PORT AND OCEAN ENGINEERING UNDER ARCTIC CONDITIONS
TECHNICAL UNIVERSITY OF NORWAY

ICE FORCES ON AN ISOLATED CIRCULAR PILE

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This paper presents a mathematical model for calculating the forces that a moving ice cover will exert against an isolated pile.

The model requires knowledge of the relationship between strain rate in the ice and the rate at which the ice cover is moving relative to the pile. An experimental and a theoretical approach to the derivation of this relationship are advanced.

It is assumed that compressive failure of the ice in the zone adjacent to the pile controls the thrust on the pile. It is further assumed that the mode of failure in this zone is comparable to that for uniaxial compression tests in the laboratory. The model takes into account the influence of ice type, strain rate, pile geometry and temperature on the thrust on the pile.

INTRODUCTION

The prediction of the force that ice can exert on a structure is a design problem that has still not been solved adequately. This is due to lack of knowledge of the deformation behaviour and strength of ice, insufficient information concerning the characteristics of ice covers associated with the design condition, and lack of appreciation of the interaction between ice and structures. The theoretical investigation of a number of relatively simple cases would result in useful progress towards establishing design criteria. If the assumptions are properly chosen, the calculations should give the maximum loads that could be expected. These studies could also provide a basis for further refinement of the design method based on additional field, laboratory, and theoretical studies.

One situation that is of interest is that of the force developed on a rigid circular pile by a laterally-moving ice cover. In this paper a method is presented for calculating the load for the condition of continuous contact between the ice and half the circumference of the pile. The model takes into account the strain rate dependence of the resistance of ice to deformation, and the effect of temperature.

DEFORMATION BEHAVIOUR OF ICE

It is necessary to have some knowledge of the deformation behaviour of ice to properly understand the interaction between an ice cover and a structure. Under certain loading conditions ice behaves in a viscoelastic manner. If it is deformed at a constant rate of strain, it exhibits an upper yield stress. The strain associated with yield depends on the type of ice being deformed and the rate of deformation. For the more resistant types of ice and the rates of deformation that should be considered in design, the strain at yield is about 2×10^{-3} (1).

If the stress or rate of strain imposed on the ice exceeds a fairly critical value, crack formation is initiated (2). This cracking activity causes a deterioration of the structure and contributes to the occurrence of yield or failure. For the loads of interest for design, the deterioration of the structure by crack formation would be so extensive at yield that the ice could not be assumed to have the same deformation properties as in the uncracked state. The cracks would have so relieved internal constraints, however, that it would be reasonable to assume that the uncracked portions have a resistance to deformation of about the same value as that observed in an unconfined compression test.

Failure or yield in an unconfined compression test at the rates of strain under consideration occurs by the formation of a fault zone in which there is a marked increase in cracking activity. This zone is approximately parallel to the plane of maximum shear. For strain rates less than that associated with the ductile-to-brittle transition in behaviour, the material in the zone remains intact, but the deformation of the specimen is concentrated in this region (i.e., the strain becomes non-uniform). At rates of strain greater than that associated with the ductile-to-brittle transition in behaviour, the formation of the zone is abrupt and results in catastrophic failure. It is assumed that a similar ductile or brittle failure behaviour takes place for the situation under consideration, and that the maximum load is associated with compressive yield or failure of the ice immediately adjacent to the pile.

It is necessary to establish the relationship between the strain rate in the ice immediately adjacent to the pile and the rate of penetration of the cover if the foregoing assumptions are to be used in calculations of the maximum load that can be exerted. This relationship could be established by measurement. The small amount of information available from field studies indicates that the shape of the load-penetration curve for approximately constant rate of penetration is similar to that of the stress-strain curve from constant strain rate tests (3). If yielding in the immediate vicinity of the pile has the same strain dependence as in the unconfined constant strain rate tests, comparison of these laboratory results with field observations on load and penetration would give the dependence of the strain rate in this area on the rate of penetration. There is, unfortunately, insufficient published information to allow this comparison to be made.

In a later section of this paper a theoretical relationship between the rate of penetration and the rate of strain in the ice is developed, assuming elastic behaviour. The assumption of elastic behaviour should be reasonable for a good proportion of the load build-up prior to yield at the rates of penetration of interest for design.

SOLUTION FOR PILE LOADING

The situation under consideration is shown in Figure 1. A pile in contact with the ice cover over the range $\theta = \pm \eta$, is subject to a force F_x acting through its centre. The boundary conditions that apply are

$$\sigma_r(a, \theta) = \sigma_r'(a, \theta) \qquad 2\pi - \eta \leq \theta \leq \eta \tag{1}$$

$$\sigma_r(a, \theta) = \sigma_r'(a, \theta) = 0 \qquad \eta \leq \theta \leq 2\pi - \eta \tag{2}$$

$$\tau_{r\theta}(a, \theta) = \tau_{r\theta}'(a, \theta) = 0 \qquad 0 \leq \theta \leq 2\pi \tag{3}$$

$$u_r(a, \theta) = u_r'(a, \theta) \qquad 2\pi - \eta \leq \theta \leq \eta \tag{4}$$

where the primed quantities refer to the pile and the unprimed to the ice.

Following Noble and Hussain (4), the stress function for the plane strain, elastic case that is appropriate for the boundary conditions is

$$\varphi = -\frac{F_x}{4\pi(1-\nu)}\left[2(1-\nu)r\theta\sin\theta - (1-2\nu)\left[r\ell nr + \frac{a^2}{2r}\right]\cos\theta\right] + D_o\ell nr + \sum_2^\infty\left[\frac{1}{r^n} - \frac{n+1}{n-1}\left(\frac{1}{a^2}\right)\frac{1}{r^{n-2}}\right]D_n\cos n\theta \tag{5}$$

where D_n are constants, a is the radius of the pile, and ν is Poisson's ratio. In the ice the radial stress is

$$\sigma_r(r, \theta) = -\frac{F_x}{4\pi(1-\nu)}\left[\frac{4(1-\nu)}{r} - \frac{(1-2\nu)}{r}\left[1 - \frac{a^2}{r^2}\right]\right]\cos\theta + \frac{D_o}{r^2} + \sum_2^\infty\left[\frac{n+2}{a^2r^n} - \frac{n}{r^{n+2}}\right](n+1)D_n\cos n\theta \tag{6}$$

and the radial strain is

$$2G\epsilon_r(r, \theta) = \frac{F_x}{4\pi(1-\nu)}\frac{1}{r}\left[(3-4\nu) + (1-2\nu)\frac{a^2}{r^2}\right]\cos\theta + \frac{D_o}{r^2} + \sum_2^\infty\left[\frac{n+2(1-2\nu)}{a^2r^n} - \frac{n}{r^{n+2}}\right]\cos n\theta \tag{7}$$

where G is the shear modulus. Equation (6) indicates that the radial stresses are compressive, and it was found that the circumferential stresses are tensile. Equations (6) and (7) form the basis for the solution presented for the pile loading problem. The foregoing derivation has been for the plane strain case. In the ice cover at some distance from the pile a plane stress situation would exist.

