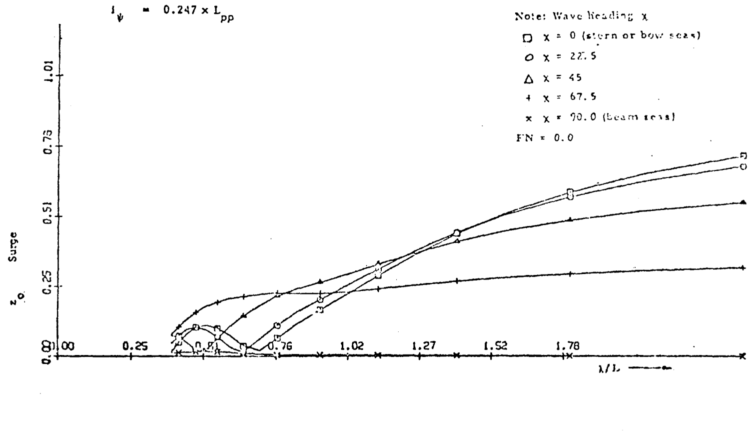
Note:

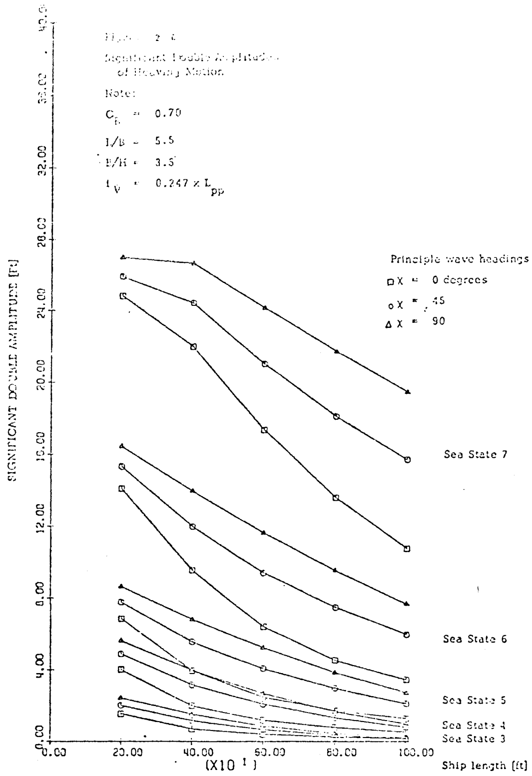
C_B = 0.70

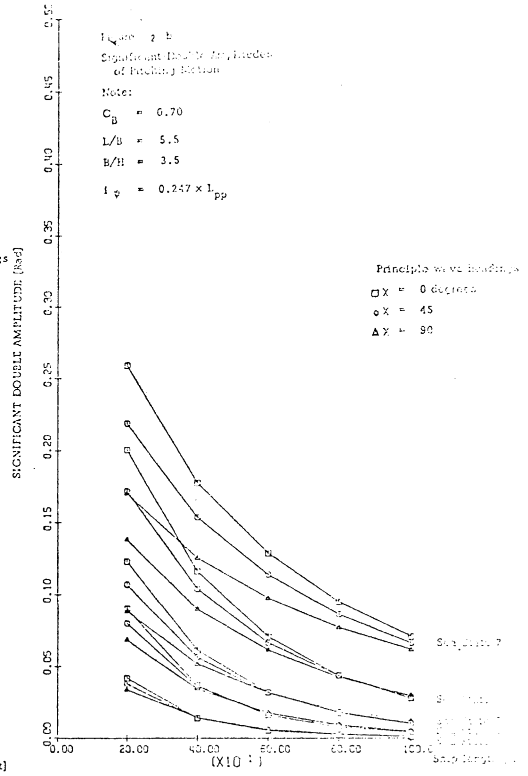
L/B = 5.5

B/H = 3.5

i_ψ = 0.247 x L_{pp}




Note:
C_B = 0.70
L/B = 5.5
B/H = 3.5
i_ψ = 0.247 x L_{pp}
Principle wave headings
□ X = 0 degrees
○ X = 45
△ X = 90
Sea State 7
Sea State 6
Sea State 5
Sea State 4
Sea State 3
SIGNIFICANT DOUBLE AMPLITUDE [ft]
(X10 ¹)
Ship length [ft]


Note:
C_B = 0.70
L/B = 5.5
B/H = 3.5
i_ψ = 0.247 x L_{pp}
SIGNIFICANT DOUBLE AMPLITUDE [Rad]
(X10 ¹)

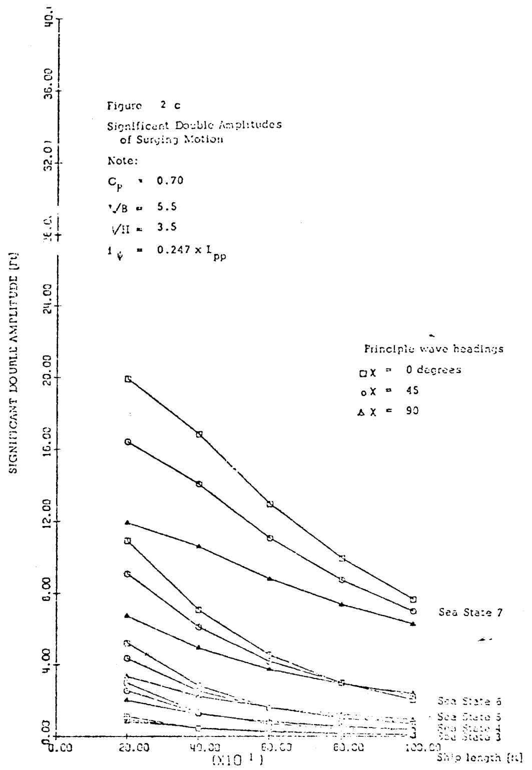


Figure 2 c
Significant Double Amplitudes
of Surging Motion
Note:
C_p = 0.70
L/B = 5.5
B/H = 3.5
i_ψ = 0.247 x L_{pp}
Principle wave headings
□ X = 0 degrees
○ X = 45
△ X = 90
Sea State 7
Sea State 6
SIGNIFICANT DOUBLE AMPLITUDE [ft]
(X10 ¹)
Ship length [ft]

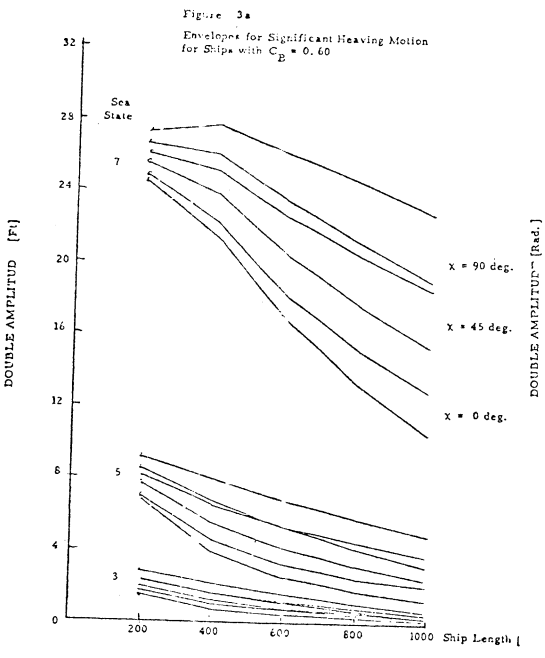


Figure 3a
Envelopes for Significant Heaving Motion
for Ships with C_B = 0.60
DOUBLE AMPLITUD [Ft]
Sea State
7
5
3
χ = 90 deg.
χ = 45 deg.
χ = 0 deg.
32
28
24
20
16
12
8
4
0
200
400
600
800
1000
Ship Length

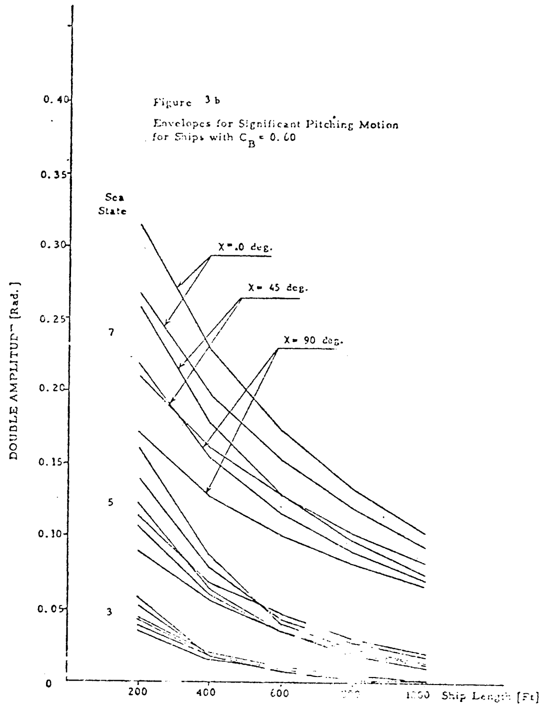


Figure 3b
Envelopes for Significant Pitching Motion
for Ships with C_B = 0.60
DOUBLE AMPLITUDE [Rad.]
Sea State
7
5
3
χ = 0 deg.
χ = 45 deg.
χ = 90 deg.
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
200
400
600
1000
Ship Length [Ft]