In the region very near the contact interface, however, the plane strain case is more appropriate (i.e., the thickness of the ice cover is assumed to be much greater than the region at the interface in which failure is initiated).

Solving for the net resultant force which an ice cover can exert involves, initially, a consideration of the stress distribution around the pile. Because the loading is dynamic the stress distribution will depend on the strain rate in the ice adjacent to the contact interface. From equation (7), the radial strain distribution at the interface is

$$2G\epsilon_r(a,\ \theta) = -\frac{F_x(2-3\nu)}{2\pi(1-\nu)}\frac{\cos\theta}{a} + \frac{D_o}{a^2} + 2(1-2\nu)\sum_{2}^{\infty}\frac{(n+1)}{n+2}\frac{D_n\cos n\theta}{a} \qquad (8)$$

Equation (8) simplifies to

$$2G\epsilon_r(a,\ \theta) = -\frac{F_x\cos\theta}{2\pi a} + \frac{D_o}{a^2} \qquad (9)$$

if Poisson's ratio has the value of $\frac{1}{2}$. This simplification is considered to be justified in the present case because of the deterioration of the ice at yield due to crack formation, and the viscoelastic behaviour of the intact material. If it is assumed that the solution of Noble and Hussain (4) is still valid, the parameter D_o can be obtained in terms of F_x, in which case equation (9) reduces to

$$2G\epsilon_r = -\frac{F_x}{2\pi a}(\cos\theta + 1.22). \qquad (10)$$

This equation is taken as being representative of the strain and strain rate distribution around the contact interface. The radial strain rate distribution, therefore, is

$$\dot{\epsilon}_r = \dot{\bar{\epsilon}}\,\frac{1.22 + \cos\theta}{2.22} \qquad (11)$$

where $\dot{\bar{\epsilon}}$ is the radial strain rate at the interface on the line of action of the force F_x. The normalized strain rate distribution from the above function is plotted in Figure 2 along with a simple sinusoidal distribution for comparison.

The unconfined compressive yield or failure strength for a given ice type is a function of both temperature and strain rate. Let it be assumed that this function is separable (i.e., $\sigma_{yp} = \sigma_1(T) \cdot \sigma_2(\dot{\epsilon})$). Over the ductile range of behaviour at a given temperature, the dependence of the yield strength on strain rate is found to have the form

$$\sigma_{yp} = \bar{\sigma} \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o} \right]^{\alpha} \tag{12}$$

where $\bar{\sigma}$ and α are experimentally determined and $\dot{\epsilon}_o$ is arbitrarily selected to maintain the dimensionless form of the equation. Combining equations (11) and (12) gives for the strain rate dependence of the stress distribution around the pile for maximum load

$$\sigma_{yp} = \bar{\sigma} \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o} \left(\frac{1.22 + \cos\theta}{2.22} \right) \right]^{\alpha} . \tag{13}$$

Gold and Krausz (1) found that for columnar-grained ice at -10°C $\bar{\sigma} = 565 \frac{kg}{cm^2}$ and $\alpha = 0.25$ over the strain rate range of 2×10^{-7} to 10^{-3} sec^{-1} where $\dot{\epsilon}_o = 1$ sec^{-1}. The maximum value for the yield strength occurred at the ductile-to-brittle transition, which was associated with a strain rate of about 2×10^{-4} sec^{-1}.

Equation (13) implicitly assumes that the maximum stress corresponding to the strain rate has been mobilized simultaneously over the full surface of contact. In reality this will probably not occur. When the load on the pile is maximum, part of the contact surface will have undergone deformation in excess of that for yield and part may not have undergone sufficient deformation to attain the maximum stress condition. The load calculated using equation (13) should, therefore, be greater than the actual load for the assumed conditions. It should not be very much greater, however, because the stress depends on the strain rate raised to the 1/4 power, and the stress at yield is relatively insensitive to the strain (i.e., the stress-strain curve has a maximum at yield).

The normalized form of equation (13) is plotted in Figure 3 for $\alpha = 0.25$. Shown also is the normalized stress for a sinusoidal stress distribution, i.e.,

$$\frac{\sigma_{yp}}{\bar{\sigma} \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o} \right]^{0.25}} = (\cos\theta)^{0.25} . \tag{14}$$

The net force per unit thickness of ice cover, f, is given by

$$f = \int_{-\eta}^{\eta} \sigma_{yp} \cos\theta \; a \; d\theta \tag{15}$$

where η is one-half the angle of contact. Values for the angle of contact, which depend on the elastic properties of the pile and ice, and are usually less than π, are given by Noble and Hussain (4).

Substituting equation (13) into (15) gives

$$f = \int_{-\eta}^{\eta} \bar{\sigma} \left[\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \left(\frac{1.22 + \cos\theta}{2.22} \right) \right]^{\alpha} a \cos\theta \; d\theta. \tag{16}$$

Sample calculations showed that the force per unit thickness of the cover was reduced by less than 5 per cent using the sinusoidal stress distribution given by equation (16) with $\alpha = 0.25$ and $\bar{\sigma} = 565$ kg/cm^2; i.e.,

$$f = \int_{-\eta}^{\eta} \bar{\sigma} \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \cos\theta \right)^{\alpha} a \cos\theta \; d\theta. \tag{17}$$

This reduction would balance, at least partially, the overestimation of the force due to the assumption that the yield stress is mobilized over the full contact surface. The calculation does indicate the relative insensitivity of the load to small changes in strain rate.

The yield strength of ice is temperature dependent. This dependence is of the form

$$\sigma_{yp} = \bar{\sigma}_o \left[\exp \frac{Q}{R} \left(\frac{1}{T} - \frac{1}{T_o} \right) \right] \left[\frac{\dot{\varepsilon}}{\dot{\varepsilon}_o} \right]^{\alpha} \tag{18}$$

where Q is the apparent activation energy
R is the gas constant (1.98×10^{-3} kcal/mole°K)
T is the temperature in degrees Kelvin and
$\bar{\sigma}_o$ is a constant for temperature equal to T_o.

Observations indicate that the apparent activation energy has about the same value as that found from creep studies, i.e., about 20 kcal/mole (5).

Modifying equation (17) to take into account temperature gives

$$f = \bar{\sigma}_o \exp\frac{Q}{R}\left[\frac{1}{T} - \frac{1}{T_o}\right]\left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o}\right]^{\alpha} a \int_{-\eta}^{\eta} (\cos\theta)^{1+\alpha}\, d\theta \qquad (19)$$

where f is now the load per unit thickness at the level in the ice cover for which the temperature is T.

Integrating equation (19) gives for the total load exerted on the pile by an ice cover of thickness h

$$F = \int_0^h f\, dz = \bar{\sigma}_o \left(\frac{\dot{\epsilon}}{\dot{\epsilon}_o}\right)^{\alpha} a \int_0^h \exp\frac{Q}{R}\left(\frac{1}{T} - \frac{1}{T_o}\right) dz \int_{-\eta}^{\eta} (\cos\theta)^{1+\alpha}\, d\theta. \qquad (20)$$

The temperature in the cover will vary from the equilibrium freezing temperature at the ice-water interface, to some value at the upper surface determined by the weather and snow cover. If it is assumed for design that the temperature decreases linearly from T_m at the ice-water interface to some value, T_s, at the surface (i.e., $T = T_s + (T_m - T_s)\frac{z}{h}$ where z is positive downwards from the surface and h is the ice sheet thickness), then equation (20) gives

$$F = \bar{\sigma}_o \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o}\right]^{\alpha} a \frac{h}{T_m - T_s} \int_{T_s}^{T_m} \exp\frac{Q}{R}\left[\frac{1}{T} - \frac{1}{T_o}\right] dT \int_{-\eta}^{\eta} (\cos\theta)^{1+\alpha}\, d\theta$$

$$= K(T)\, L(\theta)\, \bar{\sigma}_o \left[\frac{\dot{\epsilon}}{\dot{\epsilon}_o}\right]^{\alpha} ah \qquad (21)$$

where K(T), a temperature correction function, is a constant for a given temperature of the surface and the ice-water interface, and L(θ), a geometry function, is a constant for given relative elastic properties of the pile and the ice. Both of these functions can only be evaluated numerically. The temperature correction function is plotted in Figure 4 and the value of the geometry function for the case of a rigid pile is 1.80. Equation (21) indicates that the ratio of the total load to the product of the pile diameter, d, and ice thickness, h, has the following dependence on strain rate

$$\frac{F}{dh} \propto \dot{\epsilon}^{\alpha} \tag{22}$$

i.e., the normalized load on the pile is proportional to the yield strength of the ice in contact with the pile.

THEORETICAL RELATION BETWEEN STRAIN RATE AND PENETRATION RATE

To give the proper perspective to the discussion presented in this section, consider equation (21) which shows that the pile loading depends on the strain rate raised to the power α, where α is a small number (for columnar-grained river ice $\alpha = 0.25$). Even an approximate relation, therefore, between strain rate and penetration rate would be useful for predicting the load.

Integrating equation (7) for the radial strain gives

$$2Gu_r(r, \theta) = -\frac{F_x}{4\pi(1-\nu)}\left[(3-4\nu)\,\ell nr - (1-2\nu)\frac{a^2}{2r^2}\right]\cos\theta - \frac{D_o}{r}$$

$$-\sum_{2}^{\infty}\left[\frac{n+2(1-2\nu)}{n-1}\,\frac{1}{a^2 r^{n-1}} - \frac{n}{n+1}\,\frac{1}{r^{n+1}}\right](n+1)\,D_n\cos n\theta + f(\theta) \tag{23}$$

where $f(\theta)$ is an odd function of θ. The natural boundary condition to use to determine this function would be $u_r = 0$ for $r = \infty$, but it cannot be applied easily because of the ℓnr term. This difficulty can be circumvented by choosing an arbitrary radius, ℓ, for which the displacement is negligible. Applying this condition to equation (23) and assuming a Poisson's ratio of $\frac{1}{2}$ gives

$$2Gu_r(r, \theta) = -\frac{F_x}{2\pi}\,\ell n\frac{r}{\ell}\cos\theta + D_o\left[\frac{1}{\ell} - \frac{1}{r}\right]$$

$$-\sum_{2}^{\infty}\frac{n}{n-1}\left[\frac{n+1}{a^2}\left[\frac{1}{r^{n-1}} - \frac{1}{\ell^{n-1}}\right] - \frac{n-1}{r^{n+1}}\right]D_n\cos n\theta \tag{24}$$

where $\ell^2 >> a^2$.

For $r = a$ and $\theta = 0$

$$2Gu_r\,(a,\ 0) = -\frac{F_x}{2\pi}\,\ell n\,\frac{a}{\ell} + D_o\left[\frac{1}{\ell} - \frac{1}{a}\right] - \sum_{2}^{\infty}\frac{2n}{n-1}\,\frac{D_n}{a^{n+1}}\,. \tag{25}$$

Using the solution given by Noble and Hussain (4) to determine D_o and D_n in terms of F_x, and combining the result with equation (10) with $\theta = 0$ gives

$$\frac{\epsilon_r a}{u_r(a,\ 0)} = \frac{2.22}{1.22\left[\frac{a}{\ell} - 1\right] + \ell n\,\frac{a}{\ell} - 0.616} \tag{26}$$

The dependence of the strain-displacement ratio on a/ℓ is shown in Figure 5.