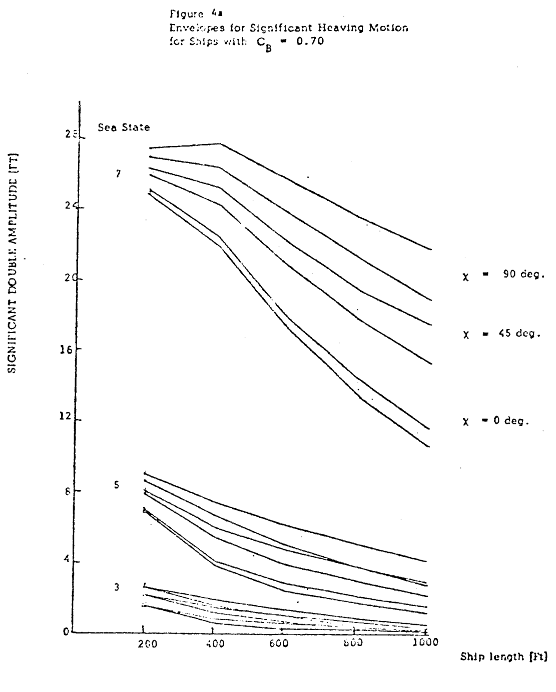


Figure 4a
Envelopes for Significant Heaving Motion
for Ships with C_B = 0.70
SIGNIFICANT DOUBLE AMPLITUDE [FT]
Sea State
7
5
3
χ = 90 deg.
χ = 45 deg.
χ = 0 deg.
16
12
8
4
0
200
400
600
800
1000
Ship length [Ft]

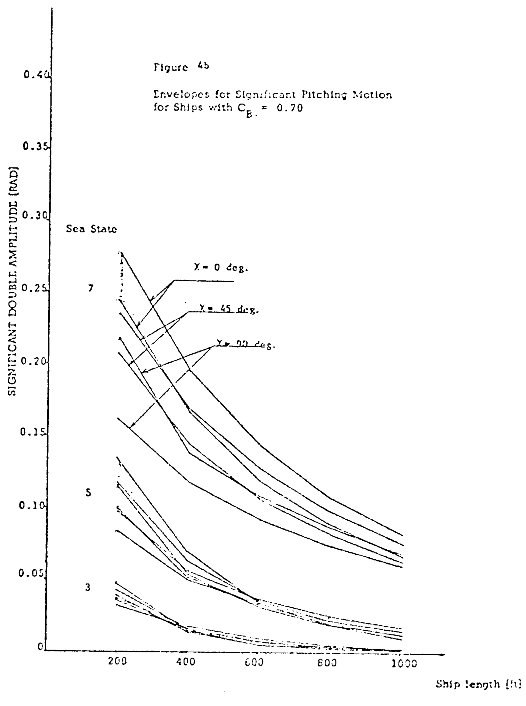
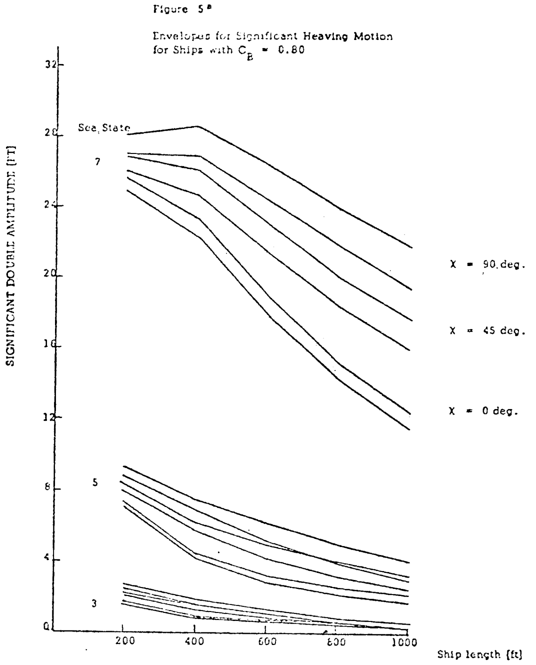
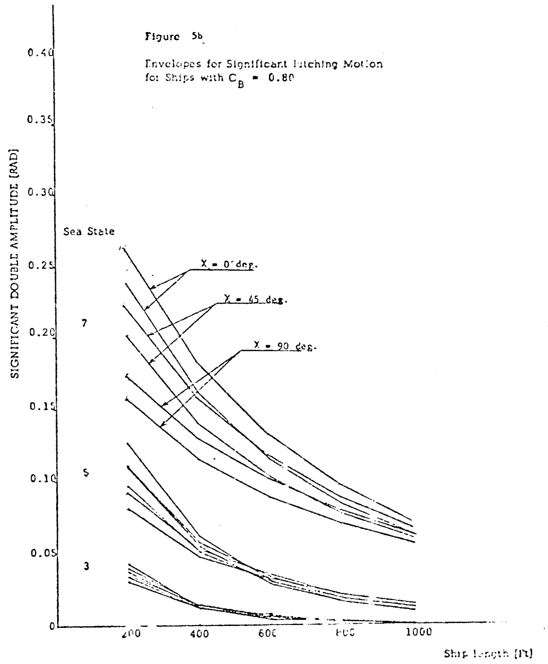