All ice covers and floes are finite. Figure 5 shows that once the size of the floe or sheet exceeds approximately 50 times the pile radius, the ratio of the strain to the displacement evaluated at the pile-ice interface is relatively insensitive to it (e.g., a decrease in a/ℓ from 5×10^{-2} to 10^{-3} causes a corresponding decrease in $\epsilon_r a/u_r$ from 0.45 to 0.25). For a given floe size the ratio is a constant.

Rearranging equation (26) and differentiating with respect to time gives

$$\dot{\epsilon}_r\,(a,\ 0) = -\frac{\dot{u}_r\,(a,\ 0)}{kd} \tag{27}$$

where d is the diameter of the pile and

$$k = 0.225\left[1.22\left(\frac{a}{\ell} - 1\right) + \ell n\,\frac{a}{\ell} - 0.616\right]. \tag{28}$$

The dependence of k on a/ℓ is shown in Figure 6. Equation (27) indicates that for a given floe size, the strain rate in the ice at the pile varies inversely with the pile diameter. Equation (27) agrees with that given by Korzhavin (6) if k is set equal to 2 (i.e., $a/\ell = 10^{-3}$).

Substituting equation (27) into equation (21) gives

$$F = \frac{K(T)\ L(\theta)}{2}\ \bar{\sigma}_o\ hd\left[\frac{\dot{u}}{kd\,\dot{\epsilon}_o}\right]^{\alpha}. \tag{29}$$

For a given material behaviour and temperature condition equation (29) reduces to

$$\frac{F}{hd} = N\left(\frac{\dot{u}}{d}\right)^{\alpha} \tag{30}$$

where N is a proportionality factor that takes into account temperature effects, strain distribution around the pile, strength properties of the ice, relative elastic properties of the ice and pile, and the geometry factor relating strain rate and penetration rate.

The normalized force, $\frac{F}{hd}$, is plotted against the strain rate in Figure 7 for the case of T_o = -10°C, $\bar{\sigma}_o$ = 565 kg cm^{-2}, α = 0.25, ice surface temperature -10°C, ice-water interface temperature 0°C and floe diameter forty times the pile diameter (i.e., $\frac{\ell}{a}$ = 40, k = 1.25). Note the strain rate of 2 x 10^{-4} sec^{-1}, which is the rate associated with the ductile-to-brittle transition for columnar-grained ice at -10°C. For strain rates less than 2 x 10^{-4} sec^{-1} the ice would be expected to behave in a ductile manner. For strain rates greater than about 2 x 10^{-4} sec^{-1}, laboratory observations indicate that the strength in simple compression is constant or even decreases with increasing rate (7). The normalized force at the strain rate associated with the ductile-to-brittle transition would, therefore, be the maximum that could occur for the assumed conditions. Bear in mind, however, that this transition strain rate is temperature dependent.

DISCUSSION

The theory developed is for cold ice (i.e., temperature less than 0°C) in intimate contact with a rigid cylindrical pile. The equations obtained should give an upper limit to the thrust that can be induced, and their form is sufficiently simple that it should be relatively easy to determine their validity through field measurements.

For many situations significant movement of the ice relative to a structure only occurs during spring breakup (e.g., many pier sites). Values of loads that have been obtained for this condition are shown in Figure 7. Neill's results (8) are for two sites with pier diameters of 0.85 and 2.3 m. The bars indicate the range in velocities and ice floe sizes observed. It should be noted that the 2.3 m pile was inclined at 23° from the vertical. The observed loads are not much smaller than the maximum predicted by equation (21) for an ice cover at 0°C throughout. Figure 7 indicates the large range in strain rates that must be taken into consideration, both in design and research.

Also shown in Figure 7 are results obtained by Nuttall and Gold (9) from laboratory tests simulating pile loading by columnar-grained ice carried out at -10°C. The theoretical curve for a temperature of -10°C throughout the ice is shown for comparison. The strain rates were evaluated at the point where creep strain was equal to the assumed strain at yield (2 x 10^{-3}). In all cases this strain occurred in the region of primary creep. Note that the observed loads are smaller than the theoretically predicted ones. If the strain rates associated with the secondary creep stage are used in the calculations, the predicted loads for given conditions are less than the ones applied.

It would be expected that an ice cover would be in intimate contact with a pile only during the very initial period of its movement relative to it. Once the failure process had been induced, contact between the ice and pile would probably no longer be continuous. It would be expected that stress concentrations would exist at the points of contact from which failure could continue with relative ease. The load resulting from such partial contact conditions would be less than that predicted assuming intimate contact.

During spring breakup the ice cover can be expected to be at 0°C throughout, and the width of the floes must be equal to or less than that of the channel. Laboratory studies indicate that the strength of melting ice is less than that which would be predicted by equation (18), (6). The foregoing factors must be taken into consideration in field investigations and when establishing design criteria. They could account, along with the possible decrease in strength with increase in strain rate in the brittle region, for the difference between the predicted and observed forces shown in Figure 7.

Equation (30) indicates that the normalized load decreases as the pile diameter increases. It might be considered that this is a geometry effect, but it is in reality a strain rate effect. This can be appreciated by referring to equation (27) which indicates that the strain rate in the ice at the pile varies inversely as the pile diameter. This possibility must be taken into consideration if the relation between rate of penetration and strain rate in the ice at the pile is to be determined by comparison of field and laboratory observations.

As the value of the exponent, α , in equation (30) is probably less than 0.5, the unit load developed on the pile will not be strongly dependent on rate of penetration and pile diameter in the ductile region. The range of diameter and rate of penetration will have to cover several decades, therefore, to determine properly the validity of the equation from field investigations.

CONCLUSIONS

It has been shown that the dependence of the thrust exerted on a circular rigid pile of diameter d by a moving ice cover of thickness h is given by

$$\frac{F}{dh} = N \left(\frac{\dot{u}}{d}\right)^{\alpha}$$

where $\dot{u}$ is the relative rate of movement between the pile and the ice, α is an exponent that depends on the type of ice, and N is a factor taking into account temperature effects, strain distribution about the pile, strength of the ice, elastic properties of the ice and pile and geometric factors. It gives an upper limit to the predicted load, is sufficiently simple to be readily checked by field measurements, and provides a basis for future developments and refinements of design criteria for ice thrust on piles.

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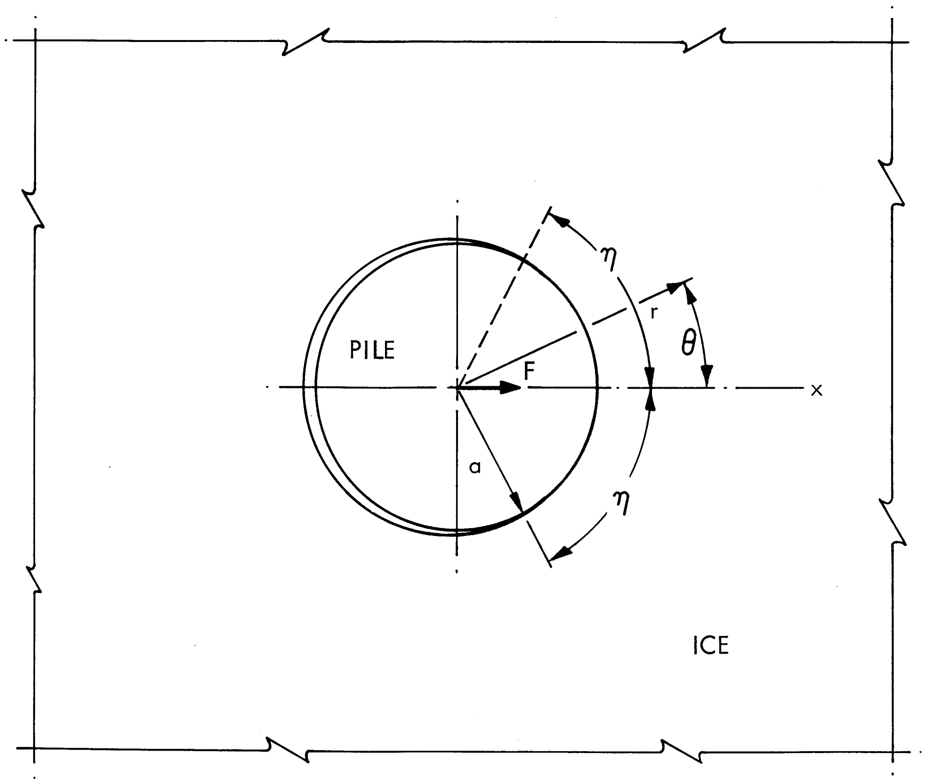
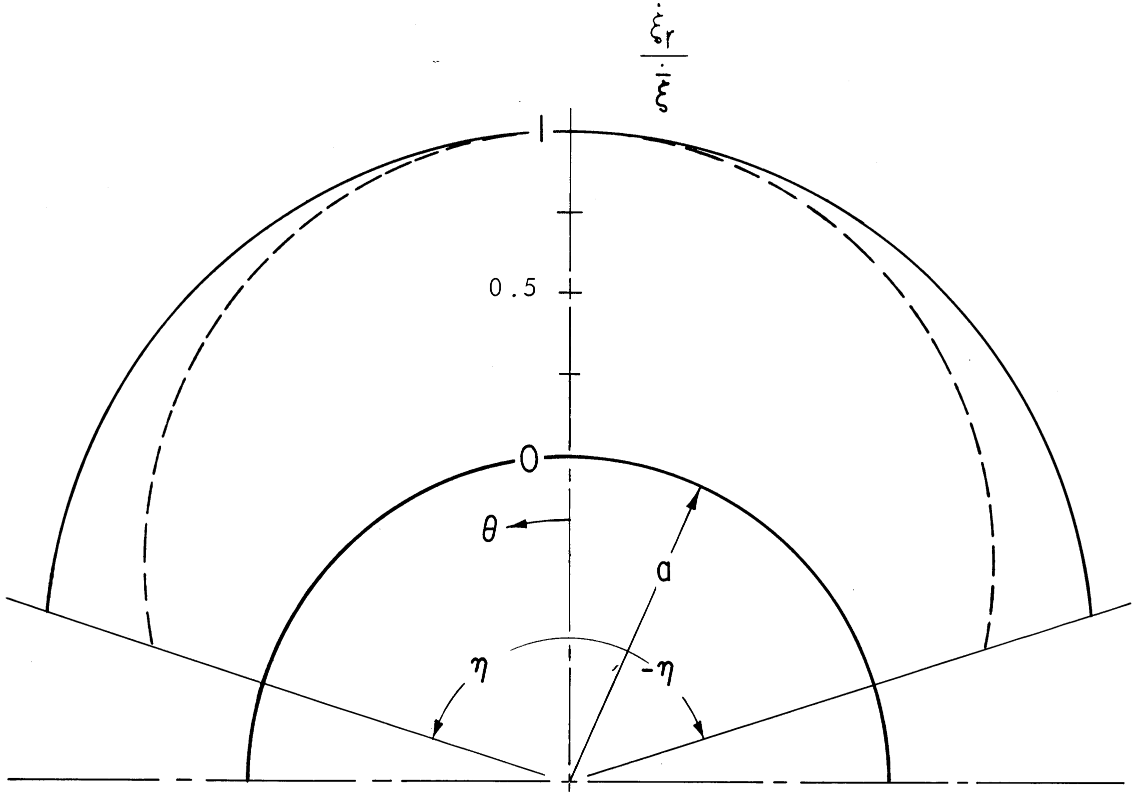