Figure 4b
Envelopes for Significant Pitching Motion
for Ships with C_B = 0.70
SIGNIFICANT DOUBLE AMPLITUDE [RAD]
Sea State
7
5
3
χ = 0 deg.
χ = 45 deg.
χ = 90 deg.
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
200
400
600
800
1000
Ship length [ft]





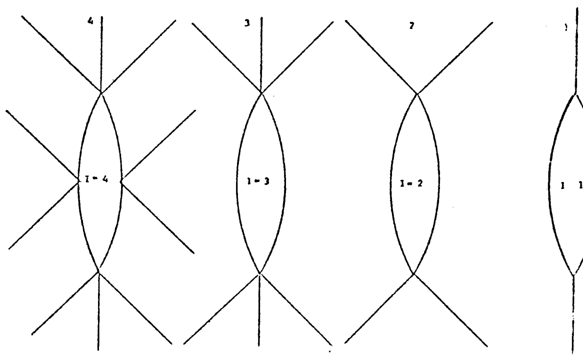


Figure 6

Discrete Representation of Mooring Line

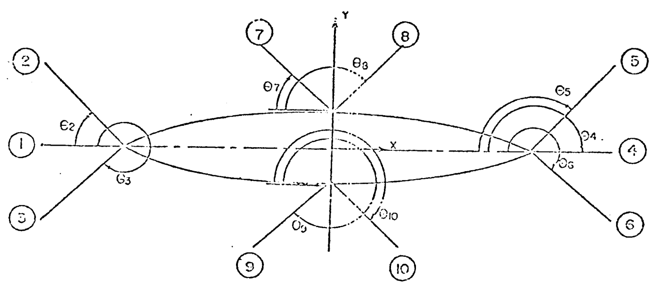


Figure 7

Forces Acting on j - th station

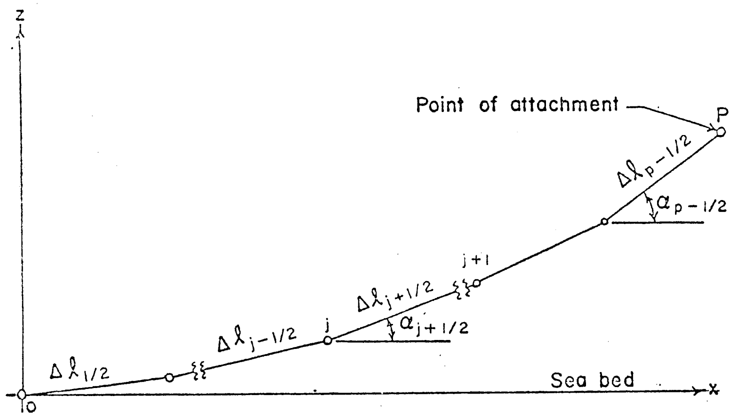


Fig. 8 : Discrete Representation of Mooring Chain

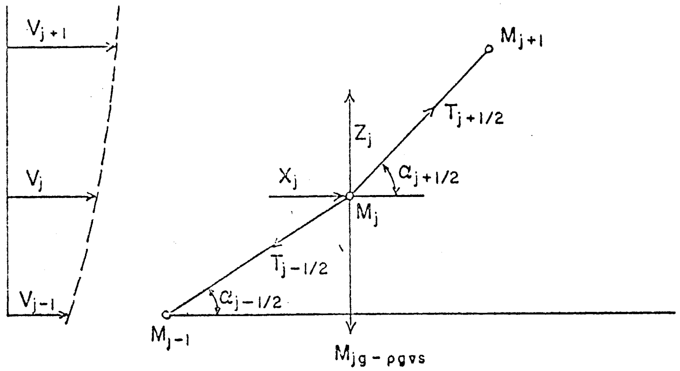


Fig. 9 · Forces acting on j-th station

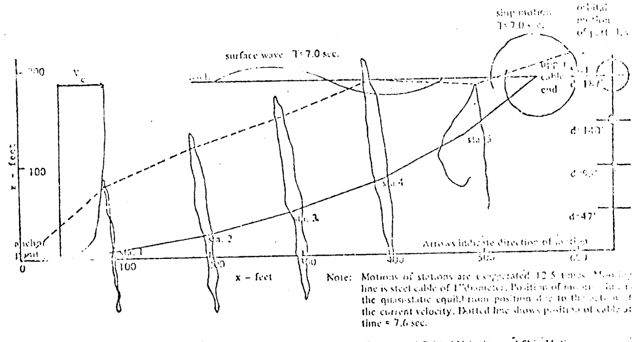