FIGURE 1 INDENTATION GEOMETRY BR 4789-1



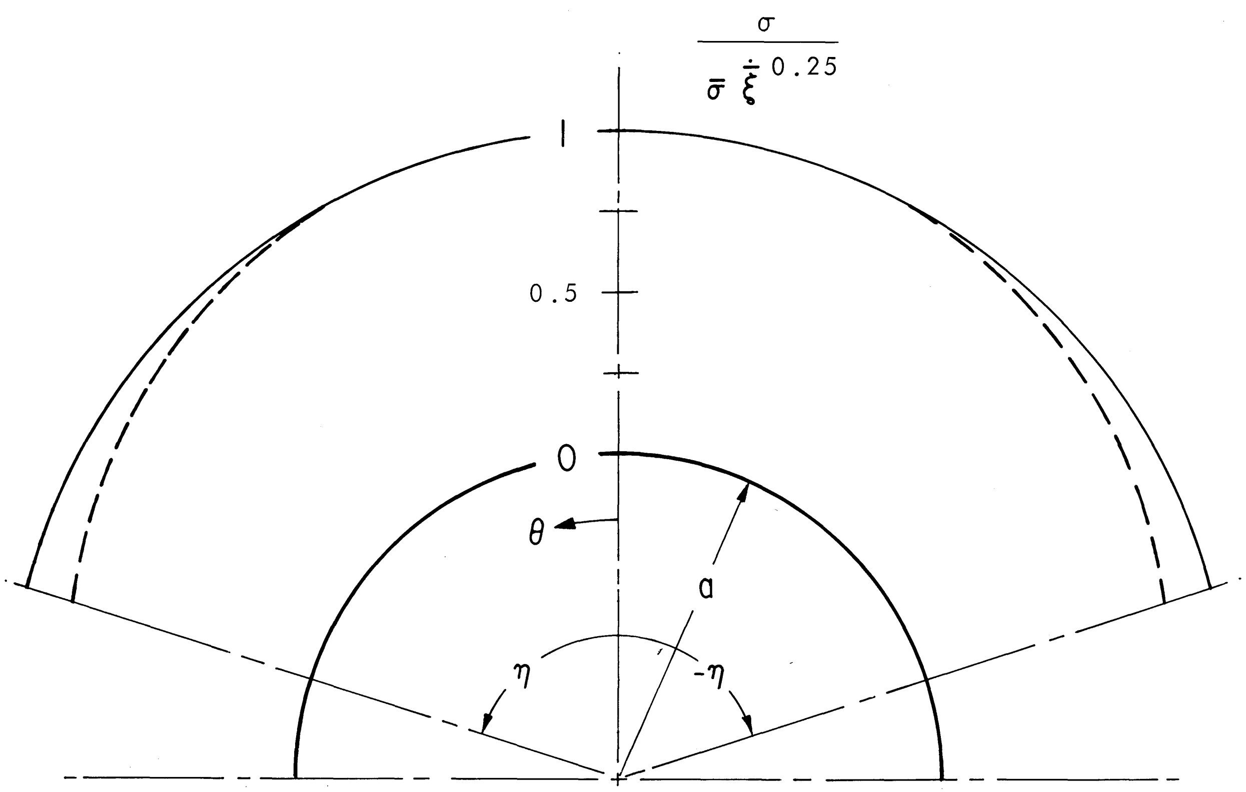
STRAIN RATE DISTRIBUTIONS

- - - SINUSOIDAL ($\cos\theta$)

—— FUNCTIONAL $\left(\frac{1.22 + \cos\theta}{2.22}\right)$

FIGURE 2 NORMALIZED STRAIN RATE DISTRIBUTION

BR4789-2



STRESS DISTRIBUTIONS

SINUSOIDAL $(\cos\theta)^{0.25}$

FUNCTIONAL $\left(\frac{1.22 + \cos\theta}{2.22}\right)^{0.25}$

FIGURE 3 NORMALIZED MAXIMUM STRESS DISTRIBUTIONS

BR4789-3

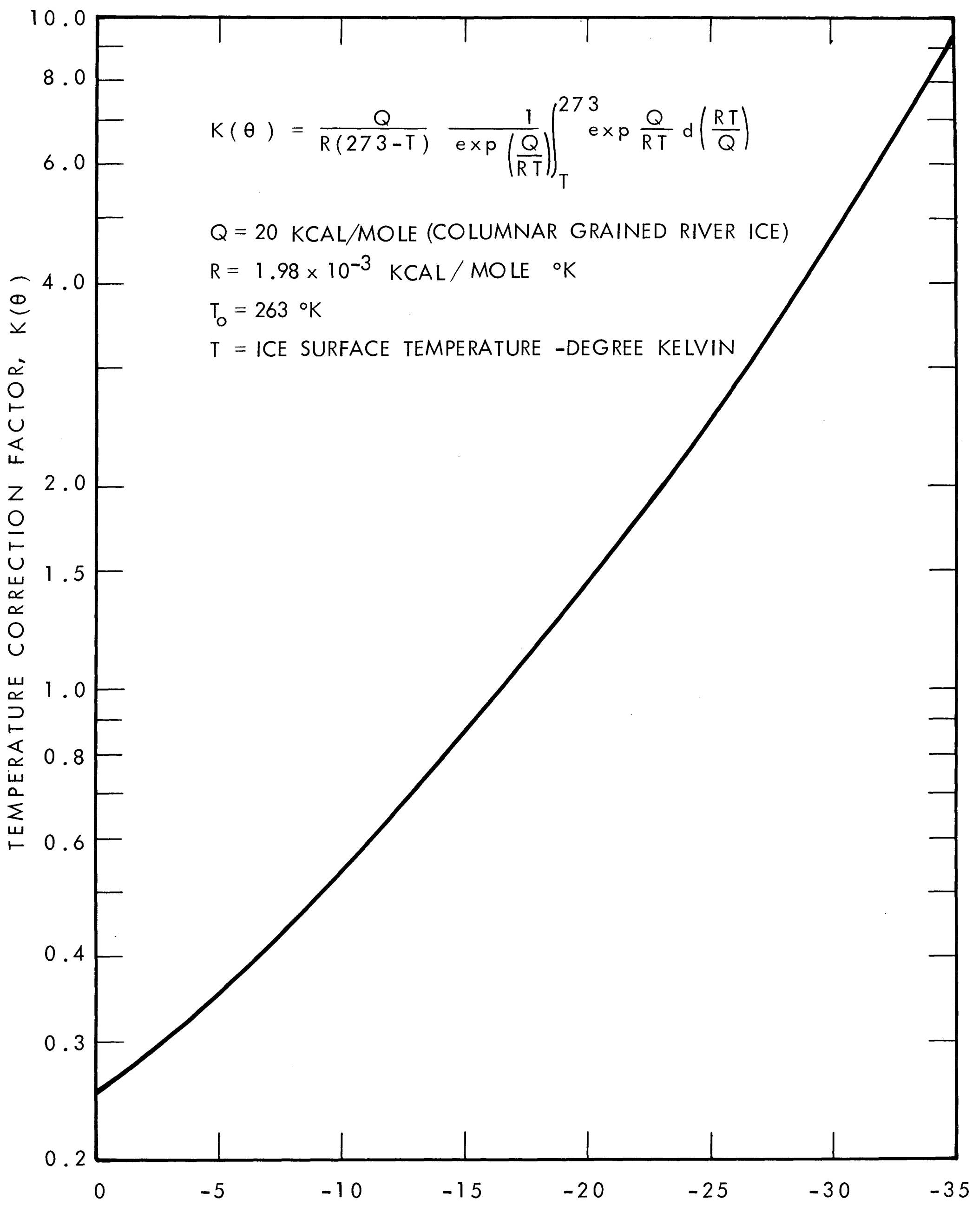


FIGURE 4

TEMPERATURE CORRECTION FACTOR FOR COLUMNAR GRAINED ICE COVERS

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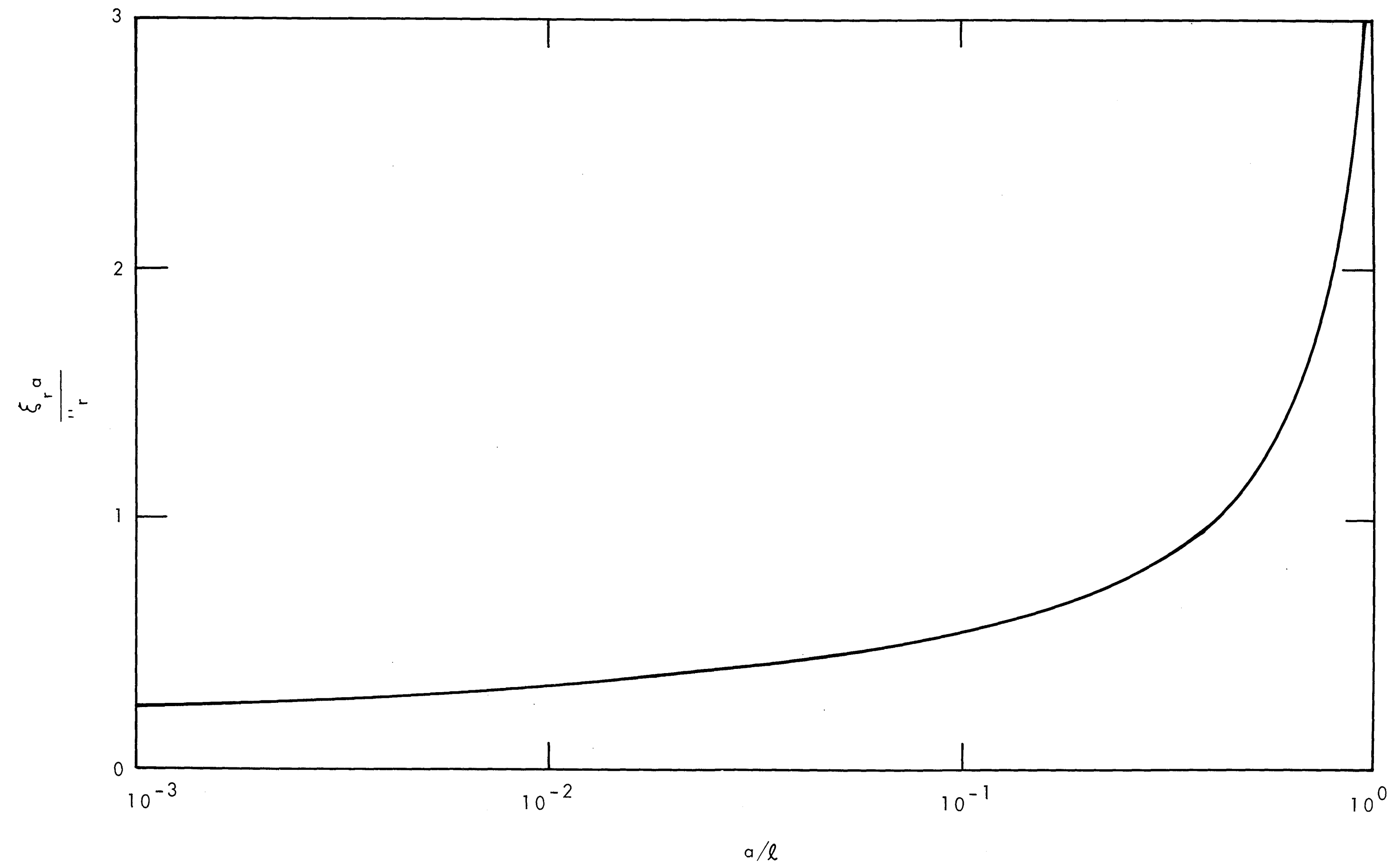

FIGURE 5

STRAIN - DISPLACEMENT RATIO, $\frac{\xi_r a}{u_r}$, VS RATIO OF SIZE OF PILE TO SIZE OF ICE FLOE, a/ℓ

BR4789-4

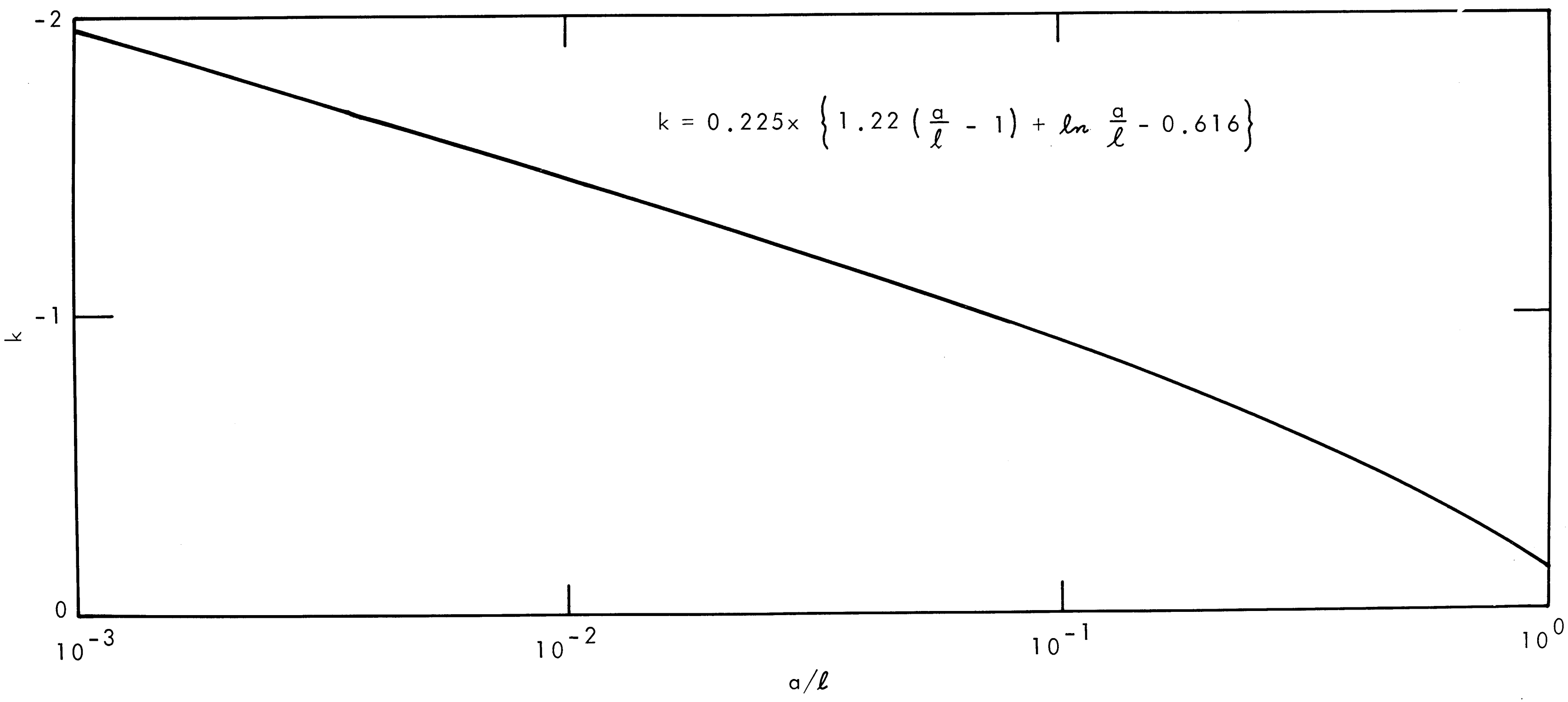

FIGURE 6

PROPORTIONALITY PARAMETER k (RELATING STRAIN RATE AND PENETRATION RATE) VS RATIO OF SIZE OF PILE TO SIZE OF ICE FLOE

BR 4789-5

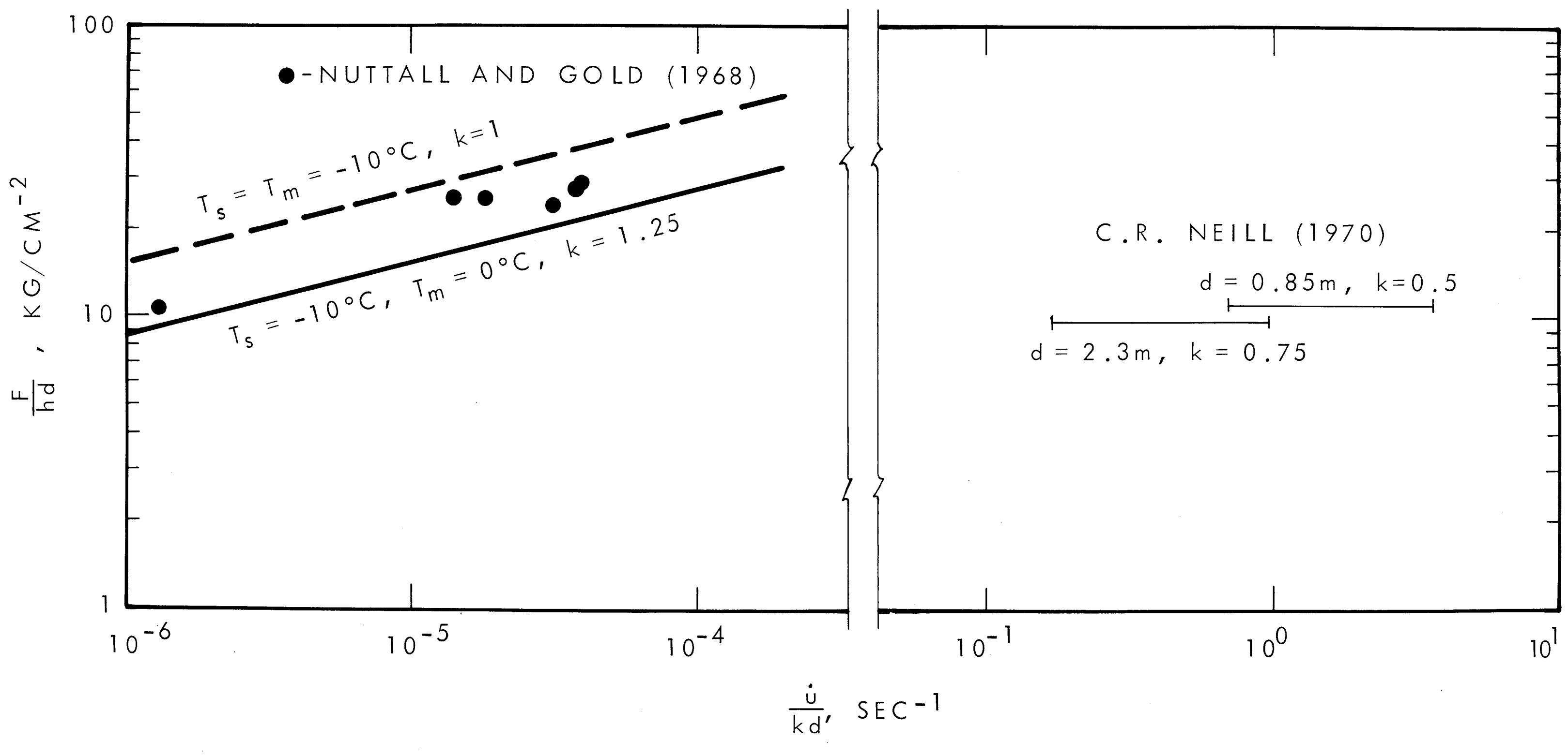

FIGURE 7 NORMALIZED PILE FORCE VS STRAIN RATE FOR COLUMNAR GRAINED ICE