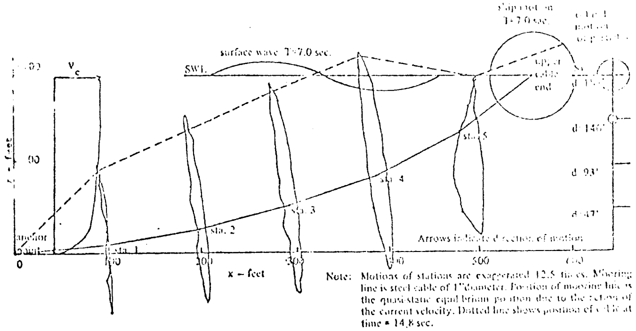


Figure 10 Dynamics of a Mooring Line in Two Dimensions, with Current and Orbital Velocity, and Ship Motion



igure 11Dynamics of a Mooring Line in Two Dimensions, with Current and Orbital Velocity, and Ship Motion.

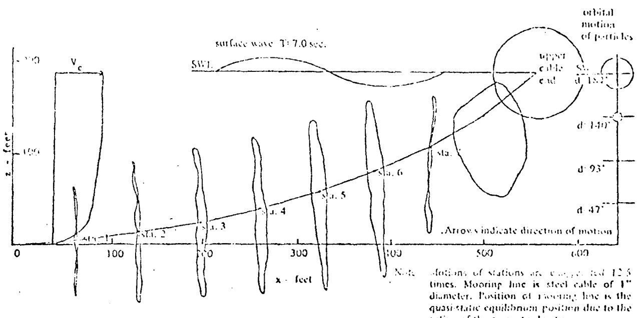


Figure 12 Dynamics of a Mooring Line in Two Dimensions, with Current and Orbital Velocity, and Ship Motion

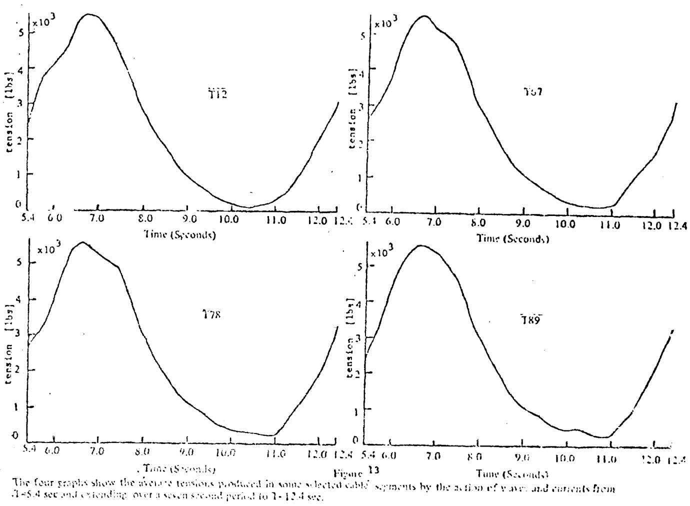


Figure 13

The four graphs show the average tensions produced in some selected cable segments by the action of waves and currents from T=5.4 sec and extending over a seven second period to T=12.4 sec